

ENERGY EFFICIENT STRATEGIES FOR SCHEDULING TRAINS ON A LINE**

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Abstract: Scheduling trains on a single railway line is treated by converting the scheduling problem to a discrete event dynamic system, and converting a greedy time-efficient strategy to an energy efficient schedule. While such a schedule cannot be referred to as a strategy dependent solely on the state of the system the strategy is easily re-computed when perturbations in the nominal schedule occur. *Copyright © 2002 IFAC.*

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1. INTRODUCTION

The need for more efficient use of existing resources in freight rail transportation is heightened by the fact that, in the US, rail freight transportation is based on diesel traction. Operating costs are thus becoming more sensitive to the increasing costs of fossil fuel, a cost that will increase in the near future. Time- and energy-efficient scheduling, and fast rescheduling, of trains can significantly contribute to the cost-effective operation of freight traffic. The problem considered here involves energy-efficient scheduling of trains on a line with single-track sections, as is the case on many freight routes in the US. The approach departs from all earlier approaches to the problem by employing a discrete event model of the traffic on the line and a heuristic, locally optimal, travel advance strategy to develop energy efficient schedules.

Discrete event models of train traffic along railway lines have been used to obtain a greedy, state-dependent, travel advance strategy (greedy TAS) that determines which train is to advance and which is to be stopped at a siding, or a station, referred to as meet and pass points (Medanic and Dorfman, 2001). The greedy TAS is a strategy, in the sense that train advances are a function of the location and the velocities of all trains in vicinity of each other. Thus, the greedy TAS can be applied whatever the location of the trains, be it on an existing schedule, or a perturbed schedule, in contrast to schedules obtained using nonlinear programming approaches that have been used in the past. The greedy TAS is a time-efficient strategy, consistently achieving an

efficiency ratio (ratio of *unobstructed time* to clear the line of all trains to *obstructed time* to clear the line of all trains using the schedule obtained by applying the greedy TAS) above 0.95 for train densities of the order of 2 trains/h from each direction, a density that suffices for most railway lines. The greedy TAS is combined here with optimal pacing velocities for individual trains to obtain an energy efficient schedule.

Energy efficient operation of trains has been the subject of intense study up to the present time (Hewlett and Pudney, 1995; Khmelnitsky, 2000; Franke, 2000). However, all these endeavors involve the energy-efficient operation of a single train, and ignore the effect of bi-directional traffic along the railway line. The optimal pacing velocities obtained by optimizing a single train can provide a significant support tool to train operations, but ignore the effect of the two-way traffic on energy efficient scheduling. On the other hand the approaches that focused on the scheduling issue have treated energy costs using constant pacing velocities, and have by and large ignored the various factors influencing the optimal velocity profile that results in minimal energy costs. The greedy TAS developed for time-efficient scheduling of traffic on a line is used here to develop an energy-efficient schedule using the greedy TAS and the optimal pacing velocities of individual trains. The energy-efficient schedule is thus obtained by decomposing the problem into a train optimization phase and train scheduling phase, and results in a schedule that is both time-efficient for a given level of energy costs, and energy-efficient for the given

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times of travel. Further simultaneous reduction of time of travel and energy costs is not possible because a trade-off between time- and energy-efficiency is unavoidable.

2. A GREEDY TRAVEL ADVANCE STRATEGY

The *energy-efficient scheduling problem* is to schedule $N = N_1 + N_2$ trains, N_1 trains traveling from O to D, N_2 trains traveling from D to O to minimize total energy cost while ensuring that all trains clear the line within a given time interval.

