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# DEADBEAT OBSERVER BASED DETECTION AND ESTIMATION OF A JUMP IN LTI SYSTEMS

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Abstract: The generalized likelihood ratio (GLR) test has been studied for the detection and estimation of a jump in linear systems, where the choices of the window size for the online implementation and of the threshold have been recognized as key problems. This paper proposes a deadbeat observer based GLR test for linear time invariant (LTI) systems. The necessary window size is automatically determined and is not greater than the McMillan degree of the system. Assuming noninformative prior information for the size of the noise variance, a marginalized GLR test is also discussed as the offline procedure to overcome the difficulty in the choice of the threshold.

Keywords: state scintillation detectors, state monitoring, fault detection, fault identification, deadbeat observers, likelihood ratio test

## 1. INTRODUCTION

The problem of detection and estimation of a jump in linear systems has been extensively studied, see (Basseville and Nikiforov, 1993; Isermann, 1984; Kerr, 1987; Willsky, 1976) for surveys. One of the most powerful methods in jump detection is the generalized likelihood ratio (GLR) test proposed in (Willsky and Jones, 1976). The GLR test applies to cases of a jump in the state of linear systems, but it requires a linearly increasing number of parallel filters and is computationally intractable. An approximated sliding window technique was also discussed in (Willsky and Jones, 1976) to obtain a constant number of filters, while the efficient choice of the window size has been recognized as the problem. The threshold depends on the noise variance and its choice has been also recognized as the problem. An alternative approach to the GLR test is to assume noninformative prior information for the scalar scaling of the noise variance, then the log likelihood ratio (LLR) can be computed by eliminating the scalar scaling by marginalization (Gustafsson, 1996).

Before introducing a new approach, we summarize the key points of the GLR test. Based on the state estimation by the Kalman filter, the residual is computed at each time instant. The residual does not depend on the initial state and is independent Gaussian sequence with/without the jump. If no jump has occurred, the mean value of the residual is 0. Once a jump occurs, the mean value of the residual is linearly dependent on the jump at each time instant, while the variance of the residual is the same. This linear dependence of the mean value can be computed utilizing the Kalman filter gain. Since the LLR becomes a function of the unknown jump and the unknown jump time, they can be estimated by maximizing the LLR over a fixed interval. As already mentioned, the choices of the window size and the threshold have been recognized as key problems.

This paper proposes a deadbeat observer based GLR test for linear time invariant (LTI) systems. Since the same relation between the residual with/without jump exists for the residual generated by the deadbeat observer, the same procedure follows for the GLR test. Compared with the Kalman filter approach, it can be shown that the small window size at most the McMillan degree of the LTI system is enough for the detection. Since the computation of the Kalman filter gain is not necessary at each time instant, the new approach would be more easily implemented for the online processing. Since the new method does not require a large window, it has the potential to provide more accurate jump time estimate. If the noise level is significantly small compared with the jump, the new method also has the potential to provide more accurate jump estimate, since the Kalman filter approach essentially requires an infinite window size for the exact jump estimation, while the new approach only requires the small window size. The variance of the residual, however, may be larger than that of the Kalman filter approach, and thus the new method would be weak in detecting the small jump.

In addition to the above demerits, the LLR is inversely proportional to the size of the noise variance. So the prior information, which may be unrealistic in several applications, on the size of the noise variance is necessary to choose the threshold. One approach to avoid this difficulty is to assume noninformative prior information for the size of the noise variance, then we can marginalize this prior information. But it should be noted that we need to discuss this marginalized GLR test as the offline procedure, therefore the choice of the window size is still a problem in this test.

This paper is organized as follows. In Section 2, we review the deadbeat observer for LTI systems and formulate the problem. In Section 3, we solve the problem over maximizing the LLR function. In Section 4, we discuss the marginalization. In Section 5, we illustrate the numerical example to demonstrate the effectiveness of our method. In Section 6, we give the conclusions.

The notations are as follows. := denotes the equal by definition. $\mathbb{R}^{n\times m}$ denotes the space of $n \times m$ real matrices. $\mathbb{R}_+$ denotes the set of nonnegative numbers. $P^T$ denotes a transpose of a matrix $P \in \mathbb{R}^{n\times m}$. $P > 0$ denotes a positive definite matrix for a symmetric matrix $P \in \mathbb{R}^{n\times n}$. $\mathbf{E}[\,\cdot\,]$ denotes the expectation for the random variable.

