

## NONLINEAR CHARGE CONTROL IN DIRECT INJECTION GASOLINE ENGINES

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Abstract: This paper develops a nonlinear charge controller for gasoline direct injection engines. The problem is to control the electronic throttle and the EGR valve to achieve the desired intake manifold pressure and the EGR flow. The control design is based on a variant of a Lyapunov design procedure, the so called Speed-Gradient approach. This methodology is reviewed, the controller for the electronic throttle and the EGR valve is derived in conjunction with an observer for the EGR flow, stability of the closed loop system is rigorously analyzed and experimental results demonstrating controller performance are reported.

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### 1. INTRODUCTION

Gasoline direct injection engine technology has been actively pursued in recent years as a potential avenue for improving fuel economy of passenger vehicles. Charge control is a critical feature of these engine management systems. It is responsible for delivering desired flow rates of air and recirculated burnt gas (for reduction of oxides of nitrogen emissions) into the engine cylinders, and for providing real-time estimates of the in-cylinder air charge and burnt gas fraction to torque and aftertreatment controllers (Kolmanovsky, *et al.*, 2000b). As compared to conventional engines challenges in the charge control for gasoline direct

injection engines include operation at high intake manifold pressure, soot deposits in the intake ports and exhaust gas recirculation (EGR) conduit caused by engine aging, and lean operation that produces exhaust containing both air and burnt gas. To address these challenges, the use of feedback and adaptation is essential (Kolmanovsky, *et al.*, 2000b).

The present paper is concerned with the analysis of a nonlinear feedback charge controller developed along the lines discussed in (Kolmanovsky, *et al.*, 2000b). Specifically, the “Speed-Gradient” methodology which underlies the development of the controller is reviewed, the stability of the controller is rigorously analyzed, and experimental results demonstrating the controller performance are reported.

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The controller design is based on an isothermal intake manifold filling and emptying model,

$$\dot{p} = c_m(W_{th} + W_{egr} - W_{cyl}), \quad (1)$$

where  $p$  is the intake manifold pressure,  $W_{th}$  is the flow rate of air through the electronic throttle,  $W_{egr}$  is the mass flow rate of the recirculated exhaust gas through the exhaust gas recirculation (EGR) valve,  $W_{cyl}$  is the total flow rate of air and burnt gas into the cylinders, and  $c_m$  is the pumping constant that depends on the intake manifold temperature. The throttle flow rate can be modelled according to the so called ‘‘orifice’’ equation as

$$\begin{aligned} W_{th} &= f_{th}(p)u_{th}, \\ f_{th} &= \phi\left(\frac{p}{p_a}\right) \cdot \frac{p_a}{\sqrt{T_a}}, \end{aligned} \quad (2)$$

where  $p_a$  and  $T_a$  are the ambient pressure and temperature (assumed constant), and  $u_{th}$  is the effective area of the throttle opening which is a known function of throttle position. The nonlinearity  $\phi$  is defined as

$$\phi(x) = \begin{cases} \gamma^{\frac{1}{2}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} & \text{if } x \leq 0.5, \\ x^{\frac{1}{\gamma}} \left\{ \frac{2\gamma}{\gamma-1} \left[1 - x^{\frac{\gamma-1}{\gamma}}\right] \right\}^{\frac{1}{2}} & \text{if } x > 0.5. \end{cases}$$

Here  $\gamma = 1.4$  is the ratio of specific heats. The EGR flow is modeled by a similar ‘‘orifice’’ equation as

$$\begin{aligned} W_{egr} &= f_{egr}(p_e, T_e)u_{egr}, \\ f_{egr} &= \phi\left(\frac{p}{p_e}\right) \cdot \frac{p_e}{\sqrt{T_e}}, \end{aligned} \quad (3)$$

where  $p_e$  is the exhaust pressure,  $T_e$  is the exhaust temperature, and  $u_{egr}$  is the EGR valve effective flow area. The cylinder flow is modelled as

$$W_{cyl} = k_0 + k_1 \cdot p, \quad (4)$$

where  $k_0$  and  $k_1$  depend on engine speed and intake manifold temperature. The control inputs are  $u_{th}$  and  $u_{egr}$  while engine speed, intake manifold temperature, and exhaust pressure  $p_e$  are treated as model parameters.

