

## REDUCING STRUCTURAL COUPLING ON A FLEXIBLE BEAM USING KALMAN FILTERS

S.A.Halsey\*, R.M.Goodall\*, J.T.Pearson\*, B.D.Caldwell†

\*Systems and Control Research Group, Loughborough University, Loughborough,  
LEICS, LE11 3TU, UK. Email: [s.a.halsey@lboro.ac.uk](mailto:s.a.halsey@lboro.ac.uk), [r.m.goodall@lboro.ac.uk](mailto:r.m.goodall@lboro.ac.uk),  
[j.t.pearson@lboro.ac.uk](mailto:j.t.pearson@lboro.ac.uk)

†BAE Systems, Warton Aerodrome, Preston, LANCS, PR4 1AX, UK.  
Email: [brian.caldwell@baesystems.com](mailto:brian.caldwell@baesystems.com)

Abstract: Structural coupling is the disturbance to the aircraft's inertial sensors caused by the flexibility in the airframe, and is particularly problematic for modern combat aircraft. The current solution uses a series of notch filters to remove the structural frequencies, but these introduce significant phase delays into the control loop. This paper presents some experimental and theoretical results from a flexible beam, and the use of Kalman filters to reduce the effects of structural coupling. This work is part of an ongoing project in conjunction with BAE Systems. Copyright © 2002 IFAC

Keywords: Structural Coupling, Aerospace, Flight Control, and Kalman Filtering

### 1. INTRODUCTION

Structural Coupling is a phenomena caused by interactions between the Flight Control System (FCS) of an aircraft, and the structural dynamics and aerodynamics of its airframe. Structural coupling can cause high frequency oscillations of the control surfaces due to propagation of signals (inertial sensors) caused by the flexibility of the airframe, through the IMU and the flight control laws.

Instability of this feedback loop could result in degradation of the structure though fatigue, and/or degradation of the performance of the FCS through saturation of the control surface actuators. Both of these effects have serious safety implications for the aircraft.

### 2. STABILITY MARGIN LIMITS

Prevention or limitation of structural coupling is an integral part of the design of the FCS. Frequency domain methods based upon the Nyquist stability criterion in conjunction with Bode and Nichols charts are commonly used in the design process.

System design stability and clearance requirements are given in the form of gain and phase margins, and exclusion regions on the Nichols chart (Caldwell

1996). These regions follow the 5dB line on the closed loop Nichols chart axes. Around the  $-180^\circ$  0dB point, this limit is bounded by  $+35^\circ$ ,  $-90^\circ$ , and  $-6$ dB. Around the  $-540^\circ$  0dB point, it is bounded by  $\pm 90^\circ$ . These two regions are shown in Figure 1.

It is the objective of the FCS designer to ensure that the frequency response of the open loop control system does not encroach on these exclusion regions under all expected flight conditions.

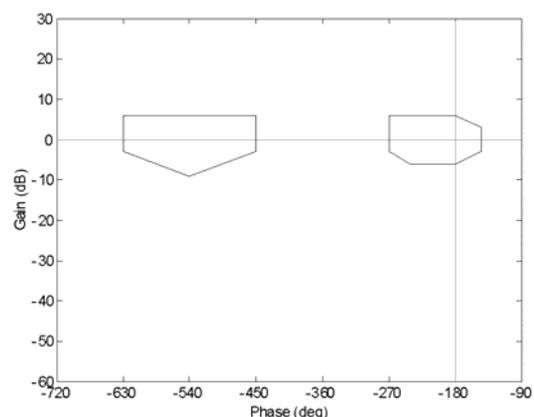


Figure 1 Stability Margin Exclusion Regions

### 3. CURRENT TECHNIQUES

At present, notch filters are used to attenuate the effects of structural coupling. The filters are placed in the control system feedback paths, and provide the FCS with a signal where the components of the feedback caused by the flexibility of the airframe have been attenuated; in the sensor output paths, reducing the structural frequency components in the sensor signals to the control loop; in the actuator demand paths, minimising the forward propagation of the high frequency signals. Figure 2 shows the placement of these filters.

Whilst the filters can achieve the desired goal of minimizing the structural coupling effects, they introduce phase lag into the control system at low frequencies. The summation of the phase lag from all the filters can lead to significant total delay, which may impinge upon the stability margins of the control loop, which must be maintained, or effectively reduce the bandwidth of the actuators, compromising the FCS performance.

In general the notch filter solution arrived at will be unique to a specific aircraft. The design requires data from aircraft models and also from ground and flight testing. In addition the mode frequencies can typically vary significantly, according to the aircraft configuration, a factor which also needs to be taken into account.