## 2. DEADBEAT OBSERVER AND PROBLEM FORMULATION

Consider the following discrete-time LTI system

$$x(t+1) = Ax(t) + Gw(t) + \delta_{t_0,t}\,\nu \tag{1}$$
$$y(t) = Cx(t) + v(t) \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state with initial condition $x(0)$. In addition $y(t) \in \mathbb{R}^q$ is the observation, and $\{w(t)\}$ and $\{v(t)\}$ are independent, zero mean, Gaussian sequences with $\mathbf{E}[w(t)w(t)^T] = W > 0$ and $\mathbf{E}[v(t)v(t)^T] = V > 0$. The term $\delta_{t_0,t}\,\nu$ represents a jump in the state. Here $t_0$ is an unknown positive integer, which assumes a value if a jump occurs and takes the value $+\infty$ if there is no jump. Also $\delta_{i,j}$ is the Kronecker delta and $\nu$ is the unknown size of the jump.

The problem is to detect and estimate the jump $\nu$ and the jump time $t_0$ from the given sequence of observations $\{y(t)\}$.

Assuming that the system is observable, one may pick a matrix $H$ such that

$$\Lambda := A - HC \tag{3}$$

is nilpotent by the pole shifting. Then there exists a positive integer $\lambda(\leq n)$ such that

$$\Lambda^{\lambda-1} \neq 0 \quad \text{and} \quad \Lambda^{\lambda} = 0. \tag{4}$$

Here we denote $0^0 = I$ for simplicity. Now consider the deadbeat observer of the form

$$\hat{x}(t+1) = A\hat{x}(t) + H\varepsilon(t) \tag{5}$$
$$\varepsilon(t) := y(t) - C\hat{x}(t) \tag{6}$$

where $\hat{x}(t)$ is the state estimate and $\varepsilon(t)$ is the residual. Let

$$e(t) := x(t) - \hat{x}(t)$$

denote the state estimation error. Then, for any initial value $x(0)$, the state estimation error satisfies

$$e(\lambda) = \sum_{t=1}^{\lambda} \Lambda^{t-1}(Gw(\lambda - t + 1) + Cv(\lambda - t + 1))$$

at time $\lambda$, therefore the initial value $x(0)$ does not effect the state estimation error $e(t)$ for time $t \geq \lambda$. Since $\{w(t)\}$, $\{v(t)\}$ are independent, zero mean Gaussian sequences, the residual $\{\varepsilon(t)\}$ is also an independent, zero mean, Gaussian sequence with

$$\begin{aligned}&\mathbf{E}[\varepsilon(t)\varepsilon(t)^T]\\ &= V + \sum_{t=1}^{\lambda} C\Lambda^{t-1}(GWG^T + CVC^T)(\Lambda^T)^{t-1}C^T\\ &=: S\end{aligned} \tag{7}$$

for $t \geq \lambda$. For the initialization it is also assumed that the jump occurs after time $\lambda$, i.e. $t_0 \geq \lambda$.

## 3. GLR TEST

In order to detect and estimate the jump $\nu$ and the jump time $t_0$, we will consider the hypotheses:

$\mathbf{H}_0$: no jump occurred
$\mathbf{H}_1$: jump occurred

at each time $t = \tau$. First we consider the effect of jump $\nu$ for the residual. Let $\varepsilon_0(t)$ denote the no-jump case ($\mathbf{H}_0$, i.e. $\tau \neq t_0$), and denote $\varepsilon_1(t)$ as the jump case ($\mathbf{H}_1$, i.e. $\tau = t_0$). Then we have

$$\varepsilon_1(t) = \begin{cases} \varepsilon_0(t) + \varphi_\tau^T(t)\nu & \text{for } \tau+1 \le t \le \tau+\lambda \\ \varepsilon_0(t) & \text{for } \tau+\lambda+1 \le t \end{cases}$$

where

$$\varphi_\tau^T(t) := C\Lambda^{t-\tau-1}. \quad (8)$$

It follows that the residual does not contain any information about the jump $\nu$ after time $t = \tau+\lambda$. So we will observe the residual over time interval $\tau + 1 \le t \le \tau + \lambda$. Over this time interval, the residual $\varepsilon(t)$ is Gaussian with a zero mean and with variance $S$ if $\mathbf{H}_0$, and is Gaussian with mean $\varphi_\tau^T(t)\nu$ and with variance $S$ if $\mathbf{H}_1$. Namely,