## 2. CONTROL DESIGN METHODOLOGY

Consider a nonlinear control system of the form

$$\dot{x} = f(x) + g(x)u, \quad (5)$$

where  $x$  is the  $n$ -vector state and  $u$  is the  $m$ -vector control. Given the desired equilibrium of (5),  $x_d$ ,

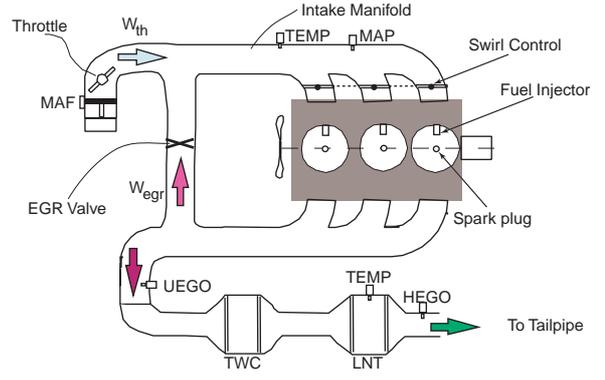


Fig. 1. Schematics of a DISI engine.

suppose the feedforward control  $u_d$  can be used to maintain this equilibrium, i.e.,

$$f(x_d) + g(x_d)u_d = 0.$$

Suppose now the input  $u$  is generated by applying, in addition to the above feedforward control  $u_d$ , a proportional feedback,  $u_p$ , and an integral feedback,  $\theta$ . Furthermore, suppose the effect of uncertainties and parameter variations can be represented by a constant disturbance  $w$  additive to the control input. Thus

$$u = u_d + u_p + \theta + w. \quad (6)$$

The feedback components  $u_p$  and  $\theta$  are determined with the help of Speed-Gradient (SG) approach (Fradkov and Pogromsky, 1999; Fradkov, 1990), which is reviewed next.

To be specific, consider a one step ahead model-predictive control-based derivation of the SG controller. Let  $Q(x)$  be the so called *goal* function. It satisfies  $Q(x_d) = 0$ ,  $Q(x) \geq 0$ , and is chosen so that the requirement

$$Q(x(t)) \rightarrow 0 \quad \text{as } t \rightarrow \infty \quad (7)$$

captures the control design objectives. For example, one possible choice for  $Q$  is

$$Q = \sum_{i=1}^n \gamma_i (x_i - x_{d,i})^2,$$

where the weights  $\gamma_i$  reflect the relative importance of different state channels. Assuming that  $u(t)$  is constant on  $[t, t + \Delta t]$ ,  $Q(x(t + \Delta t))$  can be approximated for small  $\Delta t$  as

$$\begin{aligned} Q(x(t + \Delta t)) &\approx Q(x(t)) + \dot{Q}(t)\Delta t \\ &= Q(x(t)) + \left( L_f Q(x(t)) + L_g Q(x(t)) \right) u(t)\Delta t, \end{aligned}$$

where,

$$L_f Q(x) \triangleq \frac{\partial Q}{\partial x}(x) f(x), \quad L_g Q(x) \triangleq \frac{\partial Q}{\partial x}(x) g(x).$$

