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A SOLUTION TO THE INSENSITIVE H_2 PROBLEM AND ITS APPLICATION TO AUTOMOTIVE CONTROL DESIGN

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Abstract: The parametric Insensitive H_2 control (IH_2) problem is a useful method to deal with some difficult applied control problems. Unfortunately, it is as yet unsolved. Some important applications exist however in the case where a single parameter of the process to control is uncertain (e.g. in the electricity or automotive domain). Although the heuristic used in these applications effectively reduces the closed-loop parametric sensitivity function, it does not lead to the optimal solution. This paper pursues three objectives. First, it generalizes the existing heuristic to the multiparametric case. Secondly, the IH_2 problem is shown to be equivalent to an auxiliary standard H_2 problem with structure constraints on the feedback solution. This result allows authors to propose an original Iterative LMI based algorithm. The third and last point demonstrates an implementation of the ILMI algorithm to deal with an automotive control designs and finally compares the obtained results to the previous ones. *Copyright © 2002 IFAC*

Keywords: Linear systems, robust control, H_2 control, parametric sensitivity, Bilinear Matrix Inequalities, vehicle control.

1. INTRODUCTION

The robustness theory has produced numerous robust design methods. Many of them use a criterion (H_2, H_∞, L_1, μ…) so as to quantify the quality of control. Two steps are required in this case to find the controller: 1- The definition of a pertinent criterion, 2- The resolution of an optimization problem. The robustness may be required a priori or verified a posteriori. Let us recall also that system modelling is, also, a trade-off between accuracy and usefulness. The question thus arises: how can a robust design, based on a suitable criterion, produce controllers with an adequate parametric robustness and allow that any matrix coefficient of the controlled system state space description be subjected to parameter uncertainties?. Basic versions of LQG/H_2 and H_∞ control design methods do not ensure such design requirements. In fact, this issue has motivated many contributions. Among numerous methods which try to reduce the parametric sensitivity (Eslami, 1994), one can quote the parametric LQG/LTR method proposed in (Tahk, 1987) and the "desensitized LQG" control ((Begovich, 1992), (Heniche, 1995)). Approaches based on a modified H_2 cost functional to deal with parametric uncertainties have been discussed thoroughly before (Banjerdpongchai, 1996), (Shirley, 2001)). In fact, a variety of methods have been used in practice to try to make H_2 controllers less sensitive to parameter variations in structural systems. For a survey of the most promising of these techniques, the reader is referred to (Grocott, 1994). The robust H2 control problem (Banjerdpongchai, 1996) belongs to this class of design problems. It is, in principle, the best posed design problem but probably the hardest to solve. In fact, we are interested in a design approach that must ensure the classical design specifications (nominal performance and robust stability) that is guaranteed by the nominal H_2 control, and also reduce the sensitivity of the nominal performance with regards to the system parametric variations. The principles of the IH_2 control problem were first presented in (De Larminat, 1996). However, an efficient way to deal with it failed until now. Two approaches are investigated in this paper. The first consists in solving a close problem as proposed in ((Gay, 2000), (De Larminat, 1996)): The heuristic developed in ((Gay, 2000), (De Larminat, 1996)) will be recalled and generalized to the multiparametric case. The second approach takes advantage of the (exact) reformulation of the IH_2 problem given in (Chevrel, 2001) and proposes a numerical method based on an Iterative LMI algorithm. The two approaches will then be compared. Finally, the presentation of this paper is as follows: The IH_2 problem is first presented in section 2. Section 3 generalizes the heuristic proposed in ((Gay, 2000), (De Larminat, 1996)) to the multiparametric case. In section 4, the IH_2 problem is shown to be equivalent to a structured H_2 problem derived from an auxiliary model. It is then reformulated as a linear objective optimization problem under some BMI constraints. We finally propose an original method and an associated Iterative LMI based algorithm to solve this optimization problem. Section 5 applies and compares the two different methods to deal with an automotive design problem.