$$p(\varepsilon(t)|\mathbf{H}_1) = \frac{1}{\sqrt{(2\pi)^q \det S}} \exp\left(-R(\nu,\tau;t)\right)$$
$$p(\varepsilon(t)|\mathbf{H}_0) = \frac{1}{\sqrt{(2\pi)^q \det S}} \exp\left(-\tilde{R}(t)\right)$$

where

$$R(\nu,\tau;t) := \frac{1}{2}\left(\varepsilon(t) - \varphi_\tau^T(t)\nu\right)^T S^{-1}\left(\varepsilon(t) - \varphi_\tau^T(t)\nu\right)$$
$$\tilde{R}(t) := \frac{1}{2}\varepsilon^T(t)S^{-1}\varepsilon(t).$$

The LLR between $\mathbf{H}_0$ and $\mathbf{H}_1$ can be written as

$$l(\tau,\nu) := \log \frac{\prod_{t=\tau+1}^{\tau+\lambda} p(\varepsilon(t)|\mathbf{H}_1)}{\prod_{t=\tau+1}^{\tau+\lambda} p(\varepsilon(t)|\mathbf{H}_0)} = \sum_{t=\tau+1}^{\tau+\lambda}\left(\tilde{R}(t) - R(\nu,\tau;t)\right) \quad (9)$$

We estimate the jump $\nu$ and the jump time $t_0$ by maximizing the LLR. First we estimate the jump $\nu$ as a function of the jump time $t_0$. It follows from the observability condition that

$$\sum_{t=\tau+1}^{\tau+\lambda} \varphi_\tau(t)S^{-1}\varphi_\tau^T(t) = \sum_{t=1}^{\lambda}(\Lambda^T)^{t-1}C^T S^{-1}C\Lambda^{t-1} =: U \quad (10)$$

is invertible. Then

$$\frac{\partial l(\tau,\nu)}{\partial \nu} = 0$$

gives the maximum likelihood (ML) estimate of $\nu$ as a function of $\tau$

$$\hat{\nu}(\tau) = U^{-1}\left(\sum_{t=\tau+1}^{\tau+\lambda}\varphi_\tau(t)S^{-1}\varepsilon(t)\right). \quad (11)$$

Maximization w.r.t. $\tau$ gives the GLR-test:

$$\max_\tau l(\tau,\hat{\nu}(\tau)) \underset{\mathbf{H}_0}{\overset{\mathbf{H}_1}{\gtrless}} l_0 \quad (12)$$

where $l_0$ is a user-defined threshold.

In summary, we have the following GLR test:

**Algorithm:(GLR test)** Consider the system given in (1) and (2). Suppose that

**(i)** the system is observable,
**(ii)** one and only one jump $\nu$ occurs after time $\lambda$,

where $\lambda$ is given by (4). Then $\hat{\nu}(\tau)$ defined in (11) gives the ML estimate of the jump $\nu$ at time $\tau$, where $U$, $S$, $\varphi_\tau(t)$, $\varepsilon(t)$ are defined in (10), (7), (8), (6), respectively. Substituting the ML estimate $\hat{\nu}(\tau)$ into the LLR $l(\tau,\nu)$ defined in (9), we have the estimate of the LLR $l(\tau,\hat{\nu}(\tau))$. If the LLR $l(\tau,\hat{\nu}(\tau))$ is larger than user defined threshold, there is a jump. □

**Remark:** Assumption (i) can be reduced for the jump time estimation, since this assumption is required for the pole shifting and the invertibility of $U$ in (10). If the system is detectable and the unobservable poles of $A$ lie in 0, then it is possible to shift the poles of $A$ to 0 and synthesize the deadbeat observer. If $R$ is not invertible, the ML estimate of the jump $\hat{\nu}(\tau)$ can not be determined uniquely, but the LLR $l(\tau,\hat{\nu}(\tau))$ can be computed utilizing the pseudoinverse. Note that the jump which lies in the unobservable space can not be detected without this assumption. Assumption (ii) is necessary for the initialization, while $\lambda$ is less than the McMillan degree of the LTI system and it is not a severe restriction.

**Remark:** In order to detect the sequence of jumps, a reinitialization of the state of the observer has been discussed (Willsky and Jones, 1976). Assuming that a sequence of jumps do not repeat within a small time interval of the size $\lambda$, such reinitialization is not necessary for the proposed method. This is also a merit of the new approach.