The *time-efficient scheduling problem* is a closely related problem that does not explicitly consider energy costs, but instead chooses the best train velocities and departure and arrival times to minimize a suitable time of travel related performance index. A greedy time efficient strategy is exploited here to develop an energy-efficient schedule, and is described first. The formulation used in developing the greedy TAS differs from those used in the programming approaches (see Cordeau et al, 1998 Higgins et al, 1995 and Kraft 1987) in that the departure times and train velocities in sections are assumed fixed (as opposed to belonging to pre-defined admissible ranges). The stop times at meet and pass (M&P) points, arrival times at destinations, and the complete schedule are obtained by applying the TAS and solving the discrete event dynamics. In addition, the following assumptions are made: (i) The route is fixed and defined by the vector x_d ; (ii) Velocities of all trains in all sections of the route are fixed, and given by the matrices V_L and V_R , respectively, (i.e. the element $V_L(i,m)$ is the velocity of train i traveling from O to D in section m , the element $V_R(j,n)$ is the velocity of train j traveling from D to O in section n); (iii) The departure times of the trains are given by the vectors T_{OL} and T_{OR} , and the arrival times are free, and depend on the train advance strategy; (iv) The minimal headways of trains are defined by the vectors d_L and d_R (i.e. $d_L(i)$ defines the minimal distance between train i and any train ahead of it).

Assuming constant velocities in each section of the line, the model of “system dynamics” may be written in the form

$$\begin{aligned} x(k+1) &= x(k) + \Delta t_k v_L(x(k), y(k)), \quad x(0) = x_0 \\ y(k+1) &= y(k) + \Delta t_k v_R(x(k), y(k)), \quad y(0) = y_0 \end{aligned} \quad (1)$$

where $x(k)$ is the vector of position of trains traveling from O to D, and $y(k)$ is the vector of position of trains traveling from D to O, and Δt_k is the time to the next discrete event when the nature of the dynamics changes. The time periods Δt_k in the discretization vary in duration and depend on the state of the system and train velocities, with the constant velocity of any particular train depending on

the section of the line the train is currently traversing. The next discrete event occurs the first moment a train reaches an M&P point. The trains reach M&P points asynchronously, and this leads to a discrete event system (DES).

The *greedy TAS* is a locally optimal strategy that defines which train advances and which stops at an M&P point in 11 different situations in which a meet and pass (M&P) event, a met and overtake (M&O) event, or a combined M&P/M&O event occurs. In the greedy TAS The advance of train i , moving in the O to D direction, depends on the position and velocity of only the *trains in its vicinity*, typically trains $i+1$, and $i-1$, moving in the same direction and any train j moving in the opposite direction and immediately ahead of train i , (and vice-versa for a train j moving in the D to O direction). The main components of the greedy TAS are:

- Determination of the next discrete event (the train which will first reach an M&P point, and the required time interval dt_{next}).
- Resolution of the M&P and/or M&O events at this M&P point, and possibly at other M&P points where a train is stationed at, at the current discrete event, and
- Application of the appropriate rules for a simple M&P, simple M&O, or a combined M&P with M&O event, as the case may be.

Given a state $x(k), y(k)$ at some discrete event (DE) k , the time to next M&P for each train is computed, from

$$\begin{aligned} z(i,k) &= \frac{|x(i,k) - x_d(P_L(i,k))|}{V_L(i, S_L(i,k))} \quad i=1,2,\dots,N_1 \quad (2) \\ w(j,k) &= \frac{|y(j,k) - x_d(P_R(j,k))|}{V_R(j, S_R(j,k))} \quad j=1,2,\dots,N_2 \end{aligned}$$

and adjusted by eliminating the associated components of $z(i,k)$ or $w(j,k)$ if a slower train is obstructing a faster train in reaching first an M&P point. Two vector variables, S_L and P_L , are associated with trains moving from O to D, and two vector variables, S_R and P_R , are associated with trains moving from D to O. Variables S_L, S_R are used to characterize the velocities of trains while in a section, variable P_L, P_R are used for a train at an M&P point, and are defined by

$$\begin{aligned} S_L(i,k) &= m \quad \text{if } x(i,k) \in (x_d(m), x_d(m+1)), \\ P_L(i,k) &= m \quad \text{if } x(i,k) \in [x_d(m), x_d(m+1)), \\ S_R(j,k) &= n \quad \text{if } y(j,k) \in (x_d(n), x_d(n+1)), \\ P_R(j,k) &= n \quad \text{if } y(j,k) \in [x_d(n), x_d(n+1)]. \\ V_L(i,k) &= \text{velocity of train } i \text{ in section } k \\ V_R(j,k) &= \text{velocity of train } j \text{ in section } k \end{aligned}$$

