

## AN INTERACTIVE TOOL FOR INTRODUCTORY NONLINEAR CONTROL SYSTEMS EDUCATION

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**Abstract:** In this paper a new interactive tool for control education is presented. With this tool, developed in SysQuake (Piguet, 2000), the student can easily assimilate some concepts of introductory courses in nonlinear control such as the behaviour of piecewise linear systems, stable and unstable limit cycles and the describing function method. Furthermore, the basis of more sophisticated fields such as bifurcation theory applied to control systems can also be acquired. Three application examples are included in order to show the possibilities of the tool.

**Keywords:** Control education, piecewise linear systems, describing function, limit cycles, bifurcations.

### 1. INTRODUCTION

Basic skills are learned not in isolation, but in the course of undertaking (often on a collaborative basis) higher-level "real-world" tasks. The students assume a central role as the active architect of his or her knowledge and skills, rather than passively absorbing information proffered by the teacher. The old adage "learning by doing" is today valid. The technology alone can never be a solution, but in the hands of a knowledgeable teacher, appropriately designed technology can become a useful tool. The World Wide Web and others information technology cannot be a solution to educational needs unless the creative component is included (Copinga *et al.*, 2000; Poindexter and Heck, 1999).

Because of the amazing progress in computer technology, today is possible to design "control education tools" with the following characteristics:

- Better man-machine interaction
- Natural and intuitive graphical user interfaces
- High degree of interactivity

We are at the threshold of a new era in which advanced information technologies are finding their way towards effective and efficient applications in control education (Kheir *et al.*, 1996).

### 2. THE ROLE OF INTERACTIVITY IN CONTROL EDUCATION

In order to design technical systems or simply to understand the physical laws that describe their behavior, scientists and engineers often use computers to calculate and represent graphically different magnitudes. In control engineering, these quantities include among others: time and frequency responses, poles and zeros in the complex plane, Bode, Nyquist and Nichols diagrams, phase plane, etc. Frequently, these magnitudes are closely related and constitute different visions of a single reality. The understanding of these relationships is one of the keys to achieve a good learning of the basic concepts and allows to the student to be in disposition of carrying out control-systems designs accurately.

Traditionally, the design of the systems is carried out following an iterative process. Specifications of the problem are not normally used to calculate the value of the system parameters because there is not an explicit formula that relates them directly. This is the reason for dividing each iteration into two phases. The first one, often called synthesis, consists in calculating the unknown parameters of the system taking as a basis a group of design variables (that are related with the specifications). During the second phase, called analysis, the performance of the system is evaluated and compared to the specifications. If they do not agree, the design variables are modified and a new iteration is performed. From the student's perspective, this iterative procedure can be very confusing and laborious.

It is possible, however, to merge both phases into one on which the result of modifying the parameters produces an immediate effect. In this way, the design procedure becomes really dynamic and the student perceives the gradient of the change of the performance criteria with regard to the elements he/she manipulates. This interactive capacity allows us to identify much more easily the compromises that can be achieved.

Many tools for control education have been developed over the years. Many interesting ideas and concepts were implemented by Prof. Åström and col. at Lund. In this context, it is well worth to stress the concepts of dynamic pictures and virtual interactive systems introduced by Wittenmark et al., 1998. The main objective of these tools is to make students more active and concerned in control courses.

In essence a dynamic picture is a collection of graphical windows that are handled using only the mouse. In the screen, some special elements, the active points, can be manipulated by the students, who do not have to learn or write any sentences. An additional help is that the shape of the cursor changes when it is over an active element. If the student changes any active element in the graphical windows, automatically begins an immediate recalculation and presentation to the user. In this way perceives a feeling for how their modifications affect to the result obtained.

Interactive tools, which are accessible to the students at any time through the Internet, are considered a great stimulus for developing the student's engineering intuition. These interactive tools attempt to "demystify" abstract mathematical concepts through visualization in specifically chosen examples. At the present time a new generation of software packages have generated an interesting alternative for the interactive learning of automatic control (García and Heck, 1999). These tools are based on objects that admit a direct graphic manipulation. During these manipulations, the objects are immediately updated, so that the relationship among the objects is continuously maintained. Ictools and CCSdemo (Johansson et al., 1998; Wittenmark et

al., 1998), developed in the Department of Automatic Control at Lund Institute of Technology, SysQuake in the Institut d'Automatique of the Federal Polytechnic School of Lausanne, (Piguet, 2000) and Virtual Control Lab (Sánchez et al., 2002) are good examples of this new educational philosophy of teaching automatic control.

