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KALMAN FILTERING IN SEMI-ACTIVE SUSPENSION CONTROL

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Abstract: This paper focuses on estimation of the vertical velocity of the vehicle chassis and the relative velocity between chassis and wheel. These velocities are important variables in semi-active suspension control. A model-based estimator is proposed including a Kalman filter and a non-linear model of the damper. Inputs to the estimator are signals from wheel displacement sensors and from accelerometers placed at the chassis. In addition, the control signal is used as input to the estimator. The Kalman filter is analyzed in the frequency domain and compared with a conventional filter solution including derivation of the displacement signal and integration of the acceleration signal.

Keywords: Kalman filters, semi-active dampers, vehicle suspension, automotive, frequency signal analysis

1. INTRODUCTION

Volvo Car Corporation is presently developing a controller for semi-active dampers, called continuously controlled chassis concept (4C). The 4C controller has been applied to a so-called performance concept car (PCC). This is a high performance car, implying stiff tires and stiff springs. An important part of the controller is the ability to estimate the damper velocity, i.e. the relative velocity between chassis and wheel. It is also important to be able to estimate the vertical velocity of the chassis since it affects the ride comfort for the driver. Due to the stiff tires, the bandwidth from road irregularities to damper velocities is high. This fact makes the design of the controller and the velocity estimation extra challenging.

The aim of this work is to design a filter that estimates the damper velocity and the chassis velocity. The estimation is based upon wheel displacement signals and accelerometer signals from the chassis. Two filters are proposed. First, a model-based estimator is derived including a model of the damper and a Kalman filter. The Kalman filter is based on a quarter car model. This filter is compared to a more conventional filter solution including a derivation of the wheel displacement signal, and integration of the accelerometer signal. The conventional filter includes a high-pass and a low-pass Butterworth filter respectively. This work considers both disturbances from road irregularities and from forces acting on the vehicle body. Such forces arise from e.g. braking and handling maneuvers.

State estimators have been considered in some papers, see e.g. (Hedrick *et al.*, 1994; Irmscher and Hees, 1996; Yi and Song, 1999). Hedrick *et al.* (1994) and Yi and Song (1999) assume that the damper force velocity characteristics are linear. This assumption makes it possible to design an observer based on a bilinear system. However, this assumption is not applicable in this work since the Volvo dampers operate in a range where the force velocity characteristics are non-linear.

2. QUARTER VEHICLE MODEL

The Kalman filter derived in this work is based on a so-called quarter vehicle model. This model is of course a simplification of the reality. However, it is reasonable to believe that for this filtering application this kind of model reflects important parts of the true vehicle dynamics.

Consider the quarter vehicle model pictured in Fig. 1. The coordinates are chosen such that the system is in steady state when $z_r = z_u = z_s = 0$. The vehicle dynamics are described by the following set of equations:

$$m_u \ddot{z}_u = k_t(z_r - z_u) + b_t(\dot{z}_r - \dot{z}_u) - k_s(z_u - z_s) + f_d$$
$$m_s \ddot{z}_s = k_s(z_u - z_s) - f_d + f_e$$

Here f_d represents the force generated by the damper. External forces acting on the sprung mass arising from e.g. cornering and braking are here denoted by f_e.