Thus, $P_L(i) = n$ implies $x_d(P_L(i)) = x_d(n)$ and identifies the train i as being at M&P point n , while $S_R(j,k)$

identifies the train moving from O to D as being in the k-th section, etc.

Given an arbitrary vector Γ let the two arguments α, β in the operation $[\alpha, \beta] = \min(\Gamma)$ denote the minimal component and the lexicographical order of that component in Γ . Then, given the vectors z and w , let

$$\begin{aligned} [z_{\min}, i_{\min}] &= \min(z) \\ [w_{\min}, j_{\min}] &= \min(w) \end{aligned} \quad (3)$$

characterize the train ($i_{\min}$, or $j_{\min}$) to reach first the next M&P point and the minimal time required ($z_{\min}$ or $w_{\min}$), at the current discrete event. When trains are not in the vicinity of each other all trains will advance along the line for the duration of the time interval

$$dt_{\text{next}} = \min(z_{\min}, w_{\min}) \quad (4)$$

at which time the next DE occurs (because train $i_{\min}$, or $j_{\min}$, as the case may be, reaches an M&P point, referred to as the focal M&P for that DE).

Concerning the TAS, a rule is defined for each type of M&P event, M&O event and combined M&P/M&O event involving at most three trains. Four trains cannot pass each other if they are in vicinity of each other at any one DE, and this option is excluded by the defined rules at prior discrete events. Application of the TAS requires that each train obtain information on the train in front of it traveling in the same direction, as well as closest train(s) approaching it from the opposite direction.

4. TIME-EFFICIENT PERFORMANCE OF THE GREEDY TAS

Performance measures used in analysis of the greedy TAS include: (i) the **time to clear the line** (J_1), (ii) the **total delay** of all trains (J_2), and (iii) the **maximum delay** (J_3). The **time to clear the line** criterion is defined as

$$J_1 = t_{N_a} - t_{1_d} \quad (5)$$

where t_{1_d} is the time of departure of the earliest train on the schedule, and t_{N_a} is the time of arrival of the latest train on the schedule. The criterion is well tuned to the formulation of the problem in which times of departure are fixed, because its minimum possible value, J_1^f , is the total time to clear the line when all trains travel unobstructed. (In the absence of overtakes this would correspond to the availability of double tracks over the entire line.) Given departure times and velocities the efficiency ratio is defined by

$$\eta = \frac{t_{N_a}^f - t_{1_d}^f}{t_{N_a}^{\text{ob}} - t_{1_d}^{\text{ob}}} = \frac{t_{N_a}^f - t_{1_d}^f}{t_{N_a}^{\text{ob}} - t_{1_d}^f} \quad (6)$$

where t_{1_d} is the time of departure of the first train to depart, and subscripts ob and f stand for “obstructed”

and unobstructed (i.e., “free”) travel, and so $t_{N_a}^f$ is the time of arrival of the last train to arrive.

Analysis of time-efficient performance of the greedy TAS shows that with number of trains traversing a single line in a day, from each direction, of the order of 2 per hour the greedy strategy easily determines a schedule without encountering a deadlock. Scheduling $N_1 = N_2 = 30$ trains in each direction involve on the order of 850 discrete events. Furthermore, as the number of trains is increased (from $N_1 = N_2 = 6$ to 30 in each direction), the efficiency ratio remains remarkably constant. While attempting to pack a larger number of trains from each direction will certainly lead to a deadlock, there simply is no line where a much greater number of trains from each direction needs to be scheduled in the US today. That is why local strategies work well, and are used here to develop energy-efficient schedules.

