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FURTHER DEVELOPMENTS ON ADAPTIVE OBSERVERS FOR NONLINEAR SYSTEMS WITH APPLICATION IN FAULT DETECTION

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Abstract: From previous studies of (Besançon, 1999; Besançon, 2000) on adaptive observer design for nonlinear systems, and the contribution of (Zhang, 2001) on exponential adaptive observers for linear time-varying systems, the purpose here is to propose some new observer design for a class of nonlinear systems. This observer can in turn be made adaptive, and in particular can help in detecting changes in some parameters. *Copyright ©2002 IFAC.*

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1. INTRODUCTION

So-called adaptive observers are state observers with some on-line adaptation w.r.t. unknown parameters. Various results are available for linear systems going back to the 70's (Lüders and Narendra, 1973; Lüders and Narendra, 1974, ...), as well as for nonlinear systems from the early 90's (Bastin and Gevers, 1988; Marino and Tomei, 1992, ...). Some unifying remarks on such designs were recently proposed in (Besançon, 2000), in particular emphasizing some nonlinear adaptive observer form from which an asymptotic state and parameter observer was proposed. From this, and the contribution of (Zhang, 2001) on exponential adaptive observers for linear time-varying systems, it will be shown here how those available adaptive designs can lead to new observers for a class of nonlinear systems. Such observers in particular encompass problems of state *and parameter* estimation. They can thus be used to detect changes in parameters, or to estimate constant faults which might affect the system.

The underlying previous results which will be used are first recalled in section 2, while the new design will be presented in section 3. An example illustrates the possible use of such a design in section 4, while some conclusions end the paper in section 5.

2. BACKGROUND RESULTS

In (Besançon, 2000), it has been highlighted how most systems for which an adaptive observer can be designed are systems which can be written as follows:

$$\begin{aligned} \dot{y} &= \alpha(y,\zeta,u,t) + \beta(y,\zeta,u,t)\theta; \\ &\quad u \in I\!R^m; y \in I\!R^p; \theta \in I\!R^q \\ \dot{\zeta} &= Z(y,\zeta,u,t);\ \zeta \in I\!R^r \end{aligned} \tag{1}$$

where:

(1) y is the measured output; θ denotes a vector of unknown parameters; u denotes the control input, and t denotes the dependency on any other available time-varying signal;
(2) There exist a proper decrescent positive definite $\mathcal{C}^1$ function $V(t,e)$, such that for any initial condition $x_0 = \begin{pmatrix} y_0 \\ \zeta_0 \end{pmatrix}$ for system (1), any input $u \in \mathcal{U}$, any function $y(t)$ satisfying (1) with

control u and $y(0) = y_0$, any $z, e \in I\!R^r$, and any $t \geq 0$ one has:

$$\frac{\partial V}{\partial t}(t,e) + \frac{\partial V}{\partial e}(Z(y(t), e+z, u(t), t) - \\ -Z(y(t), z, u(t), t)) \leq -\kappa(e), \quad (2)$$

for some positive definite function κ.

(3) For any initial condition $x_0 = \begin{pmatrix} y_0 \\ \zeta_0 \end{pmatrix}$ for system (1), any input $u \in \mathcal{U}$, any function $x(t) = \begin{pmatrix} y(t) \\ \zeta(t) \end{pmatrix}$ satisfying (1) with control u and $x(0) = x_0$, any $z, e \in I\!R^r$, and any $t \geq 0$ one has:

$$(i)\ \|\alpha(y(t), e+z, u(t), t) - \\ -\alpha(y(t), z, u(t), t)\| \leq \gamma_\alpha \sqrt{\kappa(e)};\ \gamma_\alpha > 0\ and \\ \|\beta(y(t), e+z, u(t), t) - \\ \beta(y(t), z, u(t), t)\| \leq \gamma_\beta \sqrt{\kappa(e)};\ \gamma_\beta > 0 \quad (3)$$

$$(ii)\ \|\beta(y(t), \zeta(t), u(t), t)\| \leq b;\ b > 0. \quad (4)$$

For such a system, an adaptive observer can be simply designed as follows (for any $k_y > 0$):

$$\begin{aligned} \dot{\hat{y}} &= \alpha(y, \hat{\zeta}, u, t) + \beta(y, \hat{\zeta}, u, t)\hat{\theta} - k_y(\hat{y} - y); \\ \dot{\hat{\zeta}} &= Z(y, \hat{\zeta}, u, t) \\ \dot{\hat{\theta}} &= -k_\theta \beta^T(y, \hat{\zeta}, u, t)(\hat{y} - y)^T;\ \ k_\theta > 0 \end{aligned} \quad (5)$$

In this design, the asymptotic convergence of the state estimation error is proved, while that of the parameter estimation error can be obtained under some classical additional assumption of persistent excitation.