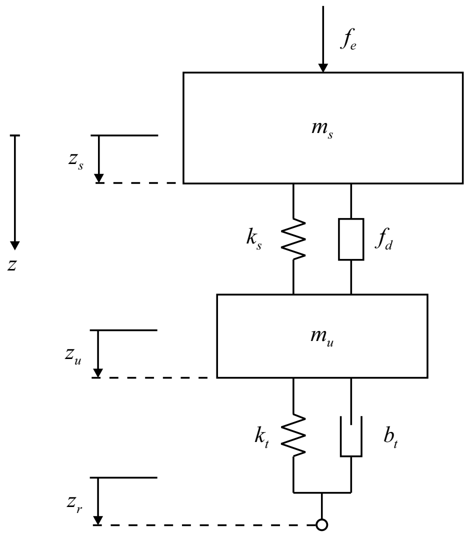

Fig. 1. Quarter model of a vehicle with an unsprung mass m_u and sprung mass m_s

3. MODEL BASED ESTIMATOR

The purpose of the estimator is to estimate the damper velocity $(\dot{z}_s - \dot{z}_u)$ and the velocity of the chassis $\dot{z}_s$. The estimated values are based on output signals from the 4C controller, sensor signals from accelerometers measuring the vertical acceleration $\ddot{z}_s$, and signals from wheel displacement sensors measuring the relative position $(z_s - z_u)$. The sensors are further described in Appendix A.

In Fig. 3 a schematic picture of the estimator is shown. The estimator includes a non-linear damper model, a linear Kalman filter, and a mapping of acceleration measurements from sensor positions to wheel positions. The damper model is described in Section 3.1 while the Kalman filter is considered in Section 3.2. The mapping of the acceleration measurements is elaborated in Appendix A.

3.1 *Damper model*

The estimator contains a simple model of the damper behavior, see Fig. 4. The damper model estimates the damper force f_d based on the output from the 4C controller (volt) and the estimated damper velocity $\frac{\mathrm{d}}{\mathrm{d}t}(\hat{z}_s - \hat{z}_u)$. The model is built up from five blocks. The damper dynamics including the valve are represented by a transfer function. A look-up table maps the damper velocity and volt input to the damper force, and a saturation block limits the velocity signal to fit within the limits of the look-up table. The two gains compensate for the fact that the damper is tilted. In this work, these gains are considered as constants. In reality, the damper ratio is a function of wheel displacement.

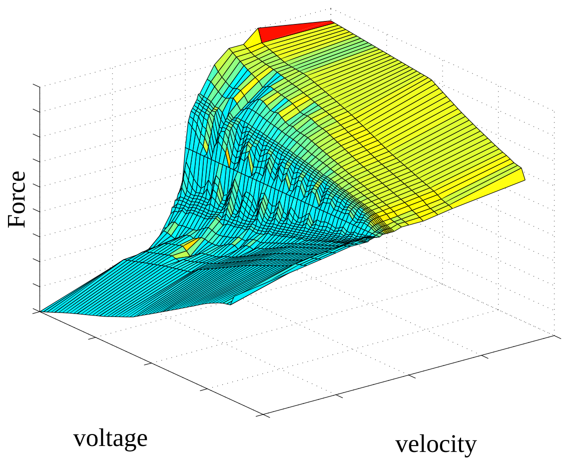

Fig. 2. Static model of damper force as a function of voltage and velocity

The static mapping from velocity and volt to force is an important characteristic of the damper. Fig. 2 shows this mapping for the semi-active dampers used in the 4C project. The opening and closing times of the valve in the damper result in dynamics in the damper. In this work these dynamics are represented by a second order system with a rise time of 10 ms. In reality the damper dynamics depend on the of the damper velocity. The dynamics are also different in the compression phase compared to the expansion phase. Furthermore, in a more complex model additional dynamics should be included due to oil elasticity.

3.2 *Kalman Filter*

The Kalman filter is a linear and discrete-time MIMO transfer function. This filter is an optimal solution to a minimization problem that is based on a plant model, i.e. the quarter car model described in Section 2, and on process and measurement noise covariance data. In addition, weighting filters are added to the model in order to achieve the desired properties of the Kalman filter.