The approximate cost can be *regularized* by augmenting a penalty on the proportional correction

and on the incremental change in the integral correction:

$$\begin{aligned} & Q(x(t)) + \left( L_f Q(x(t)) + L_g Q(x(t)) \cdot \right. \\ & \left. \cdot (u_d + u_p(t) + \theta(t) + w) \right) \cdot \Delta t + \\ & \frac{1}{2} u_p^T(t) \left( \frac{\Pi}{\Delta t} \right)^{-1} u_p(t) + \\ & \frac{1}{2} (\theta(t) - \theta(t - \Delta t))^T \Gamma^{-1} (\theta(t) - \theta(t - \Delta t)) \\ & \rightarrow \min, \end{aligned} \quad (8)$$

where  $\Pi > 0$ ,  $\Gamma > 0$ . The direct minimization of the cost (8) with respect to  $u_p(t)$  and  $\theta(t)$  yields

$$u_p = -\Pi(L_g Q)^T, \quad (9)$$

and

$$\theta(t) = \theta(t - \Delta t) - \Gamma(L_g Q)^T \Delta t.$$

The last equation is a discrete-time version of the continuous-time update law

$$\dot{\theta} = -\Gamma(L_g Q)^T. \quad (10)$$

The term  $(L_g Q)^T = \left( \frac{\partial Q}{\partial x} g \right)^T$  that appears on the right-hand of (9), (10) can also be computed as  $\nabla_u \dot{Q}$  which is the gradient of the speed of change of  $Q$ . Thus

$$u_p = -\Pi \nabla_u \dot{Q}, \quad \dot{\theta} = -\Gamma \nabla_u \dot{Q}.$$

Hence the control design (6),(9),(10) is referred to as ‘‘Speed-Gradient.’’

The transient response of the closed loop system can be shaped as desired by a proper choice of the weights in the function  $Q$ . For gasoline direct injection engines, this has been demonstrated in (Kolmanovsky, *et al.*, 2000a). Specifically, torque control mode (with torque as the primary tracking objective) or air-to-fuel ratio control mode (with air-to-fuel ratio as the primary tracking objective) can be enforced by proper selection of the weights in  $Q$ . Additionally, through the augmentation to  $Q$  of barrier functions, soft state constraints can be enforced. In (Kolmanovsky, *et al.*, 2000a) this approach was used to enforce state constraints on the air-to-fuel ratio and spark timing.

The stability analysis for the closed loop system *typically* proceeds by considering a Lyapunov function candidate of the form

$$V = Q + \frac{1}{2} (\theta + w)^T \Gamma^{-1} (\theta + w). \quad (11)$$

Its time derivative along the trajectories of the system (5), (6), (9), (10), under an additional assumption that  $\dot{w} \equiv 0$ , satisfies

$$\dot{V} = (L_f Q + L_g Q u_d) - L_g Q \Pi (L_g Q)^T.$$

The following sufficient stability conditions follows by application of the Barbalat’s lemma:

*Proposition:* Suppose that (i)  $Q(x_d) = 0$ ,  $Q(x) > 0$  if  $x \neq x_d$  and  $Q$  is twice continuously differentiable in  $\Upsilon_c = \{x : Q(x) \leq c\}$ ; (ii) the achievability condition (Fradkov and Pogromsky, 1999) holds, i.e.,

$$-\rho(Q(x) + L_g Q(x) \Lambda (L_g Q(x))^T \geq (L_f Q(x) + L_g Q(x) u_d), \quad (12)$$

for some  $m \times m$  matrix  $\Lambda \geq 0$ , a continuously differentiable function  $\rho$  that satisfies

$$\rho(0) = 0, \quad \rho(z) > 0 \text{ if } z > 0,$$

and for all  $x \in \Upsilon_c$ ; (iii)  $f, g$  are twice continuously differentiable and bounded together with the first and second derivatives in  $\Upsilon_c$ ; (iv) the initial conditions  $(x(0), \theta(0))$  satisfy

$$Q(x(0)) + \frac{1}{2} (\theta(0) + w)^T \Gamma^{-1} (\theta(0) + w) \leq c; \quad (13)$$

(v)  $\Pi > \Lambda$ ,  $\Gamma > 0$ ; (vi)  $\dot{w} \equiv 0$ . Then the closed loop system is Lyapunov stable and the closed loop trajectories satisfy

$$\begin{aligned} \lim_{t \rightarrow \infty} Q(x(t)) &= 0, \quad \lim_{t \rightarrow \infty} x(t) = x_d, \\ Q(x(t)) &\leq c \quad \forall t. \end{aligned}$$