### 3. NONLINEAR AND GLOBAL BEHAVIOR PHENOMENA IN CONTROL ENGINEERING

Classical control theory has mainly emphasized the use of linear analysis and design tools. There are at least two reasons for that. Probably the most important is that the control problem is basically a local one. The aim is to stabilize the system at the operating point for reasonably small disturbances. And in most of the practical situations it is possible to linearize the system about this operating point, and to get acceptable results from a practical point of view from that linearized version of the problem. Furthermore, the linear systems theory of dynamical systems is a body of mathematical knowledge that is very well structured, and supplies useful operative tools, that are simple to handle (for instance, the transfer function or the linear description in the state space). Most of the students of control engineering only know these linear methods, and actually with them most of the practical problems in control analysis and design can be dealt with. In most control textbooks the consideration of nonlinearities is left as a final chapter, as a sophisticated problem out of the main questions of practical interest for future control engineers. It should be added that the same happens in other branches of engineering, where also linear methods are dominant. For instance, in electrical engineering, that has used traditionally linear equations, but problems such as the tension collapse has lead to drive the attention to nonlinear problems.

In the same line, the consideration of the nonlinear effects in control systems deserves a special consideration. Nonlinear systems present behavior modes far more rich than the ones displayed by linear systems. The first point to note is that structurally stable oscillations (that is oscillations that are robust with regard to small changes in parameters or initial conditions) can only be produced by nonlinear systems. The oscillations of the so called linear harmonic oscillator are not of any practical, nor theoretical, interest. It should be emphasized to students that whenever they found oscillations they are in the realm of nonlinear systems. Furthermore the bounds of the attraction basins are associated to unstable limit cycles, a kind of phenomena that does not have a parallel in linear systems behaviour.

Another crucial difference between linear and nonlinear dynamical systems is that the former ones only show a single attractor (except in degenerate cases)

whereas a nonlinear can have many of them. Furthermore, the attractor associated to linear systems can be only a point (at infinity for unstable systems). The nonlinear systems have many kinds of attractors: points, closed curves (limit cycles, associated to periodic oscillations) and more complex geometrical objects (fractals for aperiodic behavior, as chaos).

These questions are far from merely speculative for control engineers. For instance, when an unstable open loop plant is to be stabilized linear control gives misleading results. Even if the plant can be stabilized at the operating point with a linear control plant, this control will behave adequately only locally, due to the nonlinearity associated with the actuator saturation (Stein, 1989; Aracil *et al.*, 2000). This is a significant example of the kind of situations where the classical linear control theory reaches its bounds and that should be stressed to students.

Fortunately, for many interesting problems in control systems raised by the presence of nonlinearities, as the above mentioned of the unstable plant with a saturating actuator, a quite simple and powerful analysis can be worked out. For many nonlinear problems the control engineer has the describing function as a tool to deal with them (Atherton, 1975; Cook, 1994; Khalil, 1996; Schwartz and Gran, 2001). This method has been used for decades by practical engineers. Even if the results are only approximate, they are very easy to reach and supply clues about the oscillatory and the global behavior at a very low cost. Unfortunately, the practical application of the describing function have not received enough attention by producers of software for aiding to control systems analysis and design.

The describing function method is very helpful as an analysis tool. It does not give conclusive results, but helps to predict some global phenomena that should be confirmed by simulation. It is not a true analytical tool that makes precise predictions. That is why it should be used in interaction with some simulation tool. But an interplay of simulation and the use of the describing function method allows the student of Control Engineering to reach a complete nonlinear analysis of many control problems of a great practical interest.

#### 4. DESCRIPTION OF THE TOOL

In this section the new tool, programmed with SysQuake (Piguet, 2000), is briefly described. For the sake of brevity, only the main functions are shown.

The systems that can be analyzed have the form described in Fig. 1. The linear element is any proper transfer function (future versions will allow the presence of pure time delays). The nonlinear element has the general form that appears in Fig. 2 and covers a wide variety of piecewise linear nonlinearities with

odd symmetry<sup>1</sup> including some multivalued nonlinearities (Sridhar, 1960). Among many others the following nonlinearities can be covered: dead-zone, saturation, dead-zone+saturation, relay and relay with hysteresis. The configuration of the nonlinearity is performed by means of an interactive way: the user only has to move with the mouse the active points in the "Nonlinear element" window of the tool, which are represented by small circles. The active points are identified by colors and when two or more points collapse into a single one the resultant color is the corresponding one to the movement that can still be performed. As these points are moved the effect on the other graphs can be observed. In the same window a straight line with slope  $-1/k$  (being  $k$  the static gain of the linear element) is plotted. In this way the intersections between the nonlinear element and the line are the equilibrium points of the system (Aracil *et al.*, 2000).