NOTATIONS

- $\otimes$: The Kronecker product of matrices:

$$C = A \otimes B \; := \begin{bmatrix} a_{11}B & \dots & a_{1n}B \\ & & \\ a_{m1}B & \dots & a_{mn}B \end{bmatrix}$$

2. THE IH_2 CONTROL PROBLEM

Consider the scheme of figure 1 in which G is an LTI operator with partitioned inputs and outputs and Δ is an unknown operator allowing the parametric uncertainties to be taken into account.

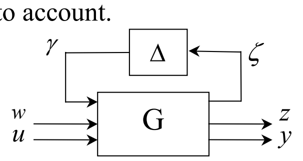

Figure 1: Linear Fractional Representation of parametric uncertainties

Let the transfer matrix $G(s)$ associated to G be defined by

$$G(s) := \begin{bmatrix} A & B_\gamma & B_w & B_u \\ C_\zeta & D_{\zeta\gamma} & D_{\zeta w} & D_{\zeta u} \\ C_z & D_{z\gamma} & D_{zw} & D_{zu} \\ C_y & D_{y\gamma} & D_{yw} & D_{yu} \end{bmatrix}$$

Let also Δ be defined by $\Delta(\zeta) = \Delta.\zeta$ with the particular form

$$\Delta = diag(\theta_1 I_{P1}, \theta_2 I_{P2}, \dots, \theta_q I_{Pq})$$

with $\theta_i \in R \;, i \in \{1, \cdots, q\}$ stacks the uncertain parameters.

Finally we introduce the feedback K (associated to the transfer matrix $K(s)$) according to figure 2.

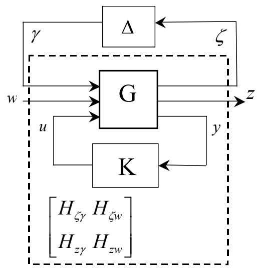

Figure 2: Closed loop transfer function

All the transfers $H_{zw}, H_{z\upsilon}, H_{\zeta\upsilon}$ and $H_{\zeta w}$ depend on the feedback K.

The closed loop transfer matrix $F_l(G(s), K(s))$ has, therefore, the partitioned form $\begin{bmatrix} H_{\zeta\gamma} & H_{\zeta w} \\ H_{z\gamma} & H_{zw} \end{bmatrix}$. Closing the "$\Delta$-loop", the final transfer z/w depends on both s and θ and can be written as

$$H = H_{zw} + H_{z\gamma}(I - \Delta H_{\zeta\gamma})^{-1} \Delta H_{\zeta w}$$

In the following sections, the sensitivity function will be considered in the neighbourhood of $\Delta = 0$ (the nominal model). The parametric sensitivity function is given by

$$\frac{\partial H}{\partial \theta} = I_q \otimes H_{z\gamma} \frac{\partial \Delta}{\partial \theta} H_{\zeta w}$$

Definition 1: (The IH_2 control problem (IH_2P))

The objective of the IH_2 control is to minimize, under the constraint of internal stability and with respect to $K(s)$, the following index

$$\boxed{\begin{aligned} J_{IH_2}(K) &= \|H_{zw}\|_2^2 + \left\| \Sigma \otimes H_{z\gamma} \frac{\partial \Delta}{\partial \theta} H_{\zeta w} \right\|_2^2 \\ &= \|H_{zw}\|_2^2 + \sum_{i=1}^{q} \sigma_i^2 \left\| H_{z\gamma} \frac{\partial \Delta}{\partial \theta_i} H_{\zeta w} \right\|_2^2 \end{aligned}} \tag{1}$$

where $\Sigma = diag\ (\sigma_1, \dots, \sigma_q)$ and each $\sigma_i \in \Re$ is a weighting parameter associated with $\frac{\partial H}{\partial \theta_i}$ - the parametric sensitivity with respect to θ_i -.