If, furthermore, the matrix  $g(x_d)$  has a full column rank, then

$$\lim_{t \rightarrow \infty} \theta(t) = -w.$$

The achievability condition (12) may be checked either analytically or numerically. The purely numerical approach proceeds by setting  $\Lambda = 0$ , and maximizing  $(L_f Q(x) + L_g Q(x) u_d)$  subject to  $\tilde{c} \leq Q(x) \leq c$ , where  $\tilde{c} > 0$ . Let  $z(\tilde{c})$  denote the maximum. By analyzing  $z(\tilde{c})$  as a function of  $\tilde{c}$ , a continuously differentiable function  $\rho$ ,  $\rho(0) = 0$ ,  $\rho(\tilde{c}) > 0$  if  $\tilde{c} > 0$ , that yields  $\tilde{z}(\tilde{c}) \leq -\rho(\tilde{c})$  may be prescribed. If such  $\rho$  can be found,  $\Lambda = 0$ ,  $\rho$  satisfy the inequality in (12) for  $x \in \Upsilon_c$ . If no such  $\rho$  can be found,  $\Lambda > 0$  needs to be considered. As a heuristic approach, tuning the controller to work well in simulations with a  $\Pi > 0$  and then trying  $\Lambda$  slightly less than  $\Pi$  works quite well. See (Kolmanovsky, *et al.*, 2000a) for a specific application.

In a situation when  $Q(x) \geq 0$  but  $Q(x) > 0$  for all  $x \neq x_d$  does not hold,  $x(t)$  may not, in general, converge to  $x_d$ . It is still possible, however, to demonstrate that  $L_f Q(x(t)) \rightarrow 0$  and  $L_g Q(x(t)) \rightarrow 0$ . Note that  $L_f Q(x) = 0$ ,  $L_g Q(x) = 0$  is a system of  $(m + 1)$  equations in  $n$  unknowns. When  $(m + 1) \geq n$ , this system may, frequently, have  $x_d$  as the unique solution, and, hence,  $x(t) \rightarrow x_d$ . If  $n = m$  and the  $m \times m$  matrix  $g(x_d)$  is nonsingular,  $\theta(t) \rightarrow -w$ . If  $n > (m + 1)$ , it is necessary to demonstrate that the closed loop trajectories are bounded and that

$L_f Q(x(t)) = 0$ ,  $L_g Q(x(t)) = 0$ ,  $\dot{x}(t) = f(x(t)) + g(x(t))u_d$ , for all  $t$ , imply that  $x(t) \equiv x_d$ . This can be checked through an observability-like condition in (Shiriaev and Fradkov, 1998).

Note that the form of the feedback (9) suggests the connection with the conventional Control Lyapunov Function methods, in particular,  $L_g V$ -techniques (Sepulchre, *et al.*, 1997). The difference is mainly in the approach:  $Q$  is selected by the designer to capture the transient performance objectives in the SG approach;  $Q$  is constructed as a Control Lyapunov Function in the methodologies covered in (Sepulchre, *et al.*, 1997). The strength of SG approach is in the strong linkage between control objectives and the selection of the function  $Q$ . This greatly helps in the tuning process. Specifically, if we are not satisfied with the transient response, we adjust the weights in  $Q$  or augment barrier functions. The weakness of SG approach is that the procedures to modify  $Q$  are not readily available if the achievability condition does not hold. The achievability condition, on the other hand, is only sufficient and the stability may be verified by other procedures. For example, numerical simulations followed up by the numerical construction of a Lyapunov function can be used.

