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ROBUST CONTROL OF HAMILTON SYSTEM AND APPLICATION TO POWER SYSTEM[1]

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Abstract: The paper investigates the robust H_∞ problem for a class of generalized forced Hamilton system with uncertainty. We begin with presenting a design approach of robust H_∞ controller and show that the L_2 gain from the disturbance input to the regulation output signal may be reduced to any given level provided that a kind of algebraic inequality has a solution. Then, by means of the proposed method, a Hamiltonian systems-like model with uncertainties is firstly presented, which can describe the power system dynamics on a full scale, and consequently a decentralized nonlinear robust H_∞ control law is achieved by construction of a Hamiltonian function for the multimachine power system. Simulations performed on a 6-machine system verified that the proposed excitation control could adapt to the conditions under large disturbance and enhance greatly the transient stability of power system compared to other types of controllers. *Copyright © 2002 IFAC*

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1. INTRODUCTION

Recently, many researchers studied a variety of problems for port-controlled Hamiltonian (PCH) system having energy dissipation and energy exchange with the environment (R. Ortega, 1998a, 1998b; B.M.J. Maschke, 1998, 1999; Fujimoto, 2000; D. Cheng, 2000, 1999; T. Shen, 2000; Z. Xi, 2001). In fact, the Hamilton function in PCH system is the total energy including potential and kinetic energy in the physical systems, and can play the role of Lyapunov function for the system. From (B.M.J. Maschke, 1999; D. Cheng, 2000, 1999; R. Ortega, 1998b), it could be seen that the Hamiltonian function approach has some advantages in the design of control law. On the other hand, as well-known, every practical engineering system in real operation is influenced by uncertainty, including unmodelled dynamics, measuring and parameter error, etc., and it is difficult to formulate a precise mathematical model to describe the system dynamics completely. Hence, the uncertainty has to be taken into account when we deal with the engineering system using Hamilton theory.

The modern power systems are characterized by strong coupling and high nonlinearity with the development of large capacity units, high voltage and long distance AC-DC transmission and the application of superconductive storage facilities, which are affected by the exogenous disturbances in operation, such as short-circuit and load-shielding. Design of effective decentralized robust controller with disturbance attenuation is an important objective in the field of control of modern power system. Recently many researchers investigated the problem and achieved many new results (Q. Lu, 1989, 1996, 2000ab; S. Mei, 1999; Y. Wang, 1997, 1998). Papers (Q. Lu, 1989, 2000a; S. Mei, 1999) proposed a decentralized nonlinear excitation control law with differential geometric methods and nonlinear H_∞ control theory, and the strict verification of its optimal and robust properties are given from a mathematical point of view; however, the physical meaning is not clear for its index of the nonlinear optimal and robust properties. Papers (Y. Wang, 1997, 1998) presented an excitation controller for multimachine system based on modern robust nonlinear control approaches, e.g. direct feedback nonlinear compensation, while it loses the practicality in engineering for the ignorance of the impact of loads in the network.

In paper (T. Shen, 2000), the authors investigated the adaptive L_2 disturbance attenuation control for Hamilton system with parametric uncertainties and applied the proposed method to power system. In paper (Z. Xi, 2001), considering the external uncertainties such as disturbance, the authors dealt with the H_∞ problem of the generalized forced Hamilton system, and present a sufficient condition which only requires the solution of a set of algebraic matrix equations, so the formidable difficulty is avoided when seeking the solution of HJI partial differential inequality in general nonlinear H_∞ control problem, and this result has applied successfully to the simple power system. However, in all these Hamilton system approaches mentioned above, there is a key problem that is to construct the Hamilton function of the system so as to transform the studied system into the form of a forced Hamilton system, although papers (B.M.J. Maschke, 1998, 1999) propose a method to realize the transformation by constructed source system and pre-set constant control. In fact, the construction of a Hamilton function is similar to that of a Lyapunov function, in which no universal law can be used, so the research results are limited to the systems in some specific forms.

As to a class of generalized forced Hamilton system with the unmodeling dynamic uncertainty, this paper proposes and studies the nonlinear robust H_∞ control problem, which includes the L_2-gain analysis, the construction of H_∞ controller, etc, and presents a sufficient condition solving the proposed problem. The study investigates a new field of the control of the generalized forced Hamilton system with uncertainty; while a new approach is initiated to transform the general nonlinear system into Hamilton system structure utilizing the thoughts of uncertainty modeling, namely, the unforced system is firstly separated into two parts, the one is easy to find out the Hamilton function, and the other one is incorporated into the uncertain perturbation function set, then the robust control methods are exploited to deal with the uncertainty (T. Shen, 1995), and finally the nonlinear robust H_∞ controller is achieved. As the application of the method above, the paper deduces the decentralized nonlinear robust control law for multimachine excitation system, and the simulation result performed on a 6-machine system demonstrates that the proposed control law enhances the transient stability of the power systems enormously and possesses superior robustness compared to that of other types of controllers.