The following three requirements are to be fulfilled when designing the controller:

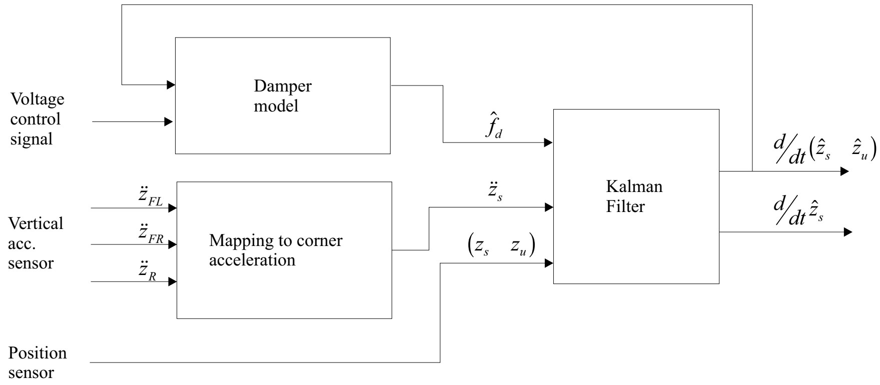

Fig. 3. Outline of the proposed estimator

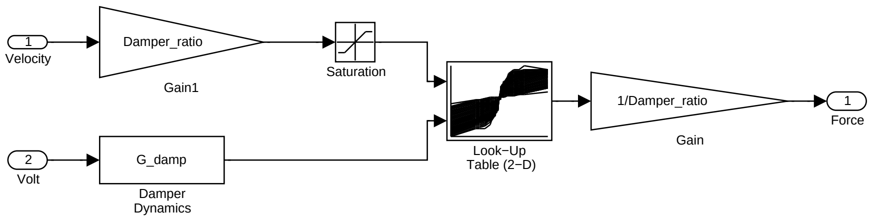

Fig. 4. Simulink block containing the damper model

- to achieve desirable bandwidth of the estimator.
- to minimize the influence of the measurement noise.
- to avoid drifting in estimation of $\dot{z}_s$ due to DC-offsets in the accelerometer signals.

Fig. 5 shows the choice of weighting filters that have been applied in the Kalman filter design. The choice of weighting filters is commented upon further down.

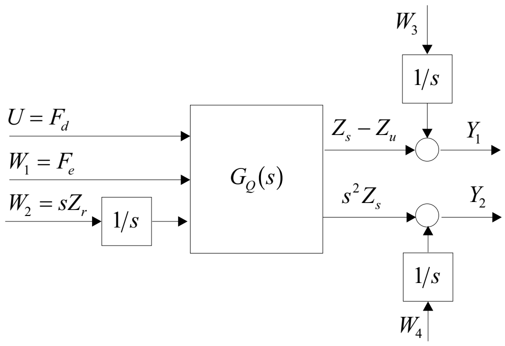

Fig. 5. Design of Kalman filter including the quarter vehicle model and additional weighting filters

The Kalman filter is tuned by choosing a suitable set of noise covariance matrices. Let the quarter vehicle model and the weighting filters pictured in Fig. 5 be described by the following state-space form:

$$x = Ax + Bu + Gw$$
$$y = Cx + Du + Hw + v$$

Here u denotes the known input, w denotes the process noise and v denotes the measurement noise, see Fig. 5. The noise covariance is then defined as

$$E\{ww'\} = Q_n,\ E\{vv'\} = R_n,\ E\{wv'\} = N_n,$$

In this work, the noise covariance matrices have been chosen in the following way:

$$Q_n = \mathrm{diag}([\,25 \cdot 10^6 \quad 4 \quad 10^{-4} \quad 10^{-4}\,])$$
$$R_n = \mathrm{diag}([\,10^{-7} \quad 10^{-8}\,])$$
$$N_n = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 10^{-2}\sqrt{10^{-7}} & 0 \\ 0 & 10^{-6} \end{bmatrix}$$

Now follows some remarks regarding the filter design:

- It is reasonable to consider the velocity of the road input $\dot{z}_r$ instead of the road position z_r in the Kalman filter design. Therefore, an integrator is placed before z_r.
- Two integrators are added to the measurement outputs. The integrator added at the acceleration measurement avoids drifting of the estimated output $\mathrm{d}/\,\mathrm{d}t\hat{z}_s$ when the $\ddot{z}_s$-signals are exposed to DC-offsets. Such effects arise when the chassis pitches or rolls. The integrator causes the Kalman filter to behave as a high-pass filter. Hence the Kalman filter do not consider DC-offsets in the sensor signals. The drawback is that the Kalman filter will not be able to detect true DC-inputs, as for example when driving in a long slope. The integrator added at position measurement improves the low-frequency behavior of the Kalman filter, especially when estimating velocities due to external force inputs f_d.
- The Kalman filter tuning is a compromise between filter bandwidth and reduction of measurement noise. If the bandwidth is too high, then the designed filter will be useless since it will

amplify measurement noise from the wheel displacement sensors too much.

- In this work a continuous-time Kalman filter is first derived. A corresponding discrete filter is then computed using linear interpolation of the inputs (triangle appx.), see the c2d-command in Matlab.

An alternative to the Kalman filter approach is to take the derivative of the position signal and to integrate the acceleration signal in order to obtain the desired velocity signals. Due to the problem of measurement noise and DC-offsets, these estimated values have to be filtered through a low-pass and high-pass filter respectively. In the following section, this filtering solution is compared with the proposed Kalman filter.

4. ANALYSIS OF KALMAN FILTER

In this section, the proposed Kalman filter is analyzed. Frequency analysis is a powerful tool for analysis of linear transfer functions. A problem with the proposed estimator is that the damper model is non-linear. Therefore, for the sake of analysis, in this section a linear passive damper will replace the semi-active damper.

The Kalman filter will be compared to another filter solution, namely a derivation and an integrator in combination with Butterworth filters. Fig. 6 shows how the vehicle model described in Section 2 is combined with the proposed Kalman filter and with the derivative/integrator filter solution. The derivative/integrator filter is designed so that the maximum gains from measurement noise to estimated outputs are equal with the corresponding gains of the Kalman filter. Note that all transfer functions in Fig. 6 are linear. Hence it is possible to analyze how the inputs f_e and $\dot{z}_r$ affects the estimated outputs in the frequency domain.

Consider the quarter model and the proposed estimators pictured in Fig. 6. Define the estimation errors e_{damp} and e_s as

$$e_{\mathrm{damp}} = (\dot{z}_s - \dot{z}_u) - \frac{\mathrm{d}}{\mathrm{d}t}(\hat{z}_s - \hat{z}_u) \qquad (1)$$

$$e_s = \dot{z}_s - \frac{\mathrm{d}}{\mathrm{d}t}\hat{z}_s \qquad (2)$$

Fig. 7 shows the Bode plot of $(f_e, \dot{z}_r) \rightarrow (e_{\mathrm{damp}}, e_s)$ for the two filters. For the sake of comparison the Bode plot of $(f_e, \dot{z}_r) \rightarrow (\dot{z}_s - \dot{z}_u, \dot{z}_s)$ is added in Fig. 7.

It is not obvious how to define bandwidth for a MIMO system like the proposed Kalman filter. In this work the bandwidth is defined as the region where the relative error $(f_e, \dot{z}_r) \rightarrow (e_{\mathrm{damp}}/(\dot{z}_s - \dot{z}_u), e_s/\dot{z}_s)$ is larger than -3dB. For this damper application the filter bandwidth has been chosen to range from 0.5 Hz to 30 Hz. Analysis shows that both the proposed filters attain the desired bandwidth.

Fig. 7 shows that the quarter car model has two resonance peaks, one representing the eigenfrequency of the unsprung mass (approx. 1 Hz) and one representing the eigenfrequency of the sprung mass (approx. 12 Hz). The Kalman filter is superior to the Butterworth filter in estimating velocities originating from forces on the sprung mass f_e, see Fig. 7. Regarding estimation of velocities originating from road irregularities the advantage of Kalman filtering over the derivative/integrator filter solution is not so clear. Fig. 7 shows that when estimating $(\dot{z}_s - \dot{z}_u)$ the Kalman filter is advantageous for frequencies about the unsprung mass eigenfrequency, while when estimating $\dot{z}_s$ the Kalman filter is advantageous for frequencies about the sprung mass eigenfrequency.