## 4. OFFLINE IMPLEMENTATION OF THE MARGINALIZED GLR TEST

Prior information about the variance of the noise may be unrealistic in several applications, as discussed in (Gustafsson, 1996). Consider a scaling $\rho$ for the noise variance as $\rho W, \rho V$, the LLR is inversely proportional to $\rho$ and the choice of the threshold is difficult without the knowledge of $\rho$. We assume that the scaling $\rho$ has the uniform distribution

$$p(\rho;a,b)=\begin{cases}\dfrac{1}{b-a} & \text{if} \quad a<\rho<b\\ 0 & \text{otherwise}\end{cases}. \tag{13}$$

By taking limit for $a \to 0$, $b \to \infty$, one can marginalize the prior knowledge about the scaling $\rho$. We will discuss this marginalized GLR test in offline with window size $N$ larger than $\lambda$. We consider the time interval 1 to $N$ without loss of generality. In addition, we assume the assumption (i), (ii) in the previous GLR test, and assume that $Nq > 2$, which is satisfied if $N > 3$ and is not a severe restriction.

The conditional distributions of the residual $\varepsilon(t)$ becomes

$$p(\varepsilon(t)|\mathbf{H}_1,\rho;a,b)=\begin{cases}\dfrac{1}{\sqrt{(2\pi)^q\det(\rho S)}}\exp\left(-\dfrac{R(\nu,\tau;t)}{\rho}\right) & \text{for } \tau\le t\le\tau+\lambda\\ \dfrac{1}{\sqrt{(2\pi)^q\det(\rho S)}}\exp\left(-\dfrac{\tilde{R}(t)}{\rho}\right) & \text{otherwise}\end{cases}$$

$$p(\varepsilon(t)|\mathbf{H}_0,\rho;a,b)=\frac{1}{\sqrt{(2\pi)^q\det(\rho S)}}\exp\left(-\frac{\tilde{R}(t)}{\rho}\right)$$

Straightforward integration yields

$$\begin{aligned}&\prod_{t=1}^{N}p(\varepsilon(t)|\mathbf{H}_1;a,b)\\ &=\int\prod_{t=\tau+1}^{\tau+\lambda}p(\varepsilon(t)|\mathbf{H}_1,\rho;a,b)p(\rho)d\rho\\ &=\frac{\int_{\frac{S_{N,\rho}(\tau)}{b}}^{\frac{S_{N,\rho}(\tau)}{a}}\rho^{\frac{Nq-2}{2}-1}\exp(-\rho)\,d\rho}{(S_{N,\rho})^{\frac{Nq-2}{2}}(b-a)\left((2\pi)^q\det S\right)^{\frac{Nq}{2}}}\end{aligned}$$

where

$$S_{N,\rho}(\tau):=\sum_{t=1}^{\tau}\tilde{R}(t)+\sum_{t=\tau+1}^{\tau+\lambda}R(\nu,\tau;t)+\sum_{t=\tau+\lambda+1}^{N}\tilde{R}(t). \tag{14}$$

Similarly,

$$\begin{aligned}&\prod_{t=1}^{N}p(\varepsilon(t)|\mathbf{H}_0;a,b)\\ &=\int\prod_{t=\tau+1}^{\tau+\lambda}p(\varepsilon(t)|\mathbf{H}_0,\rho;a,b)p(\rho)d\rho\\ &=\frac{\int_{\frac{\tilde{S}_{N,\rho}}{b}}^{\frac{\tilde{S}_{N,\rho}}{a}}\rho^{\frac{Nq-2}{2}-1}\exp(-\rho)\,d\rho}{(S_{N,\rho})^{\frac{Nq-2}{2}}(b-a)\left((2\pi)^q\det S\right)^{\frac{Nq}{2}}}\end{aligned}$$