2. ROBUST H_∞ CONTROL FOR HAMILTON SYSTEM WITH UNCERTAINTY

Considering the generalized forced Hamilton system with uncertainty as follows

$$\dot{x} = [J(x) - R(x)]\frac{\partial H}{\partial x} + \Delta f(x) + g_1(x)w + g_2(x)u \quad (1)$$
$$z = g_1^T(x)\frac{\partial H}{\partial x} + D(x)u$$

where $x^T = [x_1, x_2, \cdots, x_n] \in R^n$, $w \in R^m$ and $u \in R^s$ are the state, the disturbance and the control input, respectively. $z \in R^q$ is the regulation output, $g(x)$ is a smooth function matrix with appropriate dimention; $J(x)$ is a skew-symmetric matrix, $R(x)$ is a positive semi-definite matrix, $H: R^n \to R$ is the Hamilton function and $\frac{\partial H}{\partial x} = [\frac{\partial H}{\partial x_1}, \cdots, \frac{\partial H}{\partial x_n}]^T$. $\Delta f(x)$ represents the uncertain function, which derives from the modeling error or the parameter perturbation in the plant.

Suppose $f(0)=0$ and the uncertain function $\Delta f(x)$ can be decomposed as

$$\Delta f(x) = e_f(x)\delta_f(x) \quad (2)$$

where $e_f(x)$ and $\delta_f(x)$ are the known and unknown functions with sufficient differentiability, respectively. Also, we suppose that the uncertainty $\Delta f(x)$ belongs to the set defined by

$$\Omega_f = \{\Delta f(x) |\ \delta_f(0) = 0,\ \|\delta_f(x)\| \le \left\|\frac{\partial H}{\partial x}\right\| m_f(x), \quad \forall x \in R^n\} \quad (3)$$

where $m_f(x)$ is a given bounded function satisfying $m_f(x) > 0, \quad \forall x \neq 0$ and $m_f(x) = 0$.

Definition 2.1 The robust H_∞ control problem for system (1) is, for a given $\gamma > 0$, to construct a state feedback controller $u = U(x)$, such that the following performance is satisfied for any $\Delta f(x) \in \Omega_f$:

(i) For any given $T > 0$,

$$\int_0^T \|z\|^2 dt \le \gamma^2 \int_0^T \|z\|^2 dt + V(x_0), \quad \forall w \in L_2[0,T];$$

(ii) When $w = 0$, the corresponding closed-loop system is asymptotically stable at $x = 0$.

where x_0 is initial value of the state $x(t)$, $V(x)$ is actually the system storage function to be constructed. The controller $U(x)$ is called as robust H_∞ controller for system (1).

Definition 2.2 System (1) is said to be robust zero-state detectable if $z(x(t)) \equiv 0$ implies $x(t) \equiv 0$ for any $\Delta f(x) \in \Omega_F$ when $w(t) \equiv 0$ and $u(t) \equiv 0$.

Theorem 2.1 Suppose system (1) satisfies the following conditions

(A1) System (1) is robust zero-state detectable.

(A2) $\frac{\partial^T H}{\partial x} g_1(x)D(x) \equiv 0,$ and $rank[D(x)] = s$, $\forall x \in R^n$.

(A3) Hamilton function $H(x)$ has strict local minimum at $x = 0$.

If there exist a positive number α and some suitable scalar function $\lambda(x) > 0,\ \forall x \in R^n$ such that

$$-\alpha R(x) + \frac{\alpha^2}{2\gamma^2}[g_1(x)g_1^T(x) + \lambda^2(x)e_f(x)e_f^T(x) + \frac{1}{2}\{g_1(x)g_1^T(x)$$
$$-\alpha^2 g_2(x)[D^T(x)D(x)]^{-1}g_2^T(x)] + \frac{\gamma^2 k(x)}{2\lambda^2(x)} m_f^T(x)m_f(x)I_n\} \le 0 \quad (4)$$

then the robust H_∞ state feedback controller for system (1) is given by

$$u = U(x) = -\alpha[D^T(x)D(x)]^{-1}g_2^T(x)\frac{\partial H}{\partial x} \quad (5)$$

where I_n is the identity matrix.