### 3. CHARGE CONTROLLER DESIGN

The basic formulation of the charge control problem requires the control system to deliver the desired value of the cylinder flow and the desired burnt gas fraction in this flow. Since neither the cylinder flow nor burnt gas fraction are measured, the engine operation can be controlled using the set-points for the intake manifold pressure,  $p_d$ , and for the EGR flow,  $W_{egr,d}$ . These set-points can be backtracked from the set-points for the cylinder flow and the burnt gas fraction (Kolmanovsky, *et al.*, 2000b). The intake manifold pressure is measured but the EGR flow is not. The EGR flow can, however, be estimated along the lines discussed next.

The open-loop estimate of the EGR flow (3) depends on the exhaust manifold pressure and temperature which are not measured. The exhaust temperature of an engine is known to change with aging, hence, nominal maps predicting the exhaust temperature will ultimately become inaccurate. The EGR flow also depends on the EGR valve position. While it is measured, it may not accurately define the effective flow area of the EGR valve due to soot deposits that develop in direct injection spark ignition engines (Kolmanovsky, *et al.*, 2000b; Kolmanovsky and Siverguina, 2001). Even if soot deposits were not an issue, the EGR flow in small engines exhibits engine speed dependence in the actual experiments. Thus the

prediction of the EGR flow on the basis of (3) may not be accurate.

An alternative procedure for predicting the EGR valve flow involves a dynamic input observer (Kolmanovsky, *et al.*, 2000b),

$$\dot{\epsilon} = \alpha c_m (\hat{W}_{egr} - W_{egr,ss}), \quad (14)$$

where

$$\hat{W}_{egr,ss} = \hat{W}_{cyl} - W_{th}, \quad (15)$$

is the steady-state estimate of the EGR flow and

$$\hat{W}_{egr} = \alpha p - \epsilon, \quad (16)$$

is the dynamic estimate of the EGR flow. An accurate estimate of  $W_{cyl}$ ,  $\hat{W}_{cyl}$ , is very important for this observer to function properly. To adjust the cylinder flow for soot deposits in the intake ports an adaptation may be used (Kolmanovsky *et al.*, 2000b; Kolmanovsky and Siverguina, 2001).

We form the *goal* function as

$$Q = \frac{1}{2}\gamma_1 (p - p_d)^2 + \frac{1}{2}\gamma_2 (\hat{W}_{egr} - W_{egr,d})^2,$$

where  $\gamma_1$  and  $\gamma_2$  are weighting factors. The time rate of change of  $Q$  along the trajectories of (5), (14), (16) satisfies

$$\begin{aligned} \dot{Q} &= \gamma_1 (p - p_d) (c_m (W_{th} + W_{egr} - W_{cyl})) \\ &+ \gamma_2 (\hat{W}_{egr} - W_{egr,d}) \cdot \alpha \cdot c_m \cdot (W_{egr} - \hat{W}_{egr}). \end{aligned} \quad (17)$$

If  $W_{th}$ ,  $W_{egr}$  are treated as control inputs, then  $W_{egr,d} + W_{th,d} = W_{cyl,d}$ ,  $W_{cyl} = k_0 + k_1 p$ ,  $W_{cyl,d} = k_0 + k_1 p_d$ , imply that

$$\begin{aligned} \dot{Q}|_{u=u_d} &= -\gamma_1 c_m k_1 (p - p_d)^2 \\ &- \gamma_2 \alpha c_m (\hat{W}_{egr} - W_{egr,d})^2, \end{aligned} \quad (18)$$

i.e., the achievability condition holds with  $\Lambda = 0$ . The SG controller that prescribes  $W_{th}$ ,  $W_{egr}$  can be defined with

$$\begin{aligned} \frac{\partial \dot{Q}}{\partial W_{th}} &= \gamma_1 (p - p_d) \cdot c_m, \\ \frac{\partial \dot{Q}}{\partial W_{egr}} &= c_m \gamma_1 (p - p_d) + \gamma_2 (\hat{W}_{egr} - W_{egr,d}) \alpha c_m. \end{aligned}$$