where

$$\tilde{S}_{N,\rho}:=\sum_{t=1}^{N}\tilde{R}(t). \tag{15}$$

The LLR between $\mathbf{H}_1$ and $\mathbf{H}_0$ can be written as

$$\begin{aligned}&l_{N,\rho}(\nu,\tau;a,b)\\ &:=\log\frac{\prod_{t=1}^{N}p(\varepsilon(t),\nu|\mathbf{H}_1;a,b)}{\prod_{t=\tau+1}^{\tau+\lambda}p(\varepsilon(t),\nu|\mathbf{H}_0;a,b)}\\ &=-\frac{Nq-2}{2}\left(\log S_{N,\rho}(\tau)-\log\tilde{S}_{N,\rho}\right)\\ &+\log\int_{\frac{S_{N,\rho}(\tau)}{b}}^{\frac{S_{N,\rho}(\tau)}{a}}\rho^{\frac{Nq-2}{2}-1}\exp(-\rho)\,d\rho\\ &-\log\int_{\frac{\tilde{S}_{\tau,\rho}}{b}}^{\frac{\tilde{S}_{\tau,\rho}}{a}}\rho^{\frac{Nq-2}{2}-1}\exp(-\rho)\,d\rho.\end{aligned} \tag{16}$$

In order to marginalize the prior knowledge about the size of the noise variance, we take limit for $a \to 0$, $b \to \infty$. The marginalized LLR is given by

$$\begin{aligned}l_{N,\rho}(\nu,\tau;0,\infty)&=\lim_{a\to0,b\to\infty}l_{N,\rho}(\tau;a,b)\\ &=-\frac{Nq-2}{2}\log\frac{S_{N,\rho}(\tau)}{\tilde{S}_{N,\rho}}.\end{aligned} \tag{17}$$

Then

$$\frac{\partial l_\rho(\nu,\tau;0,\infty)}{\partial\nu}=0$$

gives the ML estimate of the jump $\hat{\nu}(\tau)$ in (11). Substituting the ML estimate $\hat{\nu}(\tau)$ in (11) into the LLR in (17), the GLR test remains unchanged. Since $S_{N,\rho}(\tau)$ is divided by $\tilde{S}_{N,\rho}$ in (17), this GLR test does not require prior information about the size of the noise variance.

Note that a sufficiently small threshold for $\tilde{S}_{N,\rho}$ is necessary to avoid $\tilde{S}_{N,\rho} \approx 0$, which denotes "$\tilde{S}_{N,\rho}$ is close to 0" for notational simplicity. If $\tilde{S}_{N,\rho} \approx 0$, it is obvious that there is no jump. So this threshold must be defined by the numerical accuracy and not by the noise level.

Next we discuss the necessity to formulate the marginalized GLR test in offline. Assume it is formulated in online, i.e. we choose the window as the time interval $\tau+1$ to $\tau+\lambda$ for each $\tau$. Then the marginalized LLR which corresponds to (17) can be written as

$$l_{\lambda,\rho}(\nu,\tau;0,\infty)=-\frac{\lambda q-2}{2}\log\frac{\sum_{t=\tau+1}^{\tau+\lambda}R(\nu,\tau;t)}{\sum_{t=\tau+1}^{\tau+\lambda}\tilde{R}(t)}.$$

Since the jump estimate $\hat{\nu}(\tau)$ is such that $\varphi_\tau^T(t)\hat{\nu}(\tau)$ approximates $\varepsilon(t)$ over the interval $\tau+1 \le t \le \tau+\lambda$, it follow that $\sum_{t=\tau}^{\tau+\lambda} R(\hat{\nu}(t),\tau;t) \approx 0$, which yields $l_{\lambda,\rho}(\hat{\nu}(t),\tau;0,\infty) \approx \infty$ for all $\tau$. This would lead to false alarms by mistake and is crucial.

On the contrary, it can be shown that there is no such a drawback in offline. Since $\varphi_\tau^T(t)\hat{\nu}(\tau)$ can not approximate the residual $\varepsilon(t)$ outside the interval $\tau+1 \le t \le \tau+\lambda$, it follows that $S_{N,\rho}(\tau) \approx \tilde{S}_{N,\rho}$, and therefore $l_{N,\rho}(\hat{\nu}(\tau),\tau;0,\infty) \approx 0$ for $\tau \neq t_0$. Hence we can avoid unnecessary false alarms. In addition, it can be shown that the marginalized LLR test can detect a large jump. If the jump $\nu$ is large, $\sum_{t=t_0+1}^{t_0+\lambda} \tilde{R}(t) \approx \tilde{S}_{N,\rho}$ holds, while $\sum_{t=t_0+1}^{t_0+\lambda} R(\hat{\nu}(t_0), t_0; t) \approx 0$ holds and it follows that $S_{N,\rho}(t_0) \approx 0$. The marginalized LLR $l_{N,\rho}(\hat{\nu}(t_0), t_0; 0, \infty)$ takes a significantly large value and we receive an alarm. But, if the jump $\nu$ is not large, $S_{N,\rho}(t_0) \approx \tilde{S}_{N,\rho}$. Since $l_{N,\rho}(\hat{\nu}(t_0), t_0; 0, \infty) \approx 0$, we do not receive.