In the case of linear systems, (Zhang, 2001) provides a result for *exponential* convergence of the parameter estimation error, which will be shown here to be useful in extending the above results. In short, the main result of (Zhang, 2001) is as follows:
Given a system $\dot{x}(t) = A(t)x(t) + B(t)u(t) + \Psi(t)\theta$; $y(t) = C(t)x(t)$ and a gain $K(t)$ such that:

I. $\dot{\eta}(t) = (A(t) - K(t)C(t))\eta(t)$ is globally exponentially stable;
II. $\Gamma(t)$ solution of $\dot{\Gamma}(t) = (A(t) - K(t)C(t))\Gamma(t) + \Psi(t)$ satisfies the following persistent excitation condition: $\forall t \geq 0$,

$$\int_t^{t+T} \Gamma^T(\tau)C^T(\tau)\Sigma(\tau)C(\tau)\Gamma(\tau)d\tau \geq \delta I (6)$$

for some bounded positive definite $\Sigma(t)$, and some positive constants T, δ,

then the system below is a global exponential state and parameter observer:

$$\begin{aligned} \dot{\hat{x}}(t) &= A(t)\hat{x}(t) + B(t)u(t) + \Psi(t)\hat{\theta}(t) + \\ &\quad +[K(t) + \Gamma(t)\Lambda\Gamma^T(t)C^T(t)\Sigma(t)] \times \\ &\quad \times [y(t) - C(t)\hat{x}(t)] \\ \dot{\hat{\theta}}(t) &= \Lambda\Gamma^T(t)C^T(t)\Sigma(t)[y(t) - C(t)\hat{x}(t)] \end{aligned}$$

On the basis of both results which have been here recalled, some new observer design can be proposed.

3. NEW (ADAPTIVE) OBSERVER

Combining the ideas of the previous section, we can propose an observer for systems of the following form:

$$\begin{aligned} \dot{y} &= \alpha(y, \zeta, u, t) + \beta(y, \zeta, u, t)\xi; \\ \dot{\zeta} &= Z(y, \zeta, u, t); \\ \dot{\xi} &= X(y, \zeta, u, t) \\ &\quad u \in I\!R^m; y \in I\!R^p; \zeta \in I\!R^r; \xi \in I\!R^q \end{aligned} \quad (7)$$

where y still denotes the measured output, provided that conditions (2), (3), and (4) are still satisfied, and with three additional ones:

- For any $y(t), \zeta(t)$ solution satisfying (7) with $u \in \mathcal{U}$, and for any $t \geq 0$:

$$\|X(y(t), e+z, u(t), t) - X(y(t), z, u(t), t)\| \\ \leq \gamma_X \sqrt{\kappa(e)};\ \gamma_X > 0 \quad (8)$$

- For any $y(t), \zeta(t)$ solution satisfying (7) with $u \in \mathcal{U}$, and for any $t \geq 0$, $\beta(t) := \beta(y(t), \zeta(t), u(t), t)$ satisfies the persistent excitation condition, i.e.:

$$\exists T, b > 0 : \forall t \geq 0,\ \int_t^{t+T} \beta(\tau)\beta^T(\tau)d\tau \geq bId; (9)$$

- $\xi(t)$ remains bounded. (10)

Notice that (7) now includes some dynamics for what was the vector of *constant* parameters θ in (1).
For such systems we can state:

Theorem *3.1.* For a system (7) where y is measured and conditions (2), (3), (4), (8), (9), (10) are satisfied, system (11) below is a global asymptotic state observer, in the sense that for any initial conditions, $lim_{t\to\infty}\|\hat{y}(t) - y(t)\| = lim_{t\to\infty}\|\hat{\zeta}(t) - \zeta(t)\| = lim_{t\to\infty}\|\hat{\xi}(t) - \xi(t)\| = 0$.

$$\begin{aligned} \dot{\hat{y}} &= \alpha(y, \hat{\zeta}, u, t) + \beta(y, \hat{\zeta}, u, t)\hat{\xi} \\ &\quad -(k_y + k_\xi \Gamma\Gamma^T)(\hat{y} - y);\ \ k_y, k_\xi > 0 \\ \dot{\hat{\zeta}} &= Z(y, \hat{\zeta}, u, t) \\ \dot{\hat{\xi}} &= -k_\xi \Gamma^T(\hat{y} - y) + X(y, \hat{\zeta}, u, t) \\ \dot{\Gamma} &= -k_y \Gamma + \beta(y, \hat{\zeta}, u, t) \end{aligned} \quad (11)$$