Proof. Let $V(x) = \alpha H(x)$, according to the inequality (4), we have

$$\frac{\partial^T H}{\partial x}\Big\{-\alpha R(x) + \frac{\alpha^2}{2\gamma^2}[g_1(x)g_1^T(x) + \lambda^2(x)e_f(x)e_f^T(x)] + \frac{1}{2}[g_1(x)g_1^T(x) \quad (6)$$
$$-\alpha^2 g_2(x)[D^T(x)D(x)]^{-1}g_2^T(x)] + \frac{\gamma^2 k(x)}{2\lambda^2(x)} m_f^T(x)m_f(x)I_n\Big\}\frac{\partial H}{\partial x} \le 0$$

Thus we obtain

$$\frac{\partial^T V}{\partial x}[(J(x) - R(x))\frac{\partial H}{\partial x} + g_2(x)U(x) + \Delta f(x)] + \frac{\partial^T V}{\partial x} g_1(x)w$$
$$\le -\frac{1}{2}\|z\|^2 - \frac{1}{2\gamma^2}\frac{\partial^T V}{\partial x} g_1(x)g_1^T(x)\frac{\partial V}{\partial x} + \frac{\partial^T V}{\partial x} g_1(x)w - \frac{\gamma^2}{2}\|w\|^2 + \frac{\gamma^2}{2}\|w\|^2 \quad (7)$$
$$= \frac{1}{2}(\gamma^2\|w\|^2 - \|z\|^2) - \frac{\gamma^2}{2}\left\|w - \frac{1}{\gamma^2} g_1(x)\frac{\partial V}{\partial x}\right\|^2$$

This implies the dissipative inequality

$$\dot{V} \le \frac{1}{2}\{\gamma^2\|w\|^2 - \|z\|^2\}, \quad \forall w \quad (8)$$

holds along any trajectory of the closed loop system. Hence, for any given $T > 0$, we have

$$\int_0^T \|z\|^2 dt \le \gamma^2 \int_0^T \|w\|^2 dt + V(x(0)), \qquad \forall \Delta f(x) \in \Omega_f \qquad (9)$$

When w=0, for the free system

$$\dot{x} = (J(x) - R(x))\frac{\partial H}{\partial x} + g_2(x)U(x) + \Delta f(x) \qquad (10)$$

we have from the dissipative inequality (8) that

$$\dot{V} \le -\frac{1}{2}\|z\|^2 = -\frac{1}{2}\left\|g_1^T(x)\frac{\partial H}{\partial x}\right\|^2 - \frac{1}{2}\|D(x)U(x)\|^2 \qquad (11)$$

Therefore, if there exists a solution $x(t)$ whose trajectory satisfying $\frac{dV}{dt} = 0$, then from (11) we have

$$z \equiv 0, \quad g_1^T(x)\frac{\partial H}{\partial x} \equiv 0, \quad U(x) \equiv 0 \qquad (12)$$

Hence, by the robust zero-state detectability condition (A1), the asymptotically stability of (10) for any $\Delta f \in \Omega_f$ follows from Lasalle Invariant Theorem (J. J. E. Slotine, 1991).

3. DECENTRALIZED ROBUST H_∞ CONTROL FOR MULTI-MACHINE POWER SYSTEM

In this section, we study the multimachine power system using the proposed approach, and the following two points are completely original: on one hand, the nonlinear dynamical model with uncertainties is formulated to describe the power system dynamics on a full scale; and on the other hand, the decentralized control is realized by constructing Hamilton function. In fact, the constructed Hamilton function is similar to the system energy function in the sense of Lyapunov analysis in the process of transforming the multimachine power system into a forced Hamilton system. So the decentralized nonlinear robust controller designed based on the structure of Hamilton system has clear physical illustration and is applicable to the engineering field.

Consider the following n-generator disturbed power system

$$\begin{cases} \dot{\delta}_i = \omega_i - \omega_0 \\ \dot{\omega}_i = \dfrac{\omega_0}{M_i}P_{mi} - \dfrac{D_i}{M_i}(\omega_i - \omega_0) - \dfrac{\omega_0}{M_i}P_{ei} + \dfrac{\omega_0}{M_i}P_{dis} \\ \dot{E}_{qi} = \dfrac{1}{T_{doi}}[-E_{qi} + V_{fi} + V_{dis}] \qquad i = 1,\cdots,n \end{cases} \qquad (13)$$