3. GENERALIZATION OF AN EXISTING HEURISTIC TO SOLVE THE IH_2P

The heuristic proposed in (De Larminat, 1996), which we will designate by "Heuristic 1", is restricted to the case where a single parameter is uncertain. The presentation in this section generalizes this heuristic to the multiple uncertain parameters case. In fact, the problem addressed is not rigorously the IH_2P but a close one. In addition to the criterion J_{IH_2} the second derivative of the parametric sensitivity function $\left\| \frac{\partial^2 H}{\partial \theta^2} \right\|_2^2$ is considered. In the multiparametric case, this criterion can be written as:

$$\hat{J} = J_{IH_2} + \frac{1}{4} \sum_{i=1}^{q} \sigma_i^2 \left(\sum_{j=1}^{q} \sigma_j^2 \left(\left\| H_{z\gamma} \frac{\partial \Delta}{\partial \theta_i} H_{\zeta\gamma} \frac{\partial \Delta}{\partial \theta_j} H_{\zeta w} \right\|_2^2 \right) \right)$$

In fact, the approximation $\hat{J} \cong J_{IH_2}$ may hold for sufficiently small $(\sigma_i)_{1 \le i \le q}$.

As a first stage, the algorithm is initialized as follows: A standard H_2 controller (obtained for $\Sigma = 0$) denoted by K_0 is first computed. From this controller, the two transfers $H_{\zeta w}(K_0)$ and $H_{z\gamma}(K_0)$ are derived. The augmented model of figure (3) in which $H_{\zeta w}(K_0)$ and $H_{z\gamma}(K_0)$ play the role of weighting functions may then be built.

$\hat{w}_\theta{}^T = \begin{bmatrix} \hat{w}_{\theta_1} & \cdots & \hat{w}_{\theta_q} \end{bmatrix}^T$ and $\hat{z}_\theta = \begin{bmatrix} \hat{z}_{\theta_1} & \cdots & \hat{z}_{\theta_q} \end{bmatrix}^T$ are considered respectively as additional "noise" at the input and error signals at the output.

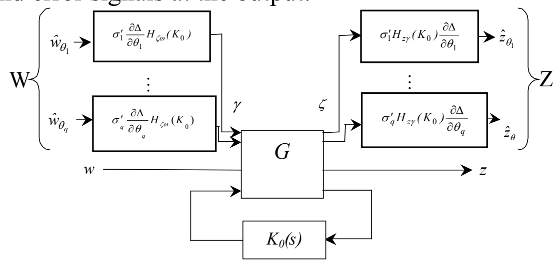

Figure 3 : *The first step of the algorithm*

An iterative procedure can then be applied. A controller K_1 is then designed that minimizes the H_2 norm of the augmented model (Z/W) of figure (3) where $Z^T = \begin{bmatrix} \hat{z}_\theta^T & z^T \end{bmatrix}^T$ and $W^T = \begin{bmatrix} \hat{w}_\theta^T & w^T \end{bmatrix}^T$.

At the i^{th} iteration, the augmented model P_i of figure (4) is derived from K_{i-1}. Controller K_i is designed in order to minimize $\hat{J}_i := \|F_l(P_i, K_i)\|_2$.

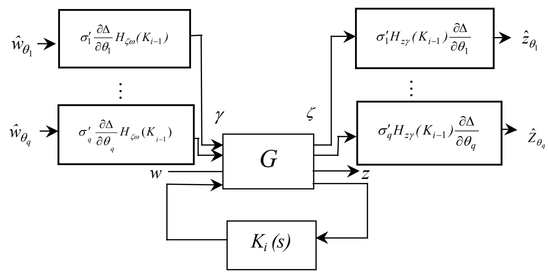

Figure 4 : *The i^{th} step of the algorithm.*

By construction the following equality holds:

$$\hat{J}_i = \|H_{zw}\|_2^2 + \left\| \Sigma' \otimes H_{z\gamma}(K_{i-1}) \frac{\partial \Delta}{\partial \theta} H_{\zeta w}(K_i) \right\|_2^2 + \left\| \Sigma' \otimes H_{z\gamma}(K_i) \frac{\partial \Delta}{\partial \theta} H_{\zeta w}(K_{i-1}) \right\|_2^2 + \sum_{i=1}^{q} \sigma_i'^2 \left(\sum_{j=1}^{q} \sigma_j'^2 \left(\left\| H_{z\gamma}(K_{i-1}) \frac{\partial \Delta}{\partial \theta_i} H_{\zeta\gamma} \frac{\partial \Delta}{\partial \theta_j} H_{\zeta w}(K_{i-1}) \right\|_2^2 \right) \right) \qquad (2)$$

where $\Sigma' = diag\ (\sigma_1', \ldots, \sigma_q') \in \Re^{q \times q}$.