When designing a model based estimator it is important to analyze robustness of the estimator. In this application, it is especially important to check robustness with respect to variances in sprung mass at the rear axle. In practice, this variance could be due to varying load in the trunk or refueling. Analysis shows that the proposed Kalman filter is robust with respect to model uncertainties.

The Kalman filter and the derivative/integrator filter have also been tested on data generated from experiments on a Volvo V70. As mentioned before, the amplitude of the measurement noise limits the filter bandwidth. Therefore, it is interesting to analyze how the proposed filters amplify measurement noise on the estimated output. This analysis is shown in Fig. 8. The analysis is performed in open-loop, i.e. the quarter model and the controller is not included in the analysis. During the experiment, the vehicle is at standstill while the engine is running idle. This fact explains the difference in noise amplitude between the front and the rear axle. Note that even if the engine is running, the sensor signals should be considered as measurement noise. The reason for this is that the wheel displacement sensors have been mounted in such a way that the vibrations have a larger impact on the measurement signal than on the actual damper displacement. Hence, better isolation of the wheel displacement sensors from engine vibration would improve estimation quality.

5. CONCLUSIONS

This paper describes the development of a model-based estimator that estimates the vertical velocity of the chassis and the relative velocity between chassis and wheel. The suggested estimator includes a Kalman filter and a model of the damper. The Kalman filter is compared to a conventional filter solution including a derivation of the wheel displacement signal and integration of the acceleration signal.

The Kalman filter is superior to the derivative/integrator filter solution in estimating velocities originating from forces on the sprung mass f_e. Analysis also shows that

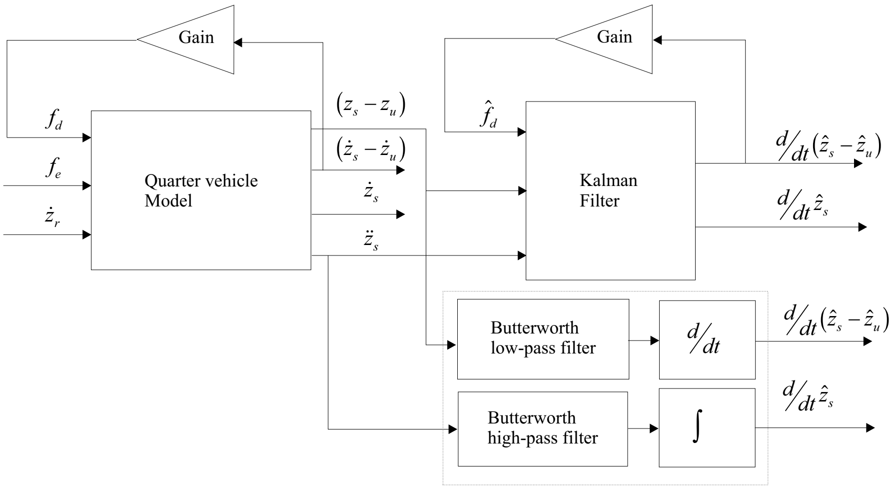

Fig. 6. Analysis of Kalman filter and derivative/integrator filter. The semi-active damper is replaced by a linear passive damper.