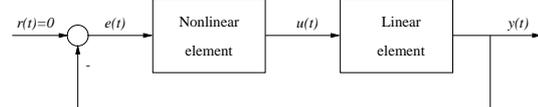


Fig. 1. Structure of the system.

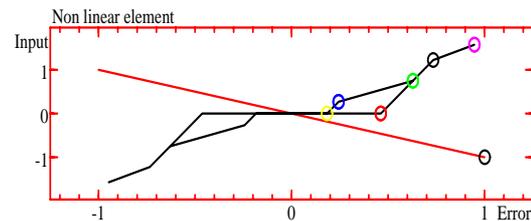


Fig. 2. Generic nonlinearity.

The rest of the windows of the tool are (Fig. 3):

- "Parameters": In this window the configuration parameters can be defined: predefined view, number of poles and zeros of the linear element, scale of other graphs, final time for the simulations (it can be negative in order to integrate backwards), precision and tolerance of the integration and the computation of the describing function.
- "Transfer function": It allows to graphically define the poles and zeros of the linear element. They can also be defined in a dialog box.
- "Nonlinear element input" and "Nonlinear element output": They show the input (a sinusoidal) and the output of the nonlinearity. The former allows to define the amplitude of the input and the second shows its response. In both graphs the frontiers between the linear zones are shown.
- "Real part describing function" and "Imaginary part describing function": They show the describing function as a function of the amplitude.

<sup>1</sup> The tool can be extended to asymmetrical nonlinearities with the use of the dual input describing function (Cook, 1994).

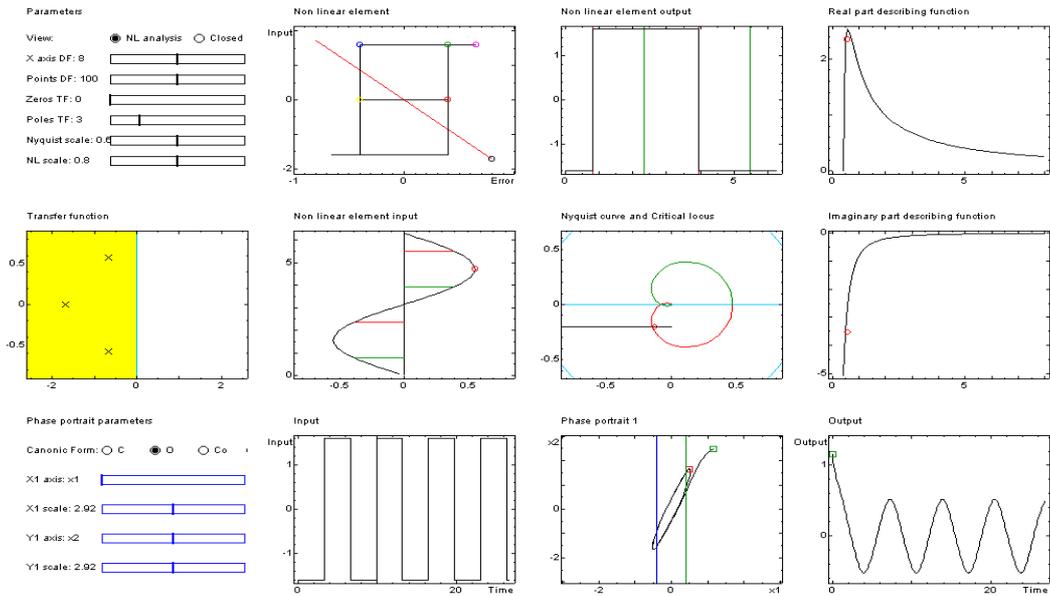


Fig. 3. Graphs for the first example.

The point corresponding to the amplitude chosen in the “Nonlinear element input” window is highlighted.