In recent years, with the development of more sophisticated and aerodynamically unstable aircraft, so the complexity of the notch filter solution has increased. The Jaguar and Tornado aircraft both required 1 notch filter on each feedback axis. In comparison to these two aircraft, the EAP (Experimental Aircraft Programme) aircraft required 7 notch filters on the pitch axis alone.

The design is an iterative process, involving a trade off between the desired filter attenuation and the phase lag introduced by the filters. This process can become very time consuming, and any alterations to the aircraft specification can result in the need to redesign the whole system of filters.

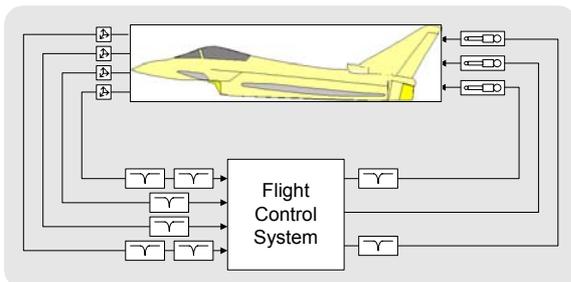


Figure 2 Notch Filter FCS Loop

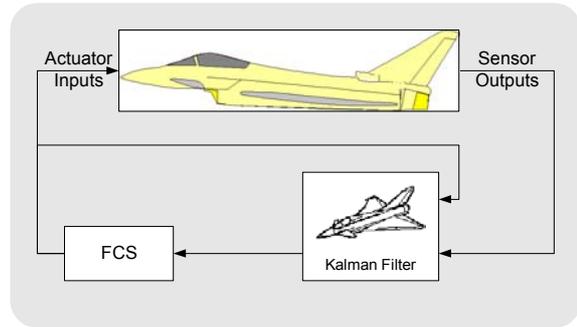


Figure 3. Kalman Filter FCS Loop

### 4. PROPOSED TECHNIQUE

As an alternative to the use of notch filters, it is proposed that a model-based approach could be used. The Kalman filter provides a state estimate based upon a combination of information from system inputs and outputs, and a model of the system (Kalman 1960).

A modern FCS may provide command and/or stability augmentation functions based on measurements of the aircraft motion. The measurements available to the FCS (generally accelerometers and angular rate gyroscopes) contain both rigid and flexible modes. The Kalman filter can use this information combined with a model of the aircraft to provide an estimate of the rigid motion of the aircraft, free from the corruption of the flexible modes. Figure 3 shows this arrangement.

This approach has a number of potential benefits. These include:

- a) Reduced Phase Lag
- b) Simpler design process
- c) Greater robustness to configuration changes

These are explained in more detail below:

#### 4.1. Phase Lag Reduction

Since the Kalman filter approach uses information from the inputs as well as the outputs from the system, preliminary theoretical analysis and computer simulations have shown that this adds less phase lag into the system than a comparable notch filter design.

#### 4.2. Simpler Design Process

The Kalman filter uses a mathematical model of the aircraft in order to generate the estimate. This model is part of the process that is used to design the notch filters.

Since the mathematical equations of the Kalman filter remain unchanged from system to system, only the system parameters change. The process to design the filter will therefore be much simpler, since it would involve developing the model of the aircraft and inserting this into the filter, as opposed to developing the model, and then designing the filters to accommodate the model.

Simplicity largely arises out of increased robustness; fewer design cases are required. It also extends to the design of the control laws, since a reduction in the phase lag introduced by the filters leads to easier clearance.

### 4.3. Configuration Change Flexibility

The notch filters are fixed in the aircraft, and so need to be designed to cover all possible changes in frequency dependent upon the current aircraft configuration. The Kalman filter approach could be used in two ways to overcome this need for flexibility:

First the model in the Kalman filter could easily be updated at the start of each flight. Alternatively, the uncertainty parameters in the Kalman filter can be adjusted to provide greater robustness to these changes.

## 5. REPRESENTATIVE MODEL

Previously published work (Pearson, et al., 2000) examined the possibilities of using the proposed solution on a simple model with a single flexible mode – this showed that the concept could be used with success.

This work has been extended to a more complicated model, which provides a better representation of an aircraft. In addition the model has been designed on a scale that allows experimental validation of the work through the use of a test rig with similar parameters (Section 6).

The beam model is based upon Benoulli-Euler theory (Bishop and Johnson, 1960). This allows an unlimited number of flexible modes to be developed, though only two flexible modes have been modelled in this work.