where the subscription i stands for the i-th generator, $P_{ei} = G_{ii}E'^2_{qi} + E'_{qi}\sum_{j=1, j\ne i}^{n} B_{ij}E'_{qj}\sin(\delta_i - \delta_j)$ is the active power, $E_{qi} = E'_{qi} + I_{di}(x_{di} - x'_{di})$ is the load-free voltage, and $I_{di} = -B_{ii}E'_{qi} - \sum_{j=1, j\ne i}^{n} B_{ij}E'_{qj}\cos(\delta_i - \delta_j)$ is the d-axis current. δ_i, ω_i and E'_{qi} denote the rotor angle (in radian), the rotor speed (in rad/sec) and the internal transient voltage (in per unit), respectively. V_{fi} is voltage of the field circuit, the control variable in per unit. M_i is the inertia coefficient of a generator set, in seconds; D_i is damping constant, in per unit; T_{doi} is field circuit time constant, in second; B_{ii} is the self-admittance of the i-th node, B_{ij} is the mutual-admittance between the i-th and j-th nodes, and G_{ii} is the self-conductance of the i-th node, in per unit respectively; x_{di} and x'_{di} are d-axis synchronous reactance and transient reactance of a generator respectively, in per unit; P_{mi} is mechanical power, in per unit; P_{dis} identifies torque disturbance acting on rotating shaft of the generator set, and V_{dis} denotes the electromagnetism disturbance entering the excitation winding .

Consider a desired equilibrium $(\delta_1^s, \omega_0, E'^s_{q1}; \cdots; \delta_i^s, \omega_0, E'^s_{qi}; \cdots; \delta_n^s, \omega_0, E'^s_{qn})$ of the studied system (13), and set

$$\begin{cases} x_{i1} = \delta_i - \delta_i^s \\ x_{i2} = \omega_i - \omega_0 \\ x_{i3} = E'_{qi}(t) - E'^s_{qi} \qquad i = 1,\cdots,n \end{cases} \qquad (14)$$

Denote

$$\begin{aligned} V_{fi}^s &= (1/T_{d0i})(E'^s_{qi} + (x_{di} - x'_{di})I_{di}^s) \\ I_{di}^s &= -B_{ii}E'^s_{qi} - \sum_{j=1, j\ne i}^{n} B_{ij}E'^s_{qi}\cos(\delta_i^s - \delta_j^s) \end{aligned} \qquad (15)$$

Assuming

$$P_{mi} = P_{ei0} = G_{ii}E'^{s2}_{qi} + E'^s_{qi}\sum_{j=1, j\ne i}^{n} B_{ij}E'^s_{qj}\sin(\delta_i^s - \delta_j^s), \ 1 \le i \le n \qquad (16)$$

we have

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = -\dfrac{D_i}{M_i}x_{i2} + \dfrac{\omega_0}{M_i}\Big\{G_{ii}E'^{s2}_{qi} - G_{ii}E'^2_{qi} + \sum_{j=1, j\ne i}^{n} B_{ij}[E'^s_{qi}E'^s_{qj}\sin(\delta_i^s - \delta_j^s) \\ \qquad - E'_{qi}E'_{qj}\sin(\delta_i - \delta_j)]\Big\} + \dfrac{\omega_0}{M_i}P_{dis} \qquad (i = 1,\cdots,n) \\ \dot{x}_{i3} = \dfrac{-1}{T_{doi}}x_{i3} - \dfrac{x_{di} - x'_{di}}{T_{doi}}\Big\{-B_{ii}x_{i3} + \sum_{j=1, j\ne i}^{n} B_{ij}[E'^s_{qj}\cos(\delta_i^s - \delta_j^s) \\ \qquad - E'_{qj}\cos(\delta_i - \delta_j)]\Big\} + \dfrac{1}{T_{doi}}(V_{fi} - V_{fi}^s) + \dfrac{1}{T_{doi}}V_{dis} \end{cases} \qquad (17)$$

Let $\sigma_{ij} = (\delta_i - \delta_j) - (\delta_i^s - \delta_j^s), \ \ 1 \le i, j \le n$, and construct the Hamilton function for the n-generator power system as

$$\begin{aligned} H(x) &= \sum_{i=1}^{n}\frac{1}{2}\frac{M_i}{\omega_0}x_{i2}^2 + \sum_{i=1}^{n}\frac{1}{2}\frac{1 - B_{ii}(x_{di} - x'_{di})}{(x_{di} - x'_{di})}x_{i3}^2 \\ &+ \frac{1}{2}\sum_{i=1}^{n}\int_0^{\sigma_{ij}}\sum_{j=1, j\ne i}^{n} B_{ij}(E'_{qi}E'_{qj}\sin(\delta_i - \delta_j) - E'^s_{qi}E'^s_{qj}\sin(\delta_i^s - \delta_j^s))d\sigma_{ij} \\ &= \sum_{i=1}^{n}\frac{1}{2}\frac{M_i}{\omega_0}x_{i2}^2 + \sum_{i=1}^{n}\frac{1}{2}\frac{1 - B_{ii}(x_{di} - x'_{di})}{(x_{di} - x'_{di})}x_{i3}^2 \\ &+ \sum_{i=1}^{n}\sum_{j=i+1}^{n} B_{ij}(E'_{qi}E'_{qj}(\cos(\delta_i^s - \delta_j^s) - \cos(\delta_i - \delta_j)) - E'^s_{qi}E'^s_{qj}\sigma_{ij}\sin(\delta_i^s - \delta_j^s)) \end{aligned}$$

where $x = (x_{11}, x_{12}, x_{13}; \cdots; x_{i1}, x_{i2}, x_{i3}; \cdots; x_{n1}, x_{n2}, x_{n3})$.