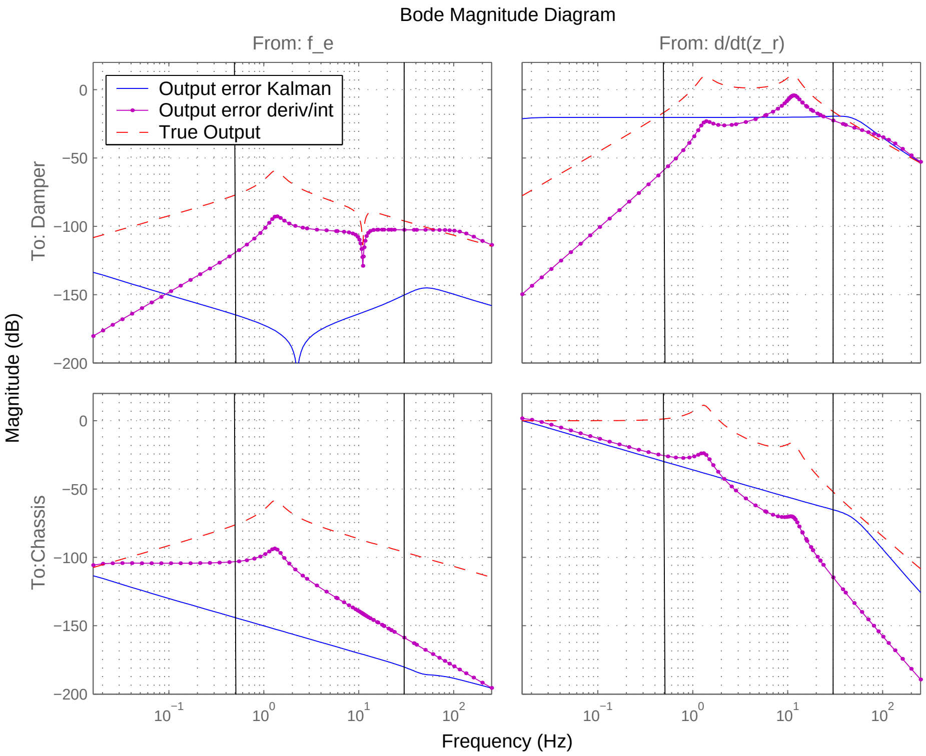

Fig. 7. Bode plot of $(f_e, \dot{z}_r) \rightarrow (e_{\mathrm{damp}}, e_s)$. The quarter model gain is also pictured. The vertical lines represent the desired bandwidth of the filter

regarding velocity estimation originating from road irregularities the advantage of the Kalman filter is more ambiguous. The Kalman solution is preferable when estimating $(\dot{z}_s - \dot{z}_u)$ for frequencies about the unsprung mass eigenfrequency, and when estimating the chassis velocity $\dot{z}_s$ for frequencies about the sprung mass eigenfrequency.

There are minor differences in computational complexity between the two proposed estimators. The Butterworth filters including the integrator have six states in total. The model-based estimator has eight states in total. In addition the model based estimator contains a table-lookup.

The estimator design presented in this paper is based on wheel displacement signals and on body acceleration signals. An alternative sensor setting is to measure the vertical acceleration both at the wheel and at the body. A corresponding strategy for Kalman filter design is applicable to this sensor setting.

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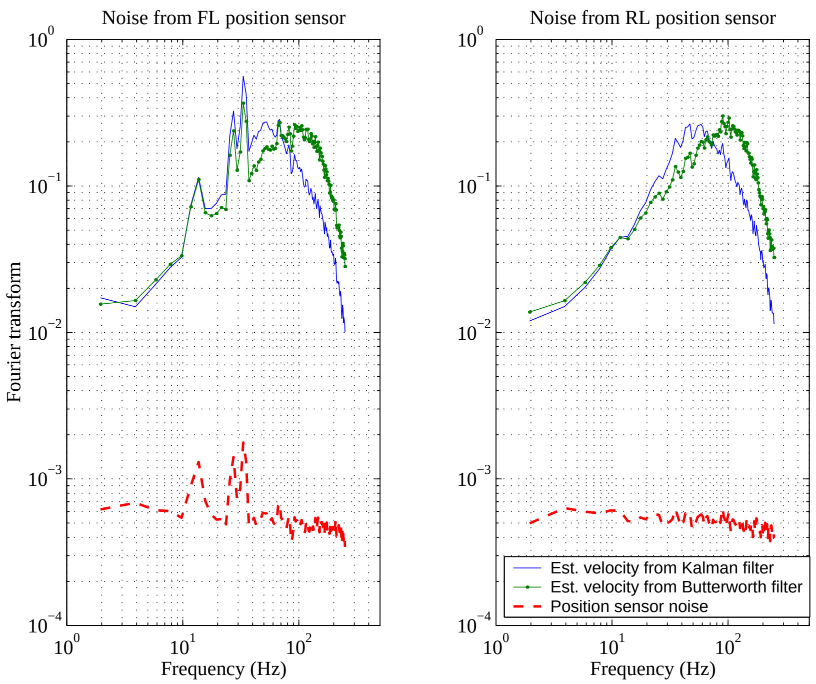