5. ENERGY-CONSERVING TAS

Consider the case when: (i) trains use maximum velocities allowed in sections of the line, which then depend on the condition of the tracks and possibly on the type of train involved, and (ii) energy costs are a convex function of train velocity. Maximum velocity travel is often how freight trains are run in the absence of efficient train schedules, the result being long stops at M&P points. Suppose the greedy TAS has been determined, and has produced departure times, and stop times at M&P points for all trains involved in the schedule, and suppose that as a result the schedule is characterized by the efficiency index η . The greedy schedule can be converted to an **efficient pacing schedule** (EPS) that will maintain the value of η , reduce total delay and maximal delay, and also reduce energy costs. However, there is no longer a strategy that can be applied when perturbations in the schedule occur, because modifications in the schedule depend on the stop times associated with the nominal schedule. On the other hand, the efficient computation time associated with re-computing a new greedy schedule allows a fast recalculation of an EPS in the perturbed case.

Once the greedy schedule is available, the EPS is obtained by introducing the following simple modifications into the greedy schedule. It is stressed that such modifications are by no means simple when the nonlinear programming approach is applied; there, the modified velocities must be found together with all other elements of the nonlinear programming solution. Let the scheduled departure times of trains be denoted by T_{dxi} , $i = 1, \dots, N_1$, and T_{dyj} , $j = 1, \dots, N_2$, and let the departure times obtained by the greedy schedule be denoted by $T_{xi}(1)$, $i = 1, \dots, N_1$ and

$T_{yj}(K+1)$, $j = 1, \dots, N_2$. Then, the EPS is obtained by introducing the following modifications into the schedule, and train velocities:

- (a) the departure times are shifted from T_{dxi} , $i = 1, \dots, N_1$, T_{dyj} , $j = 1, \dots, N_2$, to $T_{xi}(0)$, $i = 1, \dots, N_1$ and $T_{yj}(0)$, $j = 1, \dots, N_2$, and
- (b) the velocities of the trains traveling from O to D are reduced to the velocities to $V_{LP}(i,1)$, $i=1, \dots, N_1$ where

$$V_{LP}(i,1) = \frac{x_d(2) - x_d(1)}{T_{xi}(2) - T_{xi}(1) - \Delta_{xi}(2)} \leq V_L(i,1)$$

and $T_{xi}(2)$ is the time train i traveling from O to D arrives is scheduled to depart from M&P point 2, and $\Delta_{xi}(2) \geq 0$ is the stop time at the M&P point if there is a scheduled station stop there.

- (c) the velocities of the trains traveling from D to O are reduced to the pacing velocities $V_{RP}(j,K)$, $j = 1, \dots, N_2$ where

$$V_{RP}(j,K) = \frac{x_d(K+1) - x_d(K)}{T_{yj}(K) - T_{yj}(K+1) - \Delta_{yj}(K)} \leq V_R(j,K)$$

and $T_{yj}(K)$ is the time of departure of the train j traveling from D to O at M&P point K , and $\Delta_{yj}(K) \geq 0$ is the stop time of train j at the M&P K point if there is a scheduled station stop there.

This strategy eliminates superfluous delays that are due to the initial distribution of departure times, and eliminates the stopping times along the route by reducing velocities from maximal to optimal pacing velocities. This reduces energy costs but does not affect the total time to clear the line, while reducing the total delay and the maximum delay.