The beam is provided with a pair of spring / damper mechanisms offering rigid ‘pitch’ and ‘heave’ modes. The gyroscope, controller and actuator configuration provide closed loop pitch rate control which emulates the aerodynamics effects that apply to aircraft control in the longitudinal axis. The flexibility of the beam introduces flexible mode components into the sensor feedback signal, and provides the potential for

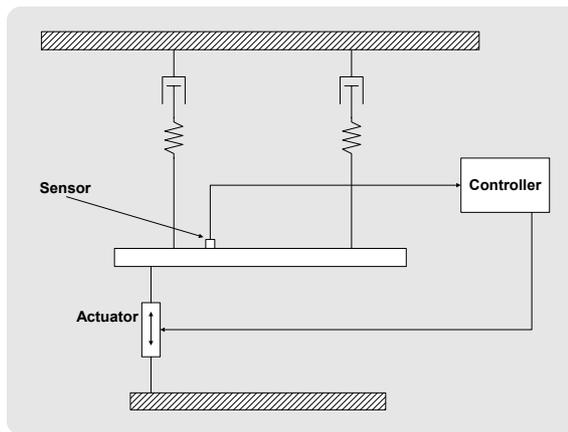


Figure 4. Beam Model

structural coupling. The arrangement of this model is shown in Figure 4.

The dimensions of the beam and the values of the springs and dampers have been selected to provide rigid and flexible mode frequencies that are representative of those found on an aircraft. These are shown in Table 1.

Table 1 Beam Mode Frequencies

	Frequency	Damping
Rigid	3.09Hz	47.4%
1 <sup>st</sup> Flexible	5.44Hz	0.8%
2 <sup>nd</sup> Flexible	15.01Hz	0.8%

### 5.1. Filter Design

As can be seen from Figure 5, whilst the first stability margins of the beam response are adequate, the effects of the flexible modes cause the frequency response of the beam model to encroach into the exclusion regions. In fact there is instability due to the response at around 15Hz, because the gain at 540° is close to unity.

In order to satisfy the restrictions imposed by the exclusion regions, some filtering is required to attenuate the effects of the flexible modes.

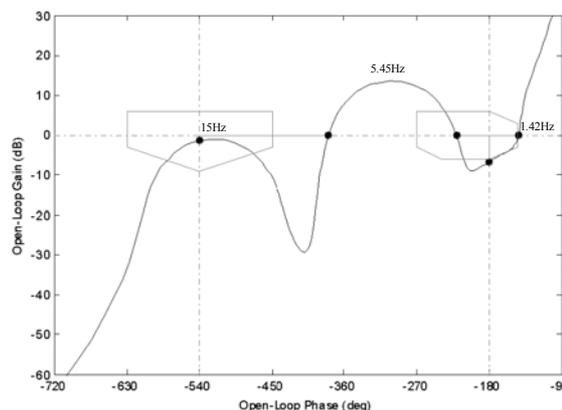


Figure 5. Unfiltered Response

The traditional notch filter approach requires two filters to be added into the system. Table 2 shows the parameters chosen for these filters.

**Table 2 Notch Filter Parameters**

	Notch Frequency	Numerator Damping	Denominator Damping
Filter 1	5.44Hz	3.5%	35.3%
Filter 2	15.0Hz	2.4%	25.6%

These filters alter the frequency response of the system to that shown in Figure 6:

The filters have improved the response; however this is at the cost of additional phase lag to the system, leading to degradation of stability margin in the 'rigid' frequency region and encroachment into the exclusion region.

The alternative approach uses a continuous Kalman-Bucy filter instead of the notch filters. This uses the same basic model of the beam, although the output from the filter uses only the rigid states of the model. The parameters of the Kalman filter have been developed from the variance of the system states (in response to a step input), and the variance of the sensor noise.

The frequency response of the system including the Kalman filter is shown in Figure 7:

The Kalman filter has removed the effects of the flexible modes from the system, and remains clear of the exclusion region. This represents an ideal solution, since the Kalman filter and the plant have identical models.

### 5.2. Effects of Errors in Model

As discussed in Section 3, the filter design process in an aircraft system has to account for some variation in the frequencies over which the filters are effective.