The proof is achieved by combining ideas of (Besançon, 2000) and results of (Zhang, 2001):
Let $e_y := \hat{y} - y$, $e_z := \hat{\zeta} - \zeta$, and $e_\xi := \hat{\xi} - \xi$ denote the observation errors. Then by (2), e_z goes to zero.
For the others, given a function f of the state, let Δf denote the error between its value at the state of the observer, and the one at the state of the system. Then:

$$\begin{aligned} \dot{e}_y &= \Delta\alpha + \Delta\beta\xi + \beta(y, \hat{\zeta}, u, t)e_\xi - (k_y + k_\xi \Gamma\Gamma^T)e_y \\ \dot{e_\xi} &= \Delta X - k_\xi \Gamma^T e_y \end{aligned}$$

Now set $\varepsilon_y := e_y - \Gamma e_\xi$. This, together with the expression of $\dot{\Gamma}$, yields:

$$\dot{\varepsilon}_y = -k_y \varepsilon_y + \Delta\alpha + \Delta\beta\xi - \Gamma\Delta X.$$

By boundedness of β, Γ is also bounded, and ξ is bounded by assumption (10). From this, together with conditions (3), (8), the dynamics of ε_y, and that of e_z, one can easily conclude that ε_y asymptotically goes to zero.

It now remains the problem of e_ξ. Injecting $e_y = \varepsilon + \Gamma e_\xi$ in its dynamics, we get:

$$\dot{e}_\xi = -k_\xi \Gamma^T \Gamma e_\xi + \mu$$

where μ goes to zero as ε_y, e_z do.

On the other hand, it has been shown in (Zhang, 2001) that if Γ satisfies (9), the homogeneous part $\dot{e}_\xi = -k_\xi \Gamma^T \Gamma e_\xi$ is exponentially stable. From (11), Γ is constructed by filtering $\beta(y, \hat{\zeta}, u, t)$ by a linear stable filter, and $\beta(y, \hat{\zeta}, u, t) = \beta(y, \zeta, u, t) + \Delta\beta$ where $\beta(y, \zeta, u, t)$ satisfies (9) and $\Delta\beta$ goes to zero. From this, Γ also satisfies (9) (see (Ioannou and Sun, 1996) for instance), and thus the homogeneous part of the equation on e_ξ is exponentially stable.

By combining this with the convergences of ε_y and e_z, we get that e_ξ goes to zero, and since Γ is bounded, e_y finally also goes to zero, which ends the proof. □

At this point notice that:

(1) If ξ is constant, namely represents some vector of unknown parameters, we come back to the structure considered in (Besançon, 2000), with an alternative observer design derived from a technical result of (Zhang, 2001).

(2) If ξ is partially constant, theorem 3.1 extends the result of (Besançon, 2000) on adaptive observer design, but with a dynamic gain for the parameter adaptation (Γ). If one is not interested in *exponential* parameter convergence, but just asymptotic convergence, one can distinguish in ξ constant parameters θ from non constant states x, and design an observer as above for the x part, and with a constant gain as in (Besançon, 2000) for the θ part.

(3) By further noting that the exponential convergence which is achieved for the ξ part, can be made as fast as desired by tuning k_ξ, one might extend even more the considered class of systems by allowing X to depend on ξ as follows:

$$\begin{aligned} \dot{y} &= \alpha(y, \zeta, u, t) + \beta(y, \zeta, u, t)\xi; \\ \dot{\zeta} &= Z(y, \zeta, u, t); \\ \dot{\xi} &= X(y, \zeta, u, \xi, t) \end{aligned} \quad (12)$$

$$u \in I\!R^m; y \in I\!R^p; \zeta \in I\!R^r; \xi \in I\!R^q$$

with similar assumptions as above, and the additional one that X must be globally Lipschitz w.r.t. ξ, uniformly w.r.t. (y, ζ, u, t).

(4) Finally, by considering the case of possible constant faults affecting the system, the proposed observer allow to detect them by simple estimation, provided that they enter the system via ξ in (12).

4. ILLUSTRATIVE EXAMPLE

In this section we present a small academic example as an illustration of state estimation or fault detection based on the design previously proposed.