- “Nyquist curve and Critical locus”: It shows the the Nyquist plot of the linear element as well as  $-1/N$  being  $N$  the describing function of the nonlinear element. The point corresponding to the amplitude chosen in the “Nonlinear element input” window is highlighted. The intersections of both curves are predicted limit cycles.
- “Phase portrait”: It shows the projection of the trajectory into the  $(x_i - x_j)$  plane, where  $x_i$  and  $x_j$  are two state variables that can be chosen in the “Phase portrait parameters” window (the state description can also be chosen among five categories). The frontiers between the linear zones are also shown. The initial condition can be changed with the mouse.
- “Input” and “Output”: They show the input and the output of the linear element.

All the output information is refreshed each time the user changes something (even when he/she drags the defining points with the mouse).

It is worthy to remark that the time integration has been programmed in such a way that the exact solution is used in the linear zones while the crossing points are searched with a tolerance that is defined by the user. Therefore, the “time step” of the simulations does not affect the accuracy of the results. Its only effect is that the computed points will be joined with lines and if the time step is too large the graphs will be sharp.

## 5. APPLICATION EXAMPLES

In this section, some examples are shown in order to demonstrate some of the possibilities of the appli-

cation. It must be taken into account that the main characteristic of it –the interactivity– is difficult to be reflected in a written text. Nevertheless, some of the potentials of the application will be illustrated.

### 5.1 Detection of limit cycles

This example shows the most classical application of the describing function method: the detection of limit cycles. Consider, as the linear part, the transfer function

$$G(s) = \frac{K}{s^3 + 3s + s + 1}$$

and a normalized relay as the nonlinear element. As it is well known the critical locus of a normalized relay is an horizontal line in the Nyquist plane.

The relay is one of the possible configurations for the nonlinearity considered in the application and can easily be constructed. Figure 3 shows the graphs corresponding to this example. The “Nonlinear element output” shows the output of the nonlinearity to a sinusoidal input whose amplitude can be defined by the user in the “Nonlinear element input” window. The resultant value of the describing function is highlighted in the real and imaginary graphs of the describing function as well as in the Nyquist plane.

Choosing appropriately the value of  $K$  (for example, moving the red line in the “Nonlinear element” window) an intersection between the Nyquist and the critical locus is produced. When this intersection occurs, the describing function method predicts a limit cycle<sup>2</sup>. The validity of this prediction can be verified in the “Input”, “Output” and “Phase portrait” graphs.

<sup>2</sup> Current work is being performed in order to on-line compute the values of the amplitude and frequency of the limit cycle as well as its stability.

With all this information as well as the possible interaction by the student, he/she can easily assimilate the basis of the describing function method.

### 5.2 Saddle-node bifurcation of limit cycles

This example is a more sophisticated case than the one presented in (Ortega *et al.*, 2000). The “Nonlinear element” window of Fig. 4 shows the type of non-linearity considered in this case, which the user can define easily. The critical locus has two branches as it appears in the “Nyquist curve and Critical locus” of the same figure. Changing the amplitude of the input sinusoidal, the user can observe how the real part of the critical locus goes from minus infinity to a value greater than  $-1$  and, then, it returns to minus infinity.

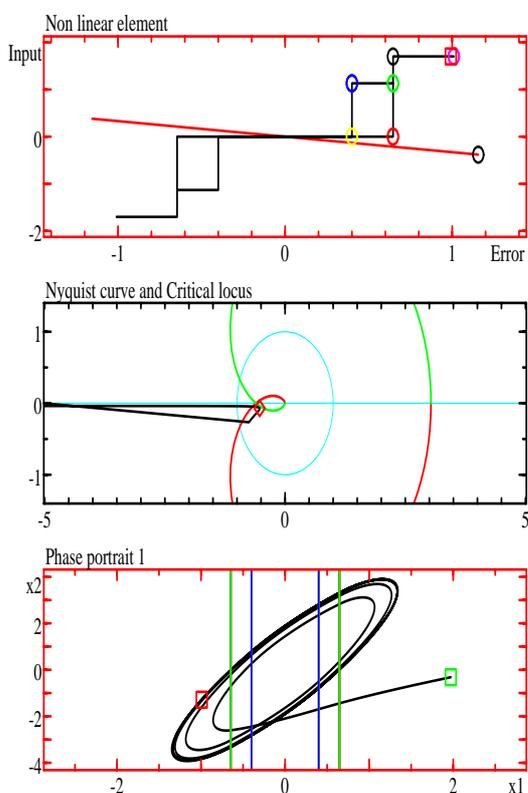


Fig. 4. Some graphs corresponding to the second example.