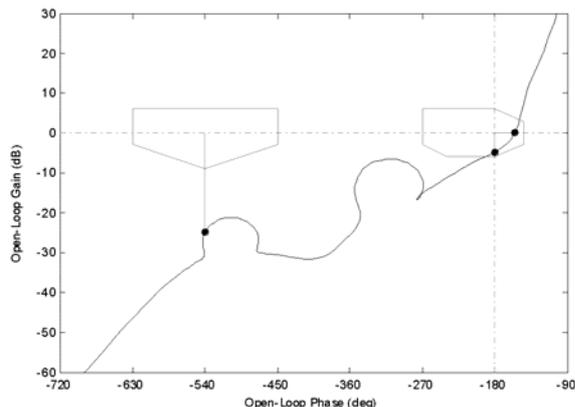


Figure 6. Notch Filtered Response

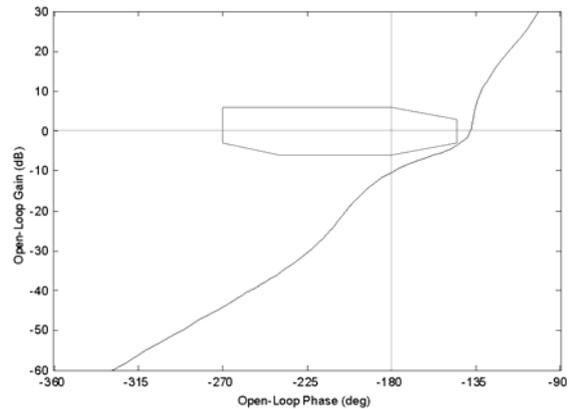


Figure 7. Kalman Filter Response

The notch filter parameters given in Table 2 were chosen to take this into account; in particular the relatively high denominator damping that broadens the range of attenuated frequencies, but also of course increases the phase lag at lower frequencies.

In order to compare the effectiveness of the filters when the flexible frequencies change, the stiffness of the beam has been varied by  $\pm 15\%$ , which results in a frequency variation of  $\pm 7.5\%$ .

For both filter types, the filters remain unchanged, expecting no errors in the model.

Figure 8 shows the frequency response for the notch-filtered system at the two extremes and also the nominal system.

The notch filter is effective except for large positive errors, when there is further encroachment into the first exclusion region.

Figure 9 shows the response of the Kalman filter to the same range of frequency variation.

Although the flexible modes can be seen when there are errors in the model, the Kalman filter successfully manages to keep the frequency responses outside the exclusion regions.

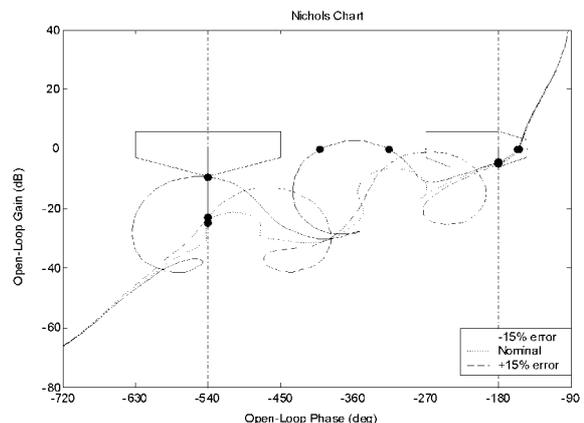


Figure 8. Notch Filter Error Response

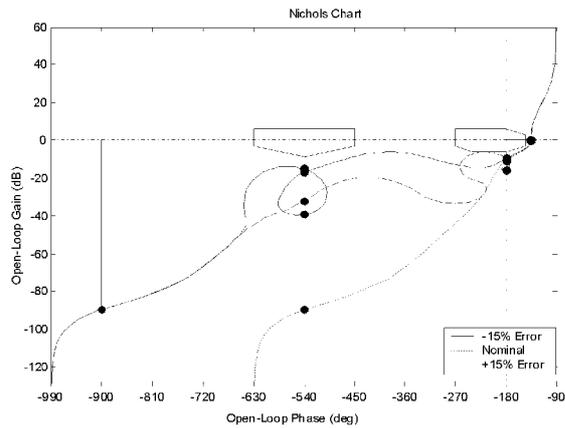


Figure 9. Kalman Filter Error Response

## 6. EXPERIMENTAL RIG

The beam model was developed with the intention of producing an experimental rig, which will allow the results obtained through modelling to be validated in a practical environment.

The rig has been designed to operate in the horizontal plane, which isolates the flexible modes from the effects of gravity. The beam is suspended on long wires, which allow it freedom of movement. Figure 10 shows a representation of the rig, and Figure 11 shows a photograph of the implementation.

Since the beam is very lightweight, friction in the system is a key consideration; both the actuator and the dampers have been selected to minimize friction. The control input to the beam is provided from a linear electro magnetic actuator. The control system emulating the aircraft pitch loop uses the actuator (representing the control surfaces) to control the angular velocity of the beam about the vertical axis, measured with a solid state gyro.