In the following, we denote the equilibrium point of system (17) by $x^s = (0,\ 0,\ 0;\ \cdots;\ 0, 0, 0;\ \cdots; 0,\ 0,\ 0)$ .In particular, let $w_i = [P_{dis}, V_{dis}]^T$, $v_i = V_{fi} - V_{fi}^s$, then system (17) can be rewritten as

$$\begin{bmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{\omega_0}{M_i} & 0 \\ -\dfrac{\omega_0}{M_i} & -\dfrac{D_i\omega_0}{M_i^2} & 0 \\ 0 & 0 & -\dfrac{x_{di} - x'_{di}}{T_{d0i}} \end{bmatrix} \begin{bmatrix} \dfrac{\partial H(x)}{\partial x_{i1}} \\ \dfrac{\partial H(x)}{\partial x_{i2}} \\ \dfrac{\partial H(x)}{\partial x_{i3}} \end{bmatrix}$$

$$+\begin{bmatrix} 0 \\ (\omega_0/M_i)G_{ii}(E'^{s2}_{qi}-E'^{2}_{qi}) \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ 1/T_{d0i} \end{bmatrix}v_i+\begin{bmatrix} 0 & 0 \\ \omega_0/M_i & 0 \\ 0 & 1/T_{d0i} \end{bmatrix}w_i \quad (18)$$

where v_i is a new control input.

Select the regulation output as

$$z_{i1}=\begin{bmatrix} \dfrac{M_i}{\omega_0}x_{i2} \\ \sum_{j=1,j\neq i}^{n}B_{ij}[E'_{qj}(\cos(\delta_i^s-\delta_j^s)-E'_{qj}\cos(\delta_i-\delta_j)]+\dfrac{1-B_{ii}(x_{di}-x'_{di})}{x_{di}-x'_{di}}x_{i3} \end{bmatrix}$$
$$=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\frac{\partial H}{\partial x_i}$$
$$z_{i2}=v_i \quad (19)$$

therefore

$$z_i=\begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\frac{\partial H}{\partial x_i}+\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}v_i \quad (20)$$

where $x_i=[x_{i1},x_{i2},x_{i3}]^T, i=1,\cdots,n$.

Theorem 3.1 Considering the multimachine system composed of (18)-(20), for any given $\gamma>0$, the robust H_∞ control problem of the system can be solved by the following control law

$$v_i=-\alpha_i(I_{di}-I_{di}^s+\frac{1}{x_{di}-x'_{di}}x_{i3}) \quad (21)$$

where $\alpha_i>0$ is the control gain, which satisfying the following inequalities

$$\begin{cases} -a_i c_i \alpha_i+\dfrac{1}{2}+\dfrac{\alpha_i^2}{2\gamma^2}(1+\lambda^2(x))+\dfrac{\gamma^2}{2\lambda^2(x)}m_f^T(x)m_f(x)\le 0 \\ -d_i\alpha_i+\dfrac{1}{2}+\dfrac{\alpha_i^2}{2\gamma^2}(1-\gamma^2)+\dfrac{\gamma^2}{2\lambda^2(x)}m_f^T(x)m_f(x)\le 0 \end{cases} \quad (22)$$

where $a_i=D_i/2M_i$, $c_i=\omega_0/2M_i$, $d_i=(x_{di}-x'_{di})/T'_{d0i}$.

Proof. Firstly, considering the uncertainties in the power system model, we have

$$\Delta f=diag(\Delta f_1,\cdots,\Delta f_n) \quad (23)$$

where

$$\Delta f_i=\begin{bmatrix} 0 \\ \dfrac{\omega_0}{M_i}G_{ii}(E'^{s2}_{qi}-E'^{2}_{qi}) \\ 0 \end{bmatrix}=\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\frac{\omega_0}{M_i}G_{ii}(E'^{s2}_{qi}-E'^{2}_{qi})=e_{f_i}\delta_{f_i}$$
$$i=1,\cdots,n$$