Taking $\Sigma' = \Sigma / \sqrt{2}$, it is hopped that the heuristic will converge as:

$$\lim_{i \to +\infty} \hat{J}_i = J_{IH_2} + \frac{1}{4} \sum_{i=1}^{q} \sigma_i^2 \left(\sum_{j=1}^{q} \sigma_j^2 \left(\left\| H_{z\gamma} \frac{\partial \Delta}{\partial \theta_i} H_{\zeta\gamma} \frac{\partial \Delta}{\partial \theta_j} H_{\zeta w} \right\|_2^2 \right) \right) := \hat{J}$$

The approximation $\hat{J} \cong J_{IH_2}$ can be considered for sufficiently small $(\sigma_i)_{1 \le i \le q}$. This method suffers, however, from some drawbacks. Even in the case where the approximation $\hat{J} \cong J_{IH_2}$ is justified, it leads to a controller of very high order, especially in the muliparametric case. At the i^{th} stage:

$$\dim(K_i) := 2q \dim(K_{i-1}) + (2q+1)n = \left(\frac{2^{i+2} q^{i+1} - 2q - 1}{2q - 1} \right) n$$

The controller order may rapidly become "unsuitable" in the multiparametric case $(q > 1)$. The procedure proposed in (Gay, 2000) in order to have constant controller order along the iterations could be generalized to the multiparametric case. It is, however, difficult to evaluate how it will affect the convergence of the algorithm – even if it seems to give some results. We will designate this second heuristic by "Heuristic 2". The question thus arises: how far will the solution be from the IH_2 optimum? .

3. A NEW IH_2P SOLUTION USING CONVEX OPTIMIZATION TOOLS

The starting point of the solution proposed in this section is the theorem (1) stated in (Chevrel, 2001). For simplicity's sake, this theorem is recalled next under the assumption that $\Sigma = I_q$. The general result, however, is straightforward. The augmented model shown in figure (5) is defined as follows:

The parametric sensitivity of the trajectory signals are denoted by

$$x_\theta = \frac{\partial x}{\partial \theta}, \ z_\theta = \frac{\partial z}{\partial \theta}, \ y_\theta = \frac{\partial y}{\partial \theta} \ \text{and} \ u_\theta = \frac{\partial u}{\partial \theta}.$$

The augmented model G_a^1 (see figure 5) with $\begin{bmatrix} w^T, u^T, u_\theta^T \end{bmatrix}^T$ as inputs and $\begin{bmatrix} z^T, z_\theta^T, y^T, y_\theta^T \end{bmatrix}^T$ as outputs is related to G as follows

$$G_a^1(s) := \left[\begin{array}{cc|ccc} A & 0 & B_w & B_u & 0 \\ A_\theta & (I_q \otimes A) & B_{w\theta} & B_{u\theta} & (I_q \otimes B_u) \\ \hline C_z & 0 & D_{zw} & D_{zu} & 0 \\ C_{z\theta} & (I_q \otimes C_z) & D_{zw\theta} & D_{zu\theta} & (I_q \otimes D_{zu}) \\ C_y & 0 & D_{yw} & D_{yu} & 0 \\ C_{y\theta} & (I_q \otimes C_y) & D_{yw\theta} & D_{yu\theta} & (I_q \otimes D_{yu}) \end{array} \right]$$