The  $W_{th}$ ,  $W_{egr}$  are, however, not the control inputs, but  $u_{th}$ ,  $u_{egr}$  are. To derive the control laws for  $u_{th}$  and  $u_{egr}$  and to analyze the stability, the nominal models for  $f_{th}$ ,  $f_{egr}$  are assumed. The integral and proportional feedback can subsequently be relied upon to compensate for the uncertainties. The SG controller that prescribes  $u_{th}$ ,  $u_{egr}$  can be defined with

$$\frac{\partial \dot{Q}}{\partial u_{th}} = \gamma_1 (p - p_d) \cdot c_m \cdot f_{th}(p), \quad (19)$$

$$\begin{aligned} \frac{\partial \dot{Q}}{\partial u_{egr}} &= c_m \gamma_1 (p - p_d) f_{egr}(p, T_e) \\ &+ \gamma_2 (\hat{W}_{egr} - W_{egr,d}) \alpha c_m f_{egr}(p, T_e). \end{aligned} \quad (20)$$

Thus the feedback is formed from the errors in the intake manifold pressure and estimated EGR flow, scaled by nonlinear gains that depend on  $f_{th}(p)$ ,  $f_{egr}(p, T_e)$ . This gain scaling may be beneficial in that it reduces the feedback gains for higher  $p$ , in the region where the actuator authority of throttle and EGR valve are close to vanishing. This may help to mitigate the effects of frequent actuator saturation and reduce the control activity.

Using (1),(2), (3), (4) and

$$f_{th}(p_d) u_{th,d} + f_{egr}(p_d, T_{e,d}) u_{egr,d} = k_0 + k_1 p_d,$$

it follows that if  $u_{th} = u_{th,d}$ ,  $u_{egr} = u_{egr,d}$  then

$$\begin{aligned} \dot{Q}|_{u=u_d} &= \gamma_1 c_m (p - p_d) \left( u_{th,d} (f_{th}(p) - f_{th}(p_d)) \right. \\ &+ u_{egr,d} (f_{egr}(p, T_e) - f_{egr}(p_d, T_{e,d})) \\ &\left. - k_1 (p - p_d) \right) + B, \end{aligned} \quad (21)$$

where

$$\begin{aligned} B &= \gamma_2 \alpha c_m (\hat{W}_{egr} - W_{egr,d}) \cdot \\ &\cdot (f_{egr}(p, T_e) u_{egr,d} - \hat{W}_{egr}) \\ &= \gamma_2 \alpha c_m (\hat{W}_{egr} - W_{egr,d}) u_{egr,d} \cdot \\ &\cdot (f_{egr}(p, T_e) - f_{egr}(p_d, T_{e,d})) \\ &- \gamma_2 \alpha c_m (\hat{W}_{egr} - W_{egr,d})^2, \end{aligned} \quad (22)$$

and  $T_{e,d}$  is the exhaust temperature corresponding in steady-state to  $p_d$ . By bounding the cross-coupling term in the expression for  $B$  we obtain that, for any  $c > 0$ ,

$$\begin{aligned} B &\leq -\gamma_2 \alpha c_m \left(1 - \frac{1}{2c}\right) (\hat{W}_{egr} - W_{egr,d})^2 \\ &+ \frac{c \gamma_2 \alpha c_m}{2} u_{egr,d}^2 (f_{egr}(p, T_e) - f_{egr}(p_d, T_{e,d}))^2. \end{aligned}$$

The objective is now to determine under what conditions the achievability condition (12) holds. First, by considering the expression for  $f_{th}$  in (2) it follows that

$$(p - p_d) (f_{th}(p) - f_{th}(p_d)) \leq 0.$$

To further analyze the expression for  $\dot{Q}$  and stability of the closed loop system the dynamic behavior of the exhaust temperature need be known. This behavior is determined not only by charge control, but also by fueling and spark timing control, i.e., it is outside of the scope of the charge control.