Example 1. We illustrate the effect of energy conservation using the case where $N_1 = N_2 = 6$ which forms one piece of a broader capacity study described in (Medanic and Dorfman, 2001). A line with 11 single track sections with total length of 210 [mi] is considered, with different maximum velocities, varying between 50-90 [mi/hr] in each section, but the same for all trains, and minimal headways set at 0.5 [mi] for all trains. The same number of trains was assumed to depart from each end of the line, with departure times of trains approximately uniformly distributed over a 24 h period. The schedule obtained using the greedy TAS is characterized by $\eta = 0.9883$ (already extremely high) with a total delay of 3.46 [h]. The actual schedule is omitted due to space restrictions.

By modifying times of departure and reducing the velocities of four trains in four sections ($V_L(4,9)=45$, reduced from 60, $V_R(3,3)=50$, from 70, $V_R(5,11)=25$, from 50 and $V_R(6,11) = 40$, from 50), a modified schedule is obtained and shown in Figure 1 in the form of the standard scheduling diagram used in railway industry, with time displayed on the

horizontal axis, and the distance (from D) on the vertical axis. Horizontal lines represent the locations of the M&P points at which passes and overtakes can take place.

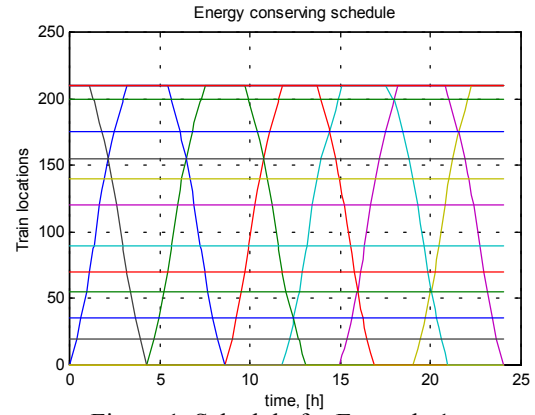


Figure 1. Schedule for Example 1

The modified schedule is characterized by $\eta = 0.9980$ with total delay down to 0.4942 (from 3.8321) and maximum delay down to 0.1057 (from 0.5869), and with lower energy costs due to lower velocities of some trains in some sections. The final departure times are $TOL = [0, 4.29, 8.59, 11.77, 14.95, 18.99]$ and $TOR = [1.12, 5.42, 9.60, 13.70, 17.45, 20.60]$. While some of the time-saving may not be relevant and may not be enacted (this example is only used to illustrative the point), replacing these perturbation in departure times with lower velocities adds to energy savings.

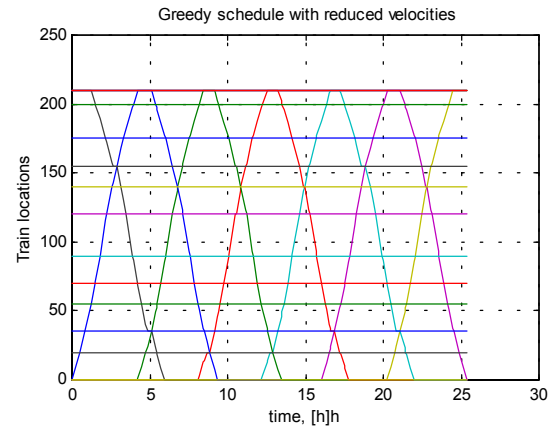


Figure 2. Schedule with reduced velocities

Further conservation of energy can be accomplished but at the expense of time of travel. As an illustration of this concept, if it is considered that energy costs associated with the velocities proposed for this schedule are too high, one can reduce the velocities to reduce the cost, and determine the effect on both the energy savings and time of travel. Reducing all velocities by a factor of 0.75 results in the velocities $\{37.5, 45, 52.5, 60, 67.5, 67.5, 60, 52.5, 45, 37.5, 37.5\}$ per section (for all trains). Applying the greedy strategy without any additional modifications results in a schedule where all the

trains clear the line in $J_1 = 25.377$ h (only 1.33 h more than with the original velocities) and with $\eta = 0.9944$, with a total delay $J_2 = 3.5627$ h and maximum delay of $J_3 = 0.73175$ h. Figure 2 displays the obtained schedule. These delays can again be reduced by applying a energy conserving modification, in this case reducing some velocities further, notably $V_R(1,9), V_R(1,3), V_R(4,3), V_R(5,9)$ and $V_R(5,3)$.