We thus consider the system described by the following representation:

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1 x_3 - x_1^3 + \theta_1 u + \theta_2 sin(x_2) \\ \dot{x}_2 &= -2x_2 + cos(x_2) + x_1 \\ \dot{x}_3 &= u - x_2^2 x_3 \\ y &= x_1 \end{aligned} \quad (13)$$

By considering that θ_1, θ_2 are basically constant ($\dot{\theta}_1 = \dot{\theta}_2 = 0$), it is easy to check that this system is under the form (12) (where $\xi = (\, x_3 \quad \theta_1 \quad \theta_2 \,)^T$), and with $u(t) = sin(5t)$ for instance, it appears that the required conditions for our observer design are satisfied. First, if parameters θ_1, θ_2 are known, theorem 3.1 provides an observer for x_1, x_2, x_3 as illustrated by simulation results of figure 1 (where $k_y = 50, k_\xi = 500$).

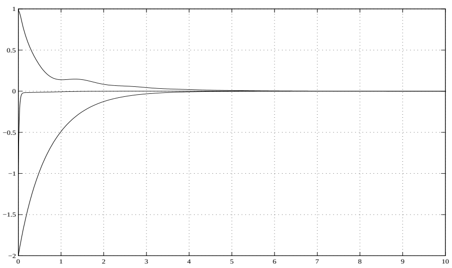

Fig. 1. State observation errors for system (13) with θ_1, θ_2 known.

If θ_1, θ_2 are unknown, as well as the whole state, the proposed observer allows to estimate them together with the state, as shown by estimation errors represented by figure 2.

Finally, if θ_1, θ_2 correspond to possible faults which might affect the system, the proposed observer provides some fault detector through the estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ as so-called "residuals". Such estimates indeed remain attracted by zero as long as no fault happens ($\theta_1 = \theta_2 = 0$), and become different from when a fault appears, indicating at the same time which parameter is faulty.

Simulations corresponding to successive faults on θ_1 and θ_2 are shown in figure 3 (including some noise affecting the system).

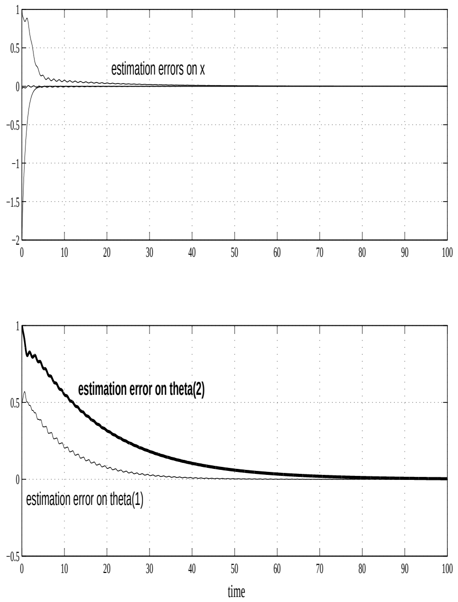

Fig. 2. Observation errors in x and θ for system (13).

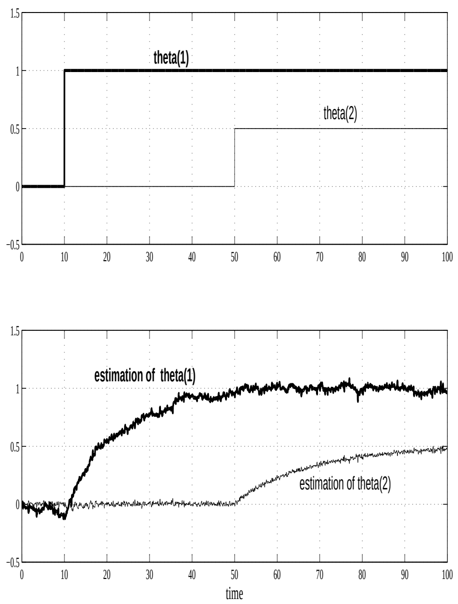

Fig. 3. Fault profiles (above) and fault detectors (below) for system (13).

5. CONCLUSION

In this paper, it has been shown how from results on *adaptive* observer design one can end up with a new *nonlinear* observer design, and how in turn this new observer provides some possible adaptive schemes if unknown parameters enter the system in a particular way, or even fault detectors if changes of parameters are considered as faults. Finally notice that the proposed new design enlarges the class of systems for which an observer can be found, in the sense that the considered structure do not fall into classical structures for which observers are available.

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