Hence, define e_f and δ_f by

$$e_f=diag\{e_{f_1},e_{f_2},\cdots,e_{f_n}\}=diag\{\frac{\omega_0}{M_1}G_{11},\frac{\omega_0}{M_2}G_{22},\cdots,\frac{\omega_0}{M_n}G_{nn}\}$$
$$\delta_f=\begin{bmatrix} \delta_{f_1} \\ \delta_{f_2} \\ \vdots \\ \delta_{f_n} \end{bmatrix}=\begin{bmatrix} E'^{s2}_{q1}-E'^{2}_{q1} \\ E'^{s2}_{q2}-E'^{2}_{q2} \\ \vdots \\ E'^{s2}_{qn}-E'^{2}_{qn} \end{bmatrix} \quad (24)$$

Then $\Delta f(x)$ can be represented as (2). Since $\delta_{f_i}(E'^s_{qi})=0$, we can determine an adequately large area $M\subset R^{3n}$ and a function $m_f(x)$ such that

$$\|\delta_{f_i}(x)\|\le\left\|\frac{\partial H}{\partial x_i}\right\|m_f^2(x), \quad \forall x\in M \quad (25)$$

holds. Thus the uncertainty set Ω_f can be determined by (3). Secondly, from Hamilton function $H(x)$, we can obtain its Hessian matrix as

$$H_{Hess}(x)=diag(H_1(x)\cdots H_i(x)\cdots H_n(x)) \quad (26)$$

whose every sub-matrix $H_i(x)$ is

$$H_i(x)=\begin{bmatrix} E'_i\sum_{j=1,j\neq i}^{n}B_{ij}E'_j\cos(\delta_i-\delta_j) & 0 & \sum_{j=1,j\neq i}^{n}B_{ij}E'_j\sin(\delta_i-\delta_j) \\ 0 & \dfrac{M_i}{\omega_0} & 0 \\ \sum_{j=1,j\neq i}^{n}B_{ij}E'_j\sin(\delta_i-\delta_j) & 0 & \dfrac{1-B_{ii}(x_{di}-x'_{di})}{(x_{di}-x'_{di})} \end{bmatrix} \quad (27)$$

At the point, from power system dynamics so based on the following facts

(1) $B_{ii}<0,\ B_{ij}>0$ for $i\neq j$.

(2) $x_{di}>x'_{di}$.

(3) There exists $\delta_0>0$, s.t. when $\left|\delta_i^s-\delta_j^s\right|<\delta_0$,

$$E'^s_i\sum_{j=1,j\neq i}^{n}B_{ij}E'^s_j\cos(\delta_i^s-\delta_j^s)\frac{1-B_{ii}(x_{di}-x'_{di})}{(x_{di}-x'_{di})}-(\sum_{j=1,j\neq i}^{n}B_{ij}E'^s_j\sin(\delta_i^s-\delta_j^s))^2>0$$

So the matrix

$$H_i(\delta_i^s,\omega_i^s,E'^s_i)=\begin{bmatrix} E'^s_i\sum_{j=1,j\neq i}^{n}B_{ij}E'^s_j\cos(\delta_i^s-\delta_j^s) & 0 & \sum_{j=1,j\neq i}^{n}B_{ij}E'^s_j\sin(\delta_i^s-\delta_j^s) \\ 0 & \dfrac{M_i}{\omega_0} & 0 \\ \sum_{j=1,j\neq i}^{n}B_{ij}E'^s_j\sin(\delta_i^s-\delta_j^s) & 0 & \dfrac{1-B_{ii}(x_{di}-x'_{di})}{(x_{di}-x'_{di})} \end{bmatrix}$$

is positive-definite, which implies $H(x^s)$ is also a positive-definite, i.e., $H(x)$ has strict minimum at x^s.

Now, using Theorem 2.1, we construct the excitation control law for every generator as follows

$$v_i=-\alpha_i[D_i^T(x)D_i(x)]^{-1}g_{2i}^T\frac{\partial H_c}{\partial x_i}$$
$$=-\alpha_i\left\{\sum_{j=1,j\neq i}^{n}B_{ij}(E'_{qj}(\cos(\delta_i^s-\delta_j^s)-\cos(\delta_i-\delta_j)+\frac{1-B_{ii}(x_{di}-x'_{di})}{x_{di}-x'_{di}}x_{i3}\right\} \quad (28)$$
$$=-\alpha_i\left\{I_{di}-I_{di}^s+\frac{1}{x_{di}-x'_{di}}x_{i3}\right\}$$