If fueling and spark strategy are prescribed, then  $T_e$  becomes a function of the pressure  $p$ . Taking the engine cycle delay into account,  $T_e$  is actually a function of the delayed value of the pressure, i.e.,  $T_e = \mathcal{T}_e(p(t - \Delta))$ , where  $\Delta$  is the engine cycle delay. Note that the exact dependence is determined by a choice of spark timing and fueling control strategy. For example, fueling being adjusted to maintain a constant air-to-fuel ratio and fueling being adjusted to maintain a constant torque would result in different dependencies of  $T_e$  on  $p(t - \Delta)$ .

Consider first the situation when the delay  $\Delta$  is negligible, which is the case at higher engine speeds. From the expressions (21) and (22) for  $\dot{Q}|_{u=u_d}$  and  $B$  it is clear that to ensure the achievability condition in the form (12) the term  $f_{egr}(p, T_e) - f_{egr}(p_d, T_{e,d})$  has to be bounded by a term dependent on the magnitude of the pressure error  $|p - p_d|$ . Suppose a  $\kappa_{egr} > 0$  can be found such that

$$|f_{egr}(p, \mathcal{T}_e(p)) - f_{egr}(p_d, \mathcal{T}_e(p_d))| \leq \kappa_{egr} |p - p_d|,$$

and

$$f_{th}(p) > 0,$$

for all  $p$  in a neighborhood of  $p_d$ . This inequality basically constrains the function  $f_{egr}(p, \mathcal{T}_e(p))$  to be Lipschitz continuous in a neighborhood of  $p_d$ ; it does hold for typical  $\mathcal{T}_e$  and  $p_d < \min\{p_a, p_e\}$ . In this case, it is straightforward to confirm that the achievability condition in the form (12) holds locally with  $\Lambda = \text{diag}(\lambda_1, 0)$  where  $\lambda_1$  is sufficiently large. Since  $\Pi > \Lambda$ , it follows that a sufficiently strong proportional feedback on the throttle to compensate for the intake manifold pressure errors can assure the closed loop stability. This condition is only sufficient and can be relaxed if more information about the function  $\mathcal{T}_e$  is available.

The situation when the delay  $\Delta$  is non-negligible is more complex. Specifically, the inequalities of the form

$$\begin{aligned} |f_{egr}(p(t), \mathcal{T}_e(p(t - \Delta))) - f_{egr}(p_d, \mathcal{T}_e(p_d))| &\leq \\ \kappa_{egr} |p(t) - p_d| + k_T |p(t - \Delta) - p_d|, & \\ f_{th}(p) > 0, & \end{aligned}$$

are assumed to hold locally, in a neighborhood of  $p_d$ , with  $\kappa_{egr} > 0$ ,  $k_T > 0$ . The achievability condition in the form (12) does not directly apply if a non-zero delay is present and one needs to use the theory of Lyapunov-Krasovskii functionals (Kolmanovskii and Nosov, 1986) to analyze the stability. The approach is to modify (11) as

$$V = Q + \frac{1}{2} (\theta + w)^T \Gamma^{-1} (\theta + w) + \gamma_3 \int_{-\Delta}^0 (p(t + \tau) - p_d)^2 d\tau$$

where  $\gamma_3 > 0$  is to be selected. Considering  $\dot{V}$  for the same SG controller defined by (9),(10),(19), (20) with  $T_e = \mathcal{T}_e(p(t - \Delta))$  we end up with

$$\dot{V} = \dot{Q}|_{u=u_d} - L_g Q \Pi (L_g Q)^T + \gamma_3 (p(t) - p_d)^2 - \gamma_3 (p(t - \Delta) - p_d)^2,$$