with

- $A_\theta = (I_q \otimes B_\gamma) \frac{\partial \Delta}{\partial \theta} C_\zeta$ • $C_{z\theta} = (I_q \otimes D_{z\gamma}) \frac{\partial \Delta}{\partial \theta} C_\zeta$ • $C_{y\theta} = (I_q \otimes D_{y\gamma}) \frac{\partial \Delta}{\partial \theta} C_\zeta$
- $B_{w\theta} = (I_q \otimes B_\gamma) \frac{\partial \Delta}{\partial \theta} D_{\zeta w}$ • $D_{zw\theta} = (I_q \otimes D_{z\gamma}) \frac{\partial \Delta}{\partial \theta} D_{\zeta w}$ • $D_{yw\theta} = (I_q \otimes D_{y\gamma}) \frac{\partial \Delta}{\partial \theta} D_{\zeta w}$
- $B_{u\theta} = (I_q \otimes B_\gamma) \frac{\partial \Delta}{\partial \theta} D_{\zeta u}$ • $D_{zu\theta} = (I_q \otimes D_{z\gamma}) \frac{\partial \Delta}{\partial \theta} D_{\zeta u}$ • $D_{yu\theta} = (I_q \otimes D_{y\gamma}) \frac{\partial \Delta}{\partial \theta} D_{\zeta u}$

Theorem 1:

The IH_2 control problem is equivalent to the constrained H_2 optimization problem, (associated to figure5), consisting in finding a stabilizing controller $K(s)$ that minimizes the criterion $J_{H_2C}(K) \ : \ = \left\| F_l(G_a^1, I_{q+1} \otimes K) \right\|_2^2$.

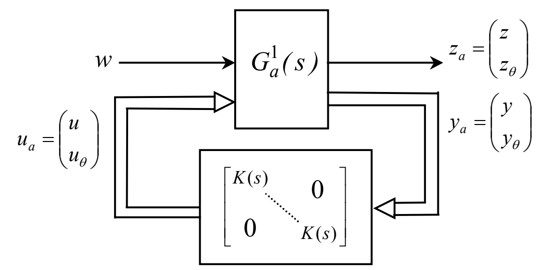

Figure 5: A structured H_2 problem

Proof: The detailed proof is given in (Chevrel, 2001). The equality $J_{H_2C}(K) = J_{IH_2}(K)$ results from the construction from the construction of the auxiliary model $G_a^1(s)$.

Using this result it is possible to show that the IH_2 control problem is equivalent to a linear objective optimization problem with BMI and LME (linear matrix equality) constraints.

Consider the following state space representation for the controller $K(s)$

$$K(s) := \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix}$$

It is commonly known that the dynamic output feedback design problem is a special case of the static one, assuming a certain structure for the plant matrices.

The new control problem described by theorem (1) is equivalent to the static output feedback H_2 optimization problem that consists in finding a stabilizing static output feedback $\hat{K}$ with the following structure

$$\hat{K} := \begin{bmatrix} \hat{A}_K & \hat{B}_K \\ \hat{C}_K & \hat{D}_K \end{bmatrix} = \begin{bmatrix} I_{q+1} \otimes A_K & I_{q+1} \otimes B_K \\ I_{q+1} \otimes C_K & I_{q+1} \otimes D_K \end{bmatrix} \quad (3)$$

for the augmented model $G_a^2(s)$ (we suppose without loss of generality (H): $D_{yu} = D_{zw} = 0$)

$$G_a^2(s) := \left[\begin{array}{ccc|cccc} A & 0 & 0 & B_w & B_u & 0 & 0 \\ A_\theta & (I_q \otimes A) & 0 & B_{w\theta} & B_{u\theta} & (I_q \otimes B_u) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_n \\ \hline C_z & 0 & 0 & 0 & D_{zu} & 0 & 0 \\ C_{z\theta} & (I_q \otimes C_z) & 0 & 0 & D_{zu\theta} & (I_q \otimes D_{zu}) & 0 \\ C_y & 0 & 0 & D_{yw} & 0 & 0 & 0 \\ C_{y\theta} & (I_q \otimes C_y) & 0 & D_{yw\theta} & 0 & 0 & 0 \\ 0 & 0 & I_n & 0 & 0 & 0 & 0 \end{array}\right]$$

$$= \begin{bmatrix} \hat{A} & \hat{B}_1 & \hat{B}_2 \\ \hat{C}_1 & 0 & \hat{D}_{12} \\ \hat{C}_2 & \hat{D}_{21} & 0 \end{bmatrix}$$