From Theorem 2.1, the control law (28) can make the L_2 gain from the disturbance w_i to regulation output z_i less than or equal to the given γ. As for the stability of the closed-loop system when $w_i=0$, from some manipulation, we have

$$\frac{dH(x)}{dt}\le-\frac{1}{2\gamma^2}\sum_{i=1}^{n}x_{i2}^2-\frac{1}{2}\sum_{i=1}^{n}\alpha_i^2\left\{I_{di}-I_{di}^s+\frac{1}{x_{di}-x'_{di}}x_{i3}\right\}$$
$$-\frac{1}{2T_{d0i}^2}(\frac{1}{\gamma^2}+1)\sum_{i=1}^{n}\left\{\frac{1}{2}\frac{1-B_{ii}(x_{di}-x'_{di})}{x_{di}-x'_{di}}x_{i3}+\sum_{j=1,j\neq i}^{n}B_{ij}E'_{qj}[\cos(\delta_i^s-\delta_j^s)-\cos(\delta_i-\delta_j)]\right\}^2$$
$$\le 0$$

So the trajectory of the closed –loop system would approach to the largest invariant set which is a subset of the following set

$$\Xi=\left\{x\ \middle|\ x_{i2}=0;\ \ I_{di}-I_{di}^s+\frac{1}{x_{di}-x'_{di}}x_{i3}=0;\ \ \frac{1}{2}\frac{1-B_{ii}(x_{di}-x'_{di})}{x_{di}-x'_{di}}x_{i3}\right.$$
$$\left.+\sum_{j=1,j\neq i}^{n}B_{ij}E'_{qj}(\cos(\delta_i^s-\delta_j^s)-\cos(\delta_i-\delta_j))=0;\quad i=1,\cdots,n.\right\}$$

It can be seen that the largest invariant subset of Ξ is just the equilibrium points of system (18)-(20), hence, according to LaSalle Invariant Theorem (Slotine J J E and Li W, 1991), the closed-loop system is asymptotically stable. Thus we could conclude that the control law (28) is a robust H_∞ one for the system.

Next, we will make the control law more practicable for engineering. According to power system dynamics (Q. Lu, 2000a), we have

$$E'_{qi}+(x_{di}-x'_{di})I_{di}=E_{qi},$$
$$E_{qi}=V_{ti}+\frac{x_{di}Q_{ei}}{V_{ti}},\quad E'_{qi}=V_{ti}+\frac{x'_{di}Q_{ei}}{V_{ti}} \quad (29)$$

where Q_{ei} is the reactive power of the *i-th* generator, and V_{ti} is its terminal voltage. Hence the control law (28) can be written as

$$v_i = -\alpha_i \left\{ \frac{1}{x_{di} - x'_{di}} (V_{ti} + \frac{x'_{di} Q_{ei}}{V_{ti}}) - \frac{1}{x_{di} - x'_{di}} E^s_{qi} \right\} \quad (30)$$

Finally we come to the excitation control law for multimachine system as follows:

$$V_{fi} = V^s_{fi} - \alpha_i \left\{ \frac{1}{x_{di} - x'_{di}} (V_{ti} + \frac{x'_{di} Q_{ei}}{V_{ti}}) - \frac{1}{x_{di} - x'_{di}} E^s_{qi}) \right\} \quad (31)$$

where the selection of the control gain α_i, which represents the disturbance attenuation level of the controller, depends on L_2-gain in the closed-loop system and the boundary function of the uncertainty set. Moreover, all variables in control law (31) are locally measurable and only related to the local generator being irrelative to the network parameters, so the decentralized control is obtained.

4. COMPUTER TEST RESULTS

A 6-machine power system is used for computer test (Fig. 1). The system data is listed in Appendix of (Q. Lu, 2000a). In the simulations, the generators'

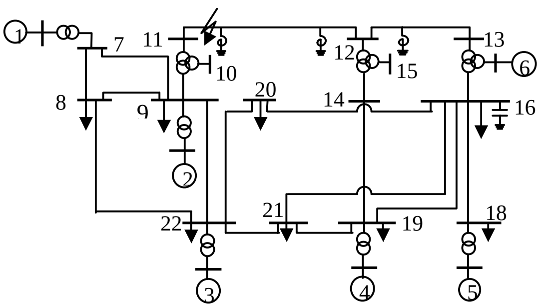

Fig. 1 Test system diagram

models are the same as in (14) and there are no turbine governors. In order to investigate the effectiveness of the suggested controller in improving transient stability, comparisons are made to several different types of excitation controllers.