where  $\dot{Q}|_{u=u_d}$  is given by (21),(22) with  $T_e = \mathcal{T}_e(p(t - \Delta))$ . The integral term in the new expression for  $\dot{V}$  provides two terms in the expression for  $\dot{V}$ :  $-\gamma_3(p(t - \Delta) - p_d)^2$  and  $+\gamma_3(p(t) - p_d)^2$ . By properly selecting  $\gamma_3$ ,  $-\gamma_3(p(t - \Delta) - p_d)^2$  can dominate those terms in the expression for  $\dot{V}$  that are bounded from below by some multiple of  $-(p(t - \Delta) - p_d)^2$ ; The latter term,  $+\gamma_3(p(t) - p_d)^2$ , contributes to increasing  $\dot{V}$  but it can be counteracted by  $-L_g Q \Pi (L_g Q)^T$ , with a proper selection of  $\Pi > 0$ . Specifically, an expression of the form  $\dot{V} \leq -\rho \cdot Q$  can be shown to hold locally if  $\Pi > \text{diag}(\lambda_1, 0)$ , where  $\lambda_1$  is sufficiently large. Thus  $p(t) \rightarrow p_d$ ,  $\dot{W}_{egr}(t) \rightarrow \dot{W}_{egr,d}$  provided a sufficiently strong proportional feedback is utilized to respond to intake manifold pressure errors. This is the same conclusion as in the delay free case.

#### 4. EXPERIMENTAL EVALUATION

Experiments have been conducted on a gasoline direct injection engine to evaluate the SG controller responses. The intake manifold pressure signal was low-pass filtered to get rid of periodic oscillations at the engine firing frequency. The commanded actuator positions for the electronic throttle and EGR valve were backtracked from the commanded by the controller effective flow areas of throttle and EGR valve. Dither and overdrive were applied on top of the base actuator position commands to deal with actuator imperfections. The dynamic observer for the EGR flow was tuned to provide a transient response qualitatively similar to that of an orifice flow model while at the same time correcting the estimate for the uncertainties in that model. Typical closed loop system responses are shown in Figures 2-3. They confirm that the closed-loop system is well-behaved.

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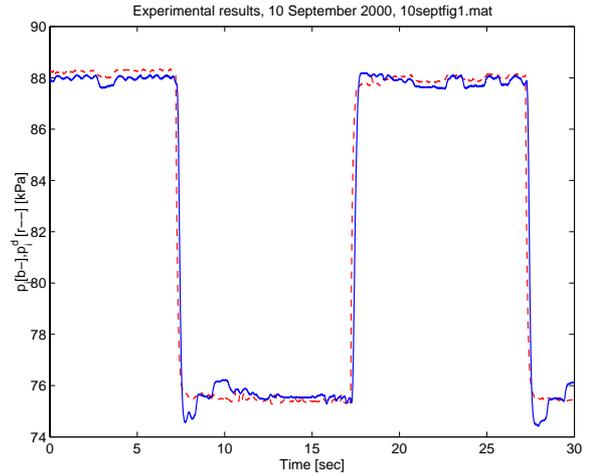


Fig. 2. Intake manifold pressure response: commanded [dash] and measured [solid].

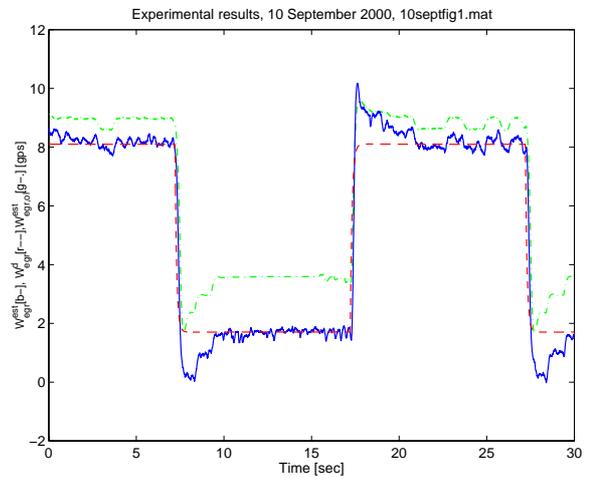


Fig. 3. Estimated EGR flow response: commanded [dash], estimated by dynamic observer [solid] and estimated by the orifice equation model [dash-dot].

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