6. ENERGY- EFFICIENT TAS

Consider now the case when the goal in scheduling is to meet target travel times for all trains while minimizing the total energy costs. The energy costs in this case are expected to be lower than with any energy conserving strategy because by assumption lower train velocities will be used throughout, and the schedule is obtained at the expense of increased travel times. The solution will depend significantly on a number of factors including targeted travel times, train composition, track condition, train velocity and curve and grade variations and section lengths. The problem is particularly meaningful over railway lines with significant grade variations where negotiation of grade variations can be a significant factor affecting energy consumption, and where the locations of M&P points may also significantly influences the optimal pacing velocities. In that case the greedy TAS is an efficient way of tying together the optimal pacing velocities of all the trains, obtained separately for each train by optimizing its travel over all sections of the line, into a complete schedule. We describe here how the two components are interconnected, and illustrate the procedure by an example.

Suppose desired travel times T_{di} , $i=1, \dots, N$ are prescribed for trains in the schedule, and suppose optimal pacing velocities $V_i(t)$, $t \in [0, T_{di}]$, have been obtained, respectively, for all the trains using, say, the approach in (Franke et al, 2000, Howlett and Pudney, 1995, Khmelnitsky, 2000). Integrating the velocities one can determine the time intervals each train needs to travel over the various sections of the line. From this one can determine the average velocities of trains in section of the line that correspond to the optimal pacing of trains along the sections. Let these be denoted by $V_L(i,m)$, $i=1, \dots, N_1$, and $V_R(j,m)$, $j=1, \dots, N_2$, and $m=1, \dots, K$. The departure times of trains in this formulation are free, but in practice will be constrained by many diverse factors; however, energy costs should be invariant with respect to departure times. The schedule covers a standard scheduling period such as a day, and the goal of scheduling is to efficiently clear the line for additional possible traffic. Thus, the scheduling period is assumed to

$$T_s \leq \frac{1}{2} \sum_{i=1}^N T_{di}$$

It may be possible to pack the trains into a shorter scheduling interval, when there is a valid reason to do so. In that case one can still start with the above T_s , and then gradually shorten T_s and observe the effect on the schedule. Given T_s and the average velocities the greedy TAS is applied to the scheduling problem and a schedule is obtained where each train employs the average pacing velocities per sections of the line, and the TAS assigns stop times to certain trains to enable all trains to meet and pass, and possibly meet and overtake, and reach their destinations. Since the stop times do not increase energy consumption this schedule results in minimal energy consumption. The time-efficiency of the schedule can be determined by the ratio η , and by the total and maximum delay. If a train is not stopped at a certain M&P point the optimal pacing velocity is adjusted, or re-computed over the two sections.

Example 2. Assume in the scenario used in Examples 1 that the railway line has a grade profile as shown in Figure 3 implying that trains traveling from O to D will require more energy to traverse sections 2,3,5,7 and 8 with an uphill grade than trains traveling from D to O. Similarly, trains traveling from D to O will require more energy to traverse sections 6, 10 and 11 that trains traveling from O to D. Consequently, trains encountering uphill sections will use lower velocities to optimize energy costs. In addition, differentiating between long and heavier trains(L), average length and weight trains (A) and short and relatively light trains (S), the long trains will use the lower velocities on the inclined sections (and on downhill section will obey speed restrictions due to track conditions irrespective of the type of train.