In this section, the marginalized GLR test has been discussed in offline. It is obvious that, if the window size $N$ is larger than $\lambda$, we can test in online. The choice of the window size $N$ is the key problem. If we fix the jump estimation window, the choice of its position, which correspond to the choice of $\tau$ over the time interval $1 \le t \le N$ in (14), is also the problem.

## 5. NUMERICAL EXAMPLE

Consider a system with

$$A = \begin{bmatrix} 0.5 & 1 \\ 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \quad (18)$$

and with the noise variance

$$W = 0.01, \quad V = 0.01. \quad (19)$$

Choose the observer gain as

$$H = \begin{bmatrix} 1.5 \\ 1 \end{bmatrix},$$

then the system matrix of the error dynamics becomes

$$\Lambda = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$

which has a multiple eigenvalue at 0.

First we demonstrate the online GLR test discussed in Section 3 by the following three jumps

$$\nu_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad \nu_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \nu_3 = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

at time

$$t_0 = 50.$$

The state jumps $\nu_1$, $\nu_2$, $\nu_3$ can be interpreted as actuator offsets, see (Caglayan, 1980; Gustafsson, 2000), and are important in practice.

The problem is to estimate the jump $\nu$ and the jump time $t_0$. For each jump $\nu_1$, $\nu_2$, $\nu_3$, Figure 1–3 show the state of the system (upper left), the output of the system (upper right), the state estimation by the Kalman filter (middle left), the state estimation by the deadbeat observer (middle right), the residual generated by the dead beat observer (bottom left), the GLR by the deadbeat observer (bottom right), respectively. First we compare the transitions of the state estimations. The second state of the system given by (18) is not directly affected by the noise and Kalman filter does not track the state quickly, while the dead observer does. Since we estimate the jump and jump time by the transition of the residual, this comparison motivates the deadbeat observer as a substitute for the Kalman filter. Next we estimate the jump and the jump time. The jump $\nu_1$ is the same level as noise, so there is no significance of the LLR at time $t_0 = 50$ (see Figure 1). For the jump $\nu_2$, the GLR test gives accurate estimate for the jump time

$$\arg\max_{\tau} l(\tau, \hat{\nu}_2(\tau)) = 50,$$

while does not give the accurate estimate for the jump

$$\hat{\nu}_2(50) = \begin{bmatrix} 0.5749 \\ 1.0078 \end{bmatrix}$$

(see Figure 2). The jump $\nu_3$ is significantly larger than the noise level, so the GLR test gives the accurate estimate for the jump time

$$\arg\max_{\tau} l(\tau, \hat{\nu}_3(\tau)) = 50,$$

and gives the more accurate estimate for the jump

$$\hat{\nu}_3(50) = \begin{bmatrix} 0.2679 \\ 9.9844 \end{bmatrix}$$

as expected (see Figure 3).

Then we compare the GLR test discussed in Section 3 and the marginalized GLR test discussed in Section 4. Assuming that true noise variance are given in (19) and that the jump $\nu_2$ occurs at time $t_0 = 50$, we compare the LLR with unknown scaling

$$\rho = 10 \quad \text{or} \quad \rho = 1\,(\text{true}) \quad \text{or} \quad \rho = 0.1$$

The LLR in Section 3 is inversely proportional to the unknown scaling $\rho$ (see upper left, upper right, lower left part of Figure 4 for $\rho = 10$, $\rho = 1$, $\rho = 0.1$ respectively), and it follows that the choice of the threshold is difficult without the knowledge of the size of the noise variance. On the other hand, the LLR in Section 4 takes the same value for all unknown scaling (see lower right part of Figure 4).

Finally the performance of the marginalized GLR test are illustrated for the jump $\nu_1$, $\nu_2$, $\nu_3$. Similar to the GLR test, there is no significance for $\nu_1$ at time $t_0 = 50$, while there is for $\nu_2$, $\nu_3$.