Theorem 2: *With assumption (H), the IH_2P is equivalent to the following linear objective optimization problem under BMI and LME constraints:*

$$\begin{cases} \min\limits_{X=X^T \geq 0, Y, \hat{K}} \; Trace\,(Y) \\ (\hat{A} + \hat{B}_2\hat{K}\hat{C}_2)^T X + X(\hat{A} + \hat{B}_2\hat{K}\hat{C}_2) + \\ \qquad (\hat{C}_1 + \hat{D}_{12}\hat{K}\hat{C}_2)^T (\hat{C}_1 + \hat{D}_{12}\hat{K}\hat{C}_{P2}) < 0 \\ Y - (\hat{B}_1 + \hat{B}_2\hat{K}\hat{D}_{21})^T X (\hat{B}_1 + \hat{B}_2\hat{K}\hat{D}_{21}) > 0 \\ \hat{K} = \begin{bmatrix} I_{q+1} \otimes A_K & I_{q+1} \otimes B_K \\ I_{q+1} \otimes C_K & I_{q+1} \otimes D_K \end{bmatrix} \end{cases} \quad (4)$$

Because of the equality constraint on the static output feedback $\hat{K}$, (4) cannot be simplified (as is usually done for the classical H_2 problem) to a convex optimization problem with LMI constraints. As a result we propose here to solve the IH_2P using the ILMI algorithm that we first developed to tackle some constrained control design problems such as the H_2 and H_∞ decentralized control (Yagoubi, 2001). The originality of this algorithm consists in using a close approximation (not too conservative) of some bilinear terms involved in (4).

Generically, the constrained controller design problem (4) involves some bilinear terms in X and K of the form

$$BL(X,K) = (M_1KM_2)^T XM_3 + M_3^T X(M_1KM_2) \quad (5)$$

where M_1, M_2 and M_3 are some constant matrices.

Consider the bilinear constraint in X and K with the special form

$$R + L(X,K) + BL(X,K) < 0 \quad (6)$$

where R is a symmetric constant matrix with the same dimensions as X, $BL(X,K)$ is defined by (5) and $L(X,K)$ is a linear term in X and K with the same dimensions as X.

Our purpose here is to approximate the bilinear term $BL(X,K)$ with a linear one. More precisely, the BMI will be approached by a set S_Q of LMIs parameterized by an auxiliary matrix named Q.

Theorem 2:

i) Assume there exist $X = X^T$, K and Q such that the following holds

$$\begin{cases} X > 0, \\ \begin{bmatrix} R + L(K,X) + \Phi & XM_3 & (M_1KM_2)^T \\ M_3^T X & -I & 0 \\ (M_1KM_2) & 0 & -I \end{bmatrix} < 0 \quad (8) \end{cases}$$

where $\Phi = Q^TQ - Q^T(XM_3 - M_1KM_2) - (XM_3 - M_1KM_2)^T Q$.

Then (X,K) satisfies the constraint (6).

ii) For the particular choice $Q = XM_3 - M_1KM_2$, the conditions (6) and (8) are equivalent.

Proof: (see (Yagoubi, 2001))

For a given matrix Q, finding (X,K) solution of the inequality (8) is straightforward as the problem is convex. In the following sections, we will designate this linear constraint, parameterized by Q, by $Lmi(Q,X,K)$. This parameterization has some advantages over existing ones. The solution of the BMI problem will coincide with the solution of a particular LMI belonging to the set S_Q. So, a “sub-problem” will be to find the parameter Q that leads to the best approximation.

The proposed ILMI algorithm is given next in a generic case. In fact, we consider the problem of minimizing a linear objective $J(X,K)$ under the BMI constraint (6).

The initialization:

-For a stabilizing controller K_0 compute X_0:

$$\begin{cases} X_0 = \arg\min\, J(X,K_0) \\ R + L(X,K_0) + BL(X,K_0) < 0 \end{cases}$$

-Compute Q_0:

$$Q_0 = X_0M_3 - M_1K_0M_2$$

The k^th iteration:

- Q_{k-1}, X_k and K_k are derived from the previous step.