Three cases are studied as follows:
Case 1: Generators 2 to 5 are installed conventional PSSs with their transfer functions given in (Q. Lu, 1996); Case 2: Generators 2 to 5 are equipped with linear optimal excitation control designed by LQR approach with feedback gains given in (Q. Lu, 1996); Case 3: The same generators are equipped with nonlinear optimal excitation controllers (NOEC) (Q. Lu, 1996);Case 4: The generators are equipped with the proposed nonlinear robust H_∞ excitation controllers (38) (NRHEC). System transients are stimulated by a three-phase short circuit fault occurred on line 11-12 close to bus 11 (see Figure 1) and cleared by tripping the faulted line in 0.1s. The simulation results for the 4 cases are shown in Figures 2 to 5 respectively where generator rotor angle response to the fault is plotted. We can see from Figures 2 and 3 that if linear PSS or LOEC is used, the system loses its synchronism soon after the fault occurs. However the system remains stable when NOEC or NDDEC is applied (see Figures 4 and 5). Comparing Figures 5 with Fig. 4, we can see that when NRHEC scheme is used, the corresponding rotor dynamics is better than that of NOEC scheme a little. Besides NRHEC has better robustness, which is significant to real power system applications. As we know, since the NOEC method is based on mathematical models with fixed structure and parameters without considering uncertainties, so the robustness of NRHEC is superior to NOEC in theory, just as the linear robust control is superior to linear optimal control (LOEC), which has been demonstrated by modern control theory.

5. CONCLUSIONS

In this paper, the nonlinear robust H_∞ control problem is studied for a class of generalized forced Hamilton system with uncertainty. The formulation of robust H_∞ controller is changed into seeking the solution of an algebra matrix inequality by considering the unmodeled dynamics in modeling the system, hence, the design problem of nonlinear robust H_∞ controller is greatly simplified. Another advantage of the proposed method is its application to a real power system which considering the network load, that is, for multimachine system whose self-admittances is not omitted, the generalized forced Hamilton system model is constructive in formulating the Hamilton function and consequently achieve the decentralized excitation control strategy. Simulation results have verified the effectiveness of the proposed excitation law.

6. ACKNOWLEDGEMENT

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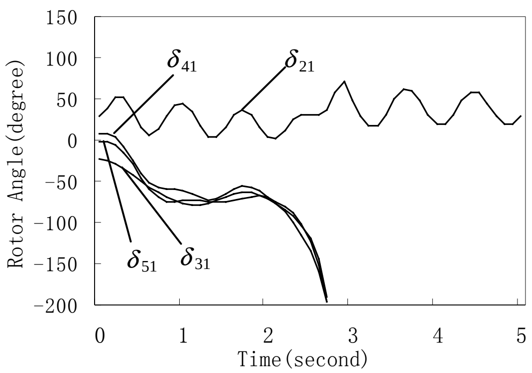

Fig. 2 Dynamic response of the system with PSS

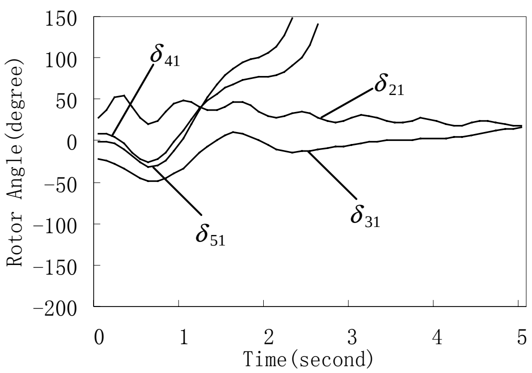

Fig. 3 Dynamic response of the system with LOEC

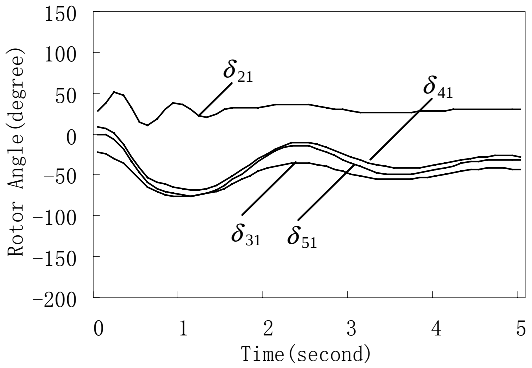

Fig. 4 Dynamic response of the system with NOEC

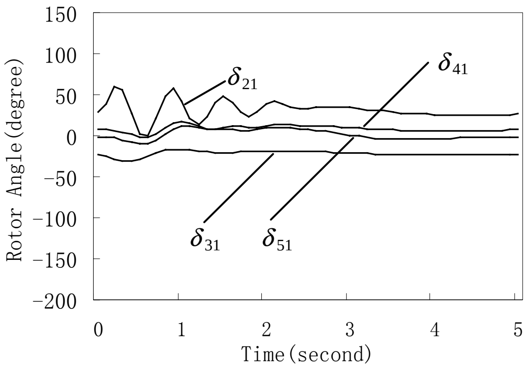

Fig. 5 Dynamic response of the system with NDHEC