Fig. 8. Analysis of noise from wheel displacement sensors that is filtered through the proposed estimators

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Appendix A

Consider the quarter model shown in Fig. 1. The following vehicle parameters for a Volvo V70 are used in this work: $k_s = 26000$ N/m, $k_t = 240000$ N/m, $b_t = 50$ Ns/m, $m_u = 50$ kg, $m_s = 450$ kg at the front, and $m_s = 340$ kg at the rear.

The following vehicle parameters are used, see Fig. A.1: $a = 1.12$ m, $b = 1.68$ m, $c = 1.55$ m, $d = 1.53$ m, $e = 0.675$ m, $f = 2.3$ m, and $g = 0.78$ m.

The task is now to calculate the chassis accelerations at the wheels based on measurements from the accelerometers shown in Fig. A.1. The calculations are done under the assumption that the chassis is rigid. Observe that the accelerometers measure the vertical acceleration perpendicular to the chassis.

It can be shown that the vertical acceleration at any point of the chassis (x_p, y_p) is given by

$$\ddot{z}_p = \ddot{z}_{\rm CG} + \alpha x_p + \beta y_p$$

Here $\ddot{z}_{\rm CG}$ denotes the vertical acceleration at the center of gravity. Let $\gamma = \ddot{z}_{\rm CG}$. Then

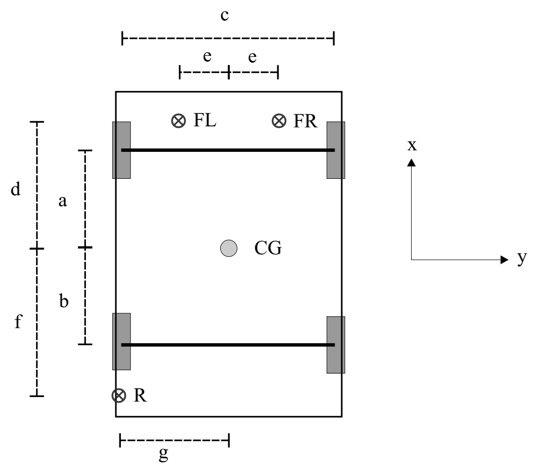

Fig. A.1. Car seen from above. The symbol $\otimes$ denotes the position of the accelerometers R, FR, and FL

$$\ddot{z}_{FL} = \alpha d - \beta e + \gamma$$
$$\ddot{z}_{FR} = \alpha d + \beta e + \gamma$$
$$\ddot{z}_{R} = -\alpha f - \beta g + \gamma$$

Hence

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} d & -e & 1 \\ d & e & 1 \\ -f & -g & 1 \end{bmatrix}^{-1} \begin{bmatrix} \ddot{z}_{FL} \\ \ddot{z}_{FR} \\ \ddot{z}_{R} \end{bmatrix}$$

The vertical acceleration at any point (x_p, y_p) of the chassis is then given by

$$\begin{aligned} \ddot{z}_p &= \alpha x_p + \beta y_p + \gamma \\ &= [\,x_p \quad y_p \quad 1\,] \begin{bmatrix} d & -e & 1 \\ d & e & 1 \\ -f & -g & 1 \end{bmatrix}^{-1} \begin{bmatrix} \ddot{z}_{FL} \\ \ddot{z}_{FR} \\ \ddot{z}_{R} \end{bmatrix} \end{aligned}$$