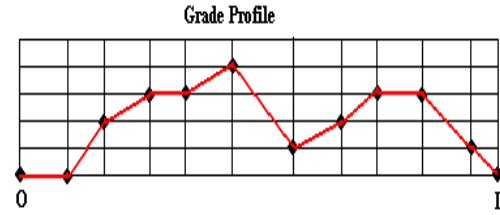


Figure 3. Illustrative grade profile

With this in mind, and trains types traveling from O to D are {S,A,L,L,A,S}, and have departure times at {0,4,8,12,16,20} hours, train types traveling from D to O are {S,A,L,L,A,S}, and have departure times at {1,5,9,13,17,21} hours, all train headways are 2 mi, and the M&P points are at mileposts {0,20,35, 55,70,90,120,140,155,175,200,210}, and suppose the optimal pacing velocities translate into the average velocities in the section of the line as shown in Table 1. The greedy TAS then provides the schedule shown in Figure 4, characterized by $\eta = 0.9897$, $J_1 = 24.3431$, $J_2 = 4.0984$, and $J_3 = 0.8476$.

Table 1. Train velocities

	Section											
V_L	70	50	60	70	60	90	60	50	70	90	80	
	50	30	40	50	40	70	40	30	50	70	60	
	60	40	50	60	50	80	50	40	60	80	70	
	50	30	40	50	40	70	40	30	50	70	60	
	60	40	50	60	50	80	50	40	60	80	70	
	70	50	60	70	60	90	60	50	70	90	80	
V_R	50	50	60	80	70	50	70	60	70	80	60	
	60	60	70	90	80	60	90	70	80	90	70	
	50	50	60	80	70	50	70	60	70	80	60	
	40	40	50	70	60	40	60	50	60	70	50	
	40	40	50	70	60	40	60	50	60	70	50	
	60	60	70	90	80	60	90	70	80	90	70	

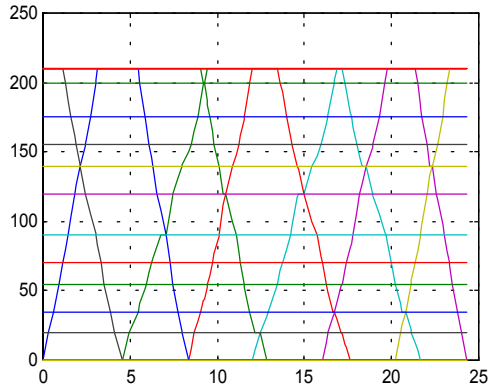


Figure 4. Energy optimal time-efficient Schedule

The time-efficient schedule of the individual energy efficient optimal pacing routes is not a consequence of the relatively large intervals between departures.

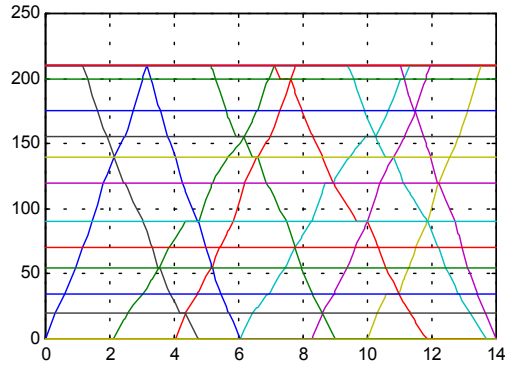


Figure 5. Tighter energy optimal schedule

When the departure times are spaced tighter, to cover a 14 [h] period, with departure times at {0,2,4,6,8,10} and {1,3,5,7,9,11} hours, the Greedy

TAS produces the schedule shown in Figure 5 which has the same energy costs and remains time-efficient, with $\eta = 0.9929$, $J_1 = 13.9927$, $J_2 = 4.5625$, and $J_3 = 0.7700$.

7. CONCLUSIONS

Local, greedy TAS combined with optimization of individual train along section s of the line offer the possibility of obtaining suboptimal energy-efficient train schedules over a line with single track sections. The TAS is easily adapted to the case where double track exist in certain sections by removing restrictions for M&P, and M&O within such sections. The local nature of the strategy raises expectations that it can be extended to railway networks, and this will be pursued.

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