-Compute Q_k:

$$Q_k = X_kM_3 - M_1K_kM_2$$

-Test $\|Q_k - Q_{k-1}\| < \varepsilon$ for a given tolerance bound. If the test is true then the algorithm is stopped and $\hat{X}_{opt} = X_k$, $\hat{K}_{opt} = K_k$. If the test is false then the pair (X_{k+1}, K_{k+1}) is computed by solving the following LMI problem

$$\begin{cases}(X_{k+1},K_{k+1}) = \arg\min\ J(X,K)\\ Lmi(Q_{k+1},X,K)\end{cases}$$

As with all local methods for solving BMIs, the choice for the initial value is important for convergence to acceptable solutions, which is a potential weakness of these methods. However this is not really a drawback in our case since we can use the standard H_2 controller (obtained for $\Sigma = 0$) K_0 .

4. COMPARISON AND TEST ON AN AUTOMOTIVE CONTROL

The practical interest of the new numerical algorithm proposed in this paper is shown in this example through a robust vehicle dynamics control as considered in (Gay, 2000). The lateral velocity V_y and the yaw velocity $\dot{\psi}$ have to be controlled through two control inputs: the yaw moment C_z that can be obtained by differential braking and the rear steering α_r (see figure (7)). The vehicle must stay near to the desired trajectory as shown in figure (6). Disturbances acting on the vehicle can be summarized into the lateral force F and the yaw moment M

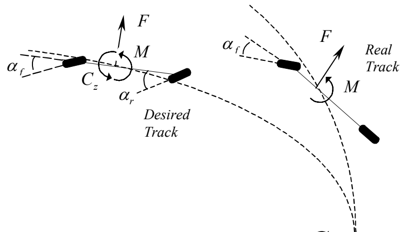

Figure 6 : Correction effects of C_z and α_r

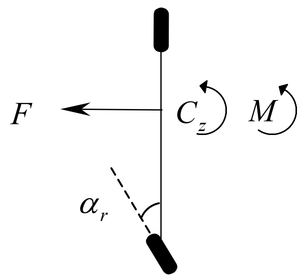

Figure 7: The "bicycle" model

The well known "bicycle model" given by (9) is used to describe the vehicle motions.

$$\begin{bmatrix}\dot{V}_y\\ \ddot{\psi}\end{bmatrix} = A\begin{bmatrix}V_y\\ \dot{\psi}\end{bmatrix} + B_r(\alpha_r + d_{\alpha r}) + B_c(C_z + d_{Cz}) \qquad (9)$$

with

$$A = \begin{bmatrix} -\frac{2\mu}{mV_x}(C_{yv}+C_{yr}) & -V_x + \frac{2\mu}{V_x}(l_2C_{yr} - l_1C_{yv}) \\ \frac{2\mu}{CV_x}(l_2C_{yr} - l_1C_{yv}) & -\frac{2\mu}{CV_x}(l_1^2C_{yv} + l_2^2C_{yr}) \end{bmatrix},$$

$$B_r = \begin{bmatrix}\frac{2\mu C_{yr}}{m}\\ \frac{-2\mu l_2 C_{yr}}{C}\end{bmatrix} \text{ and } B_c = \begin{bmatrix}0\\1\end{bmatrix}.$$

In this model, m denotes the weight, C the inertia, l_1 the front wheelbase, l_2 the rear wheelbase and (C_{yv}, C_{yr}) the nominal cornering stiffness. Note also that the model is parameterized by the road friction parameter μ which is, indeed, uncertain. The standard H_2 problem to be minimized to meet the control requirements is built following the Standard State Control methodology as adopted in (Gay, 2000). The nominal model has been taken for a constant longitudinal speed $V_x = 80km/h$. The H_2 optimal controller is denoted by $K_0(s)$. Weithing the parametric sensitivity function with $\Sigma = 0.75$, two controllers, namely $K_1(s)$ and $K_2(s)$, are derived using respectively "Heuristic 1" and the ILMI numerical method. Table (1) summarizes the results obtained:

The controller	$\|H_{zw}\|_2^2$	$\left\|\Sigma\otimes H_{z\gamma}\frac{\partial\Delta}{\partial\theta}H_{\xi w}\right\|_2^2$	J_{IH_2}	The controller order
$K_0(s)$	2.58	10.30	12.88	4
$K_1(s)$	6.13	4.30	10.43	20
$K_2(s)$	3.34	5.37	8.71	4

Table (1): numerical results

The controller $K_1(s)$ is obtained after only one iteration in order to have a controller of admissible order. In fact, for two iterations of the first heuristic presented in this paper, the controller order would be 52. It appears that the proposed ILMI algorithm gives significantly better results in term of the IH_2 criterion with a low order controller (the same as the system). The computational time is comparable for the two heuristics. A simulation test is now performed in order to observe the effect of the parametric sensitivity reduction. A lateral force step occurs at t=1s and a yaw moment step occurs at t=4s. The test is performed for two values of μ , $(\mu = 1)$ and $(\mu = 0.6)$. Figures (8) and (9) respectively report the results obtained with the standard H_2 controller and those of the IH_2 controller (i.e. $K_2(s)$ which is performed using the ILMI algorithm).

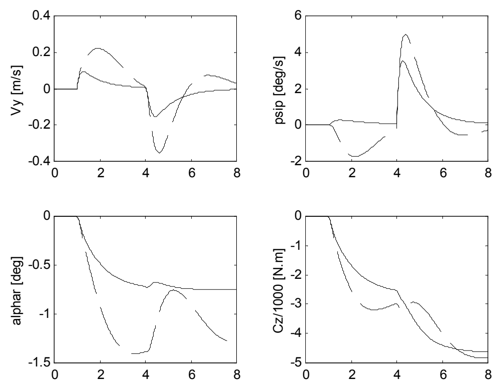

Figure 8: performances obtained with K_0 . $(\mu = 1)$ [—]and $(\mu = 0.6)$ [---]

The above diagrams show the outputs and those below show the inputs with respect to the H_2 optimal controller $K_0(s)$.

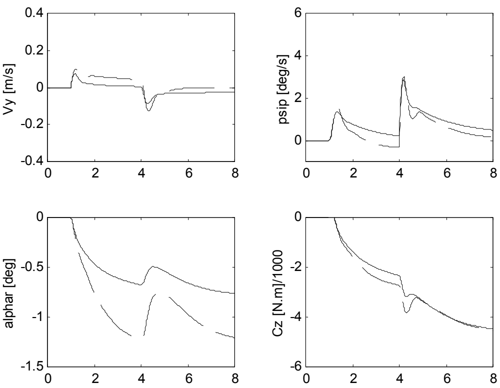

Figure 9: performances obtained with K_2. $(\mu = 1)$ *[—]and* $(\mu = 0.6)$ *[---]*

Controller $K_2(s)$ clearly improves the parametric robustness in comparison with controller $K_0(s)$. Step responses obtained with $K_2(s)$ are clearly less sensitive to the road friction parameter. This example shows the interest of the IH_2 methodology together with the efficiency of the proposed ILMI algorithm. Note also that the resulting controller is of the same order as the H_2 controller.

The ILMI algorithm has been initialized with the H_2 controller. In order to save computation time, "Heuristic 2" could also be used to find a refined starting point for the ILMI algorithm.

4. CONCLUSION

The insensitive H_2 control is an interesting way to deal with applied control design. Based on it, an efficient multivariable control design methodology can be proposed. This observation has motivated the present work. After having presented the Insensitive H_2 control problem, an existing heuristic ("Heuristic 1") has been generalized to the multiple uncertain parameters case. This heuristic suffers, however, from some drawbacks. First of all, it does not deal strictly with the IH_2 problem, but with an approximated one. Secondly it provides high order controllers. For these two reasons, the IH_2 problem has been revisited. The first step has consisted in proving that this problem is equivalent to an H_2 problem for a particular augmented plant with a structure constraint on the feedback loop. This equivalent problem has then been reformulated as a linear objective optimization problem under BMI constraints. Unfortunately, it cannot be reduced to a convex optimization problem by the usual techniques. So, the second step has consisted in proposing an original Iterative LMI numerical method to solve it. It seems to be both efficient and tractable.

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