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A MULTIVARIABLE H_∞ CONTROLLER FOR A ROTARY DRYER

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Abstract: This paper describes the design of a multivariable robust controller for a drying process. The plant consists of a co-current rotary dryer to evaporate moisture from a waste product, called *alpeorujo* or *two phase cake*, generated by olive-oil mills. A methodology to design a multivariable H_∞ controller is shown, including the selection of appropriate weighting functions for a mixed sensitivity control formulation. The performance obtained with this control strategy has been validated under simulation, using a very accurate nonlinear simulator developed in Dymola. *Copyright © 2002 IFAC*

Keywords: Control Applications, Industrial Process, Multivariable Control, Food processing

1. INTRODUCTION

This paper presents a multivariable H_∞ control of an olive waste drying process. The modern *two phase* olive-oil mills generate a waste called *alpeorujo* or *two phase cake*, which has to be dried before being fed it into the leaching system in order to extract the remaining oil. (Rubio *et al.*, 2000)

This process uses a continuous rotary dryer in which the material is tumbled, or mechanically turned over with extremely hot air which is continuously flowing in the same direction.

The prime control requirement is to maintain the outlet moisture of the product at a constant value, but it is also interesting to control the output temperature if the exhaust air is used by another process.

A robust controller is needed to keep a good performance of the temperature and moisture values due to the system behavior which depends on the operating point. A multivariable H_∞ controller is proposed, which will be robust enough to work in an appropriate mode according to the performance of the controlled variables.

This article is organized as follows. In section 2, a description of the system is presented. In section 3, the methodology used to compute the controller is shown, including scaling several linear models, estimating an unstructured uncertainty model, and the selection of appropriate weighting functions. Section 4 presents experimental results obtained with the H_∞ controller designed in the previous section. Finally, the major conclusions to be drawn are given in Section 5.

2. SYSTEM DESCRIPTION

The system considered corresponds to a *co-current rotary dryer*. The main objective of the rotary dryer is to reduce the outlet moisture of the product at a desired value by heating the product with the air passing through the dryer.

Furthermore, the exhaust air generated can be used by another auxiliary process or application. A dryer plant, as shown in Fig. 1, includes, in addition to the drum, many auxiliary elements needed for feeding the product and generating the necessary heat.

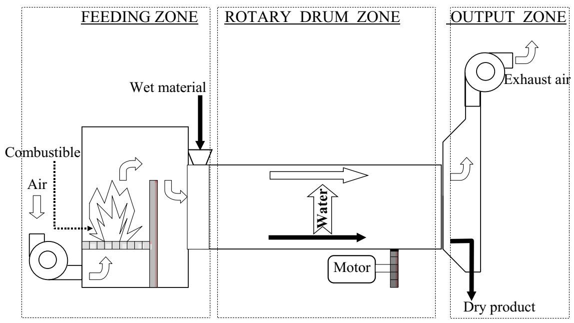


Fig. 1. Co-current rotary dryer

In a rotary dryer, the wet material is continually transported by the rotation of the drum, and dropped through a hot air current that circulates throughout the dryer. The cylinder has a continuous series of blades in its interior, so that while the drying cylinder turns, these blades take the material and throw it into the gaseous current.

With regards to the air flow direction, it is fundamental to use the solid flow in favor of the air current when a great amount of moisture proportion must be evaporated in the first stages of the dryer. This allows high temperatures to be reached in the inlet air without reaching dangerous high temperatures in the product to be dried. Since the air and solid temperatures converge when the flows reach the outlet, the temperature of the solid that leaves the cylinder can be easily controlled until it reaches its maximum value, while maintaining the advantage of having a wide range of temperatures in the first stages.

The system can be described by a set of nonlinear equations describing energy and mass balances (Holgado, 1999) (Rubio *et al.*, 2000). Due to drum's geometry, strong gradients of concentration and temperature appear. Therefore, the application of mass and energy balances give rise to appearance of partial derivatives and differential equations, which can be avoided by dividing the dryer into a finite number of elements in series (Fig.2), where in each element equations are applied.

A detailed dynamic model of the dryer plant is provided in (Holgado, 1999).

A linear model of the system is obtained by linearization at an operating point.

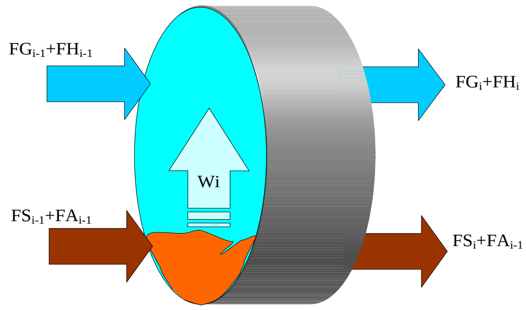


Fig. 2. Element of dryer

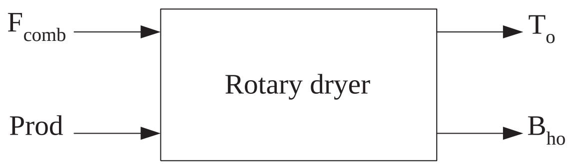


Fig. 3. MIMO system

In this case, the plant can be modelled like a multivariable system (fig.3) where the control variables are the mass flow of fuel (F_{comb}) and the mass flow of wet material(Prod), and the controlled variables are the moisture of the dry product (Bh_o) and the exhaust air temperature (T_o). A multivariable ARX model was calculated, and transfer functions were obtained using a sample time of 120 seconds. Figure 4 shows the operating point (PN) and the working space. These values were obtained from the real plant, where the variation of the fuel mass is around 8% and the variation of the mass flow of wet material is approximately 16%.

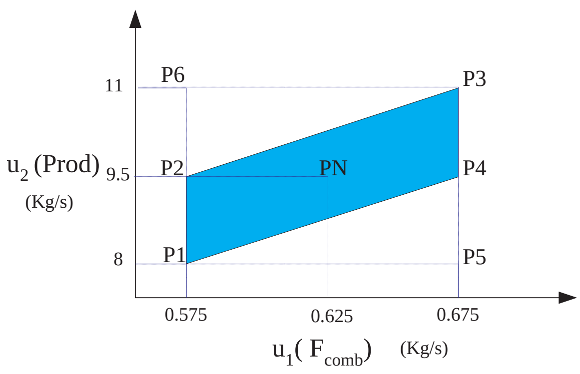


Fig. 4. Working area of control variables