The linear part has three poles –two of them are complex conjugate. The user can observe that changing the gain of the linear part, the Nyquist plot sometimes does not cross the critical locus, while for some other values of the gain two intersections occur. The stability analysis of the predicted limit cycles will reveal that the limit cycle with smaller amplitude is unstable while the one with larger amplitude is stable. When the gain of the linear part decreases, the amplitudes of the predicted limit cycles will approach. When the Nyquist plot is tangent to the critical locus the predicted limit cycles will have the same amplitude. If, then, the gain continue decreasing there will not be

limit cycles any more. This phenomenon follows the archetype known as *saddle-node bifurcation of periodic orbits* (Hale and Koçak, 1991, p. 383).

This analysis is based on the approximations inherent in the describing function method but the user can verify its validity clicking in the “Closed Loop” button and analyzing the phase portrait: all the trajectories go to zero when the value of the gain is small; on the other hand, for large values of the gain, some trajectories go to the origin and some others to a limit cycle. Changing the value of the gain and the initial conditions the predicted behaviour can be verified.

### 5.3 Detection of equilibrium points

As it has been stated in the previous section, the tool can also be used to detect equilibrium points. This example shows an intuitive case of a saddle-node bifurcation. Consider, as the linear part, the transfer function

$$G(s) = \frac{K}{s^2 + 5s + 6}$$

with  $K < 0$ . The nonlinear part will be a dead-zone and a saturation as it is shown in the “Nonlinear element” window of Fig. 5. In this case there is no intersections between the Nyquist plot and the critical locus apart from the corresponding to  $\omega = 0$  for large values of  $K$ . Therefore, the describing function method does not predict any limit cycle.

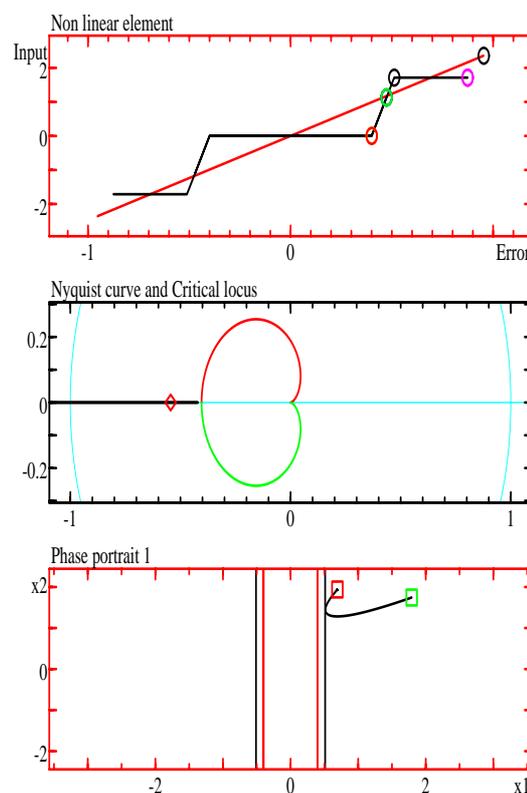


Fig. 5. Some graphs corresponding to the third example.

When the red line of the “Nonlinear element” window has a large positive slope (that is,  $K$  is negative and small) only one intersection between this curve and the nonlinear characteristic will occur. At the same time, no intersections appear between the critical locus and the Nyquist plot. In this case, only the desired equilibrium exists and it can be seen that it is (globally) stable.

On the other hand, when the red line has a small positive slope (that is,  $K$  is negative and large) five intersections between this curve and the nonlinear characteristic will occur. At the same time, one intersection appears between the critical locus and the Nyquist plot corresponding to  $\omega = 0$ . In this case, five different equilibrium points exist. The desired equilibrium is still stable (but only locally stable) while two couples with a stable equilibrium and an unstable one have been born. In the critical case (the intermediate value of  $K$  for which the equilibria emerge) the equilibrium points of each pair coincide. Therefore two simultaneous saddle-node bifurcations occur (the simultaneity is due to the symmetry of the nonlinearity). All this phenomenon can be seen moving the red line and can be corroborated by the simulations that appear in the “Phase portrait” graph.

## 6. CONCLUSIONS

In this paper, a new educational tool for preliminary nonlinear control systems has been presented. The main characteristic of this tool, developed in SysQuake, is its interactivity. The tool can help the students to understand the behaviour of piecewise linear systems, the describing function method and the basis of bifurcations theory in control systems.

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