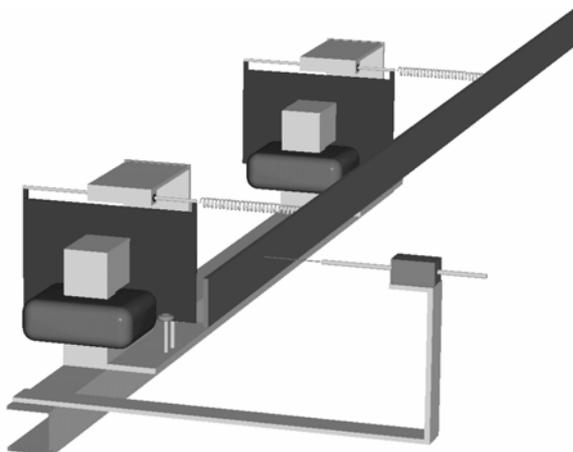


Figure 10. Drawing of Experimental Rig

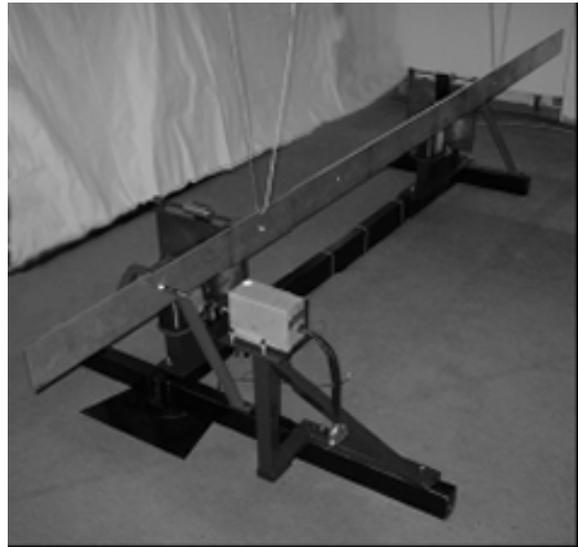


Figure 11. Photo of Experimental Rig

The dampers are also based upon electro magnetism. The movement of a copper sheet (supported on linear bearings) is damped by a magnetic field. A number of accelerometers provide information for data logging purposes.

To date the experimental work has measured the open-loop frequency response of the system (i.e. from actuator force to angular velocity) and this can be seen in Figure 12 together with the theoretical response from the model as described in Section 5. Broadly the responses are similar, although there are significant variations at low frequencies, which relate to the effects of the suspension of the experimental beam.

## 7. CONCLUSIONS

Computer simulations have show that a Kalman filter based approach has the potential to successfully estimate flexible modes without introducing worse levels of phase lag at low frequencies than a notch filter.

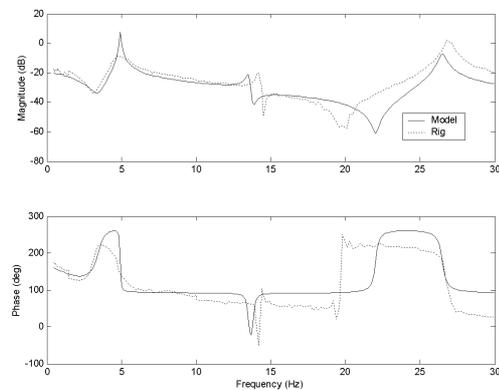


Figure 12. Comparison of Model and Rig Frequency Responses

Preliminary experimental work has validated the basic concept, and investigations are continuing to assess the robustness properties of the notch filter and Kalman filter solutions in a comprehensive manner.

Future work will include applying the ideas to a more complex model, developed in conjunction with BAE Systems, which will more closely represent the structural coupling problem.

#### ACKNOWLEDGEMENTS

The authors would like to express their gratitude to BAE Systems for their support throughout this project.

#### REFERENCES

- Bishop, R.E.D. And Johnson, D.C. (1960), *The Mechanics of Vibration*, Cambridge University Press, London
- Caldwell, B.D., (1996) FCS Design for Structural Coupling, *The Aeronautical Journal*, **100**, pp506-519
- Kalman, R.E., (1960), A New Approach to Linear Filtering and Prediction Problems, *Trans. ASME Journal of Basic Engineering*, **82**, pp35-45.
- Pearson, J.T., Goodall, R.M., Halsey, S.A., Caldwell, B.D. (2000), Kalman Filters for Reducing the Effects of Structural Coupling, *Control 2000*.