## 6. CONCLUSION

The deadbeat observer based GLR test has been studied for the detection and estimation of a jump in LTI systems. Since the proposed method only requires a small window size of at most the McMillan degree of the system, it is suitable for the online detection of the relatively large jump. Assuming the noninformative prior information for the size of the noise variance, the marginalized GLR test has been also discussed in offline to overcome the difficulty in the choice of the threshold.

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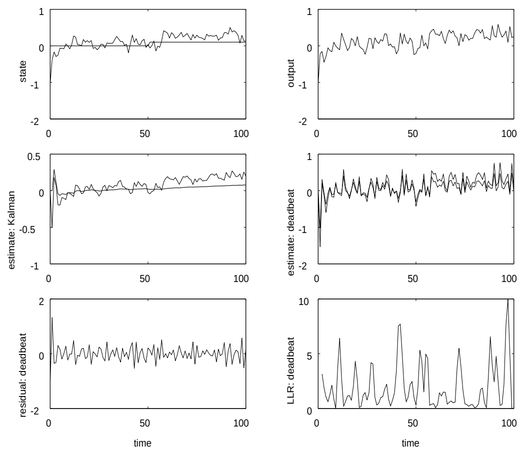


Fig. 1. GLR test for the jump $\nu_1$ at time $t_0 = 50$

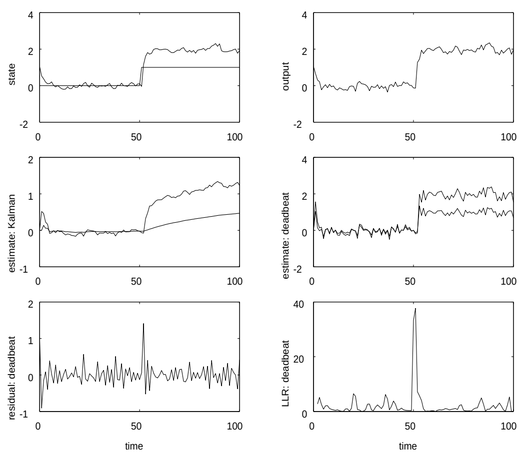


Fig. 2. GLR test for the jump $\nu_2$ at time $t_0 = 50$

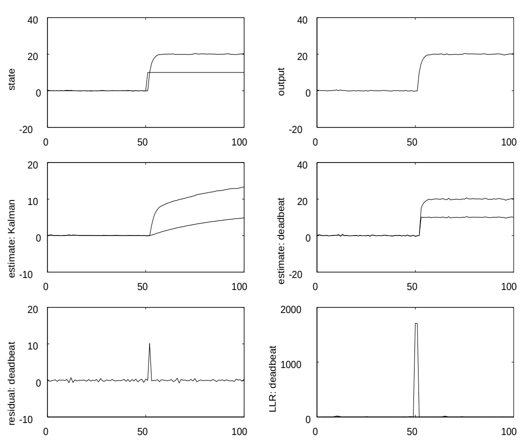


Fig. 3. GLR test for the jump $\nu_3$ at time $t_0 = 50$

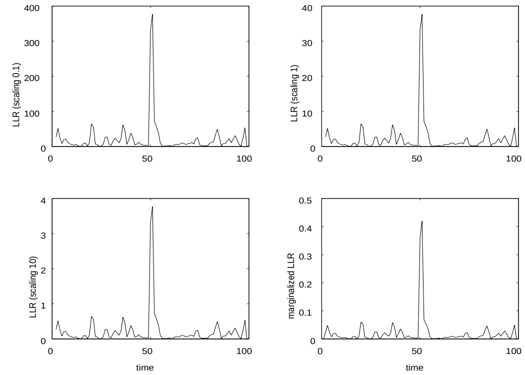


Fig. 4. GLR test for the jump $\nu_2$ with unknown scaling $\rho = 0.1$ (upper left), $\rho = 1$ (upper right), $\rho = 10$ (lower left), marginalized GLR test for the jump $\nu_2$ (lower right)

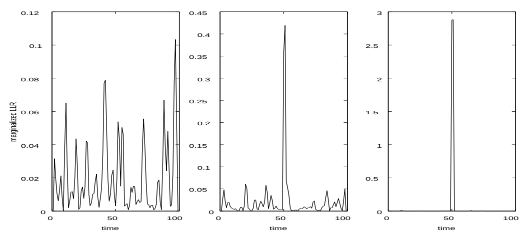


Fig. 5. marginalized GLR test for the jump $\nu_1$(left), $\nu_2$(center), $\nu_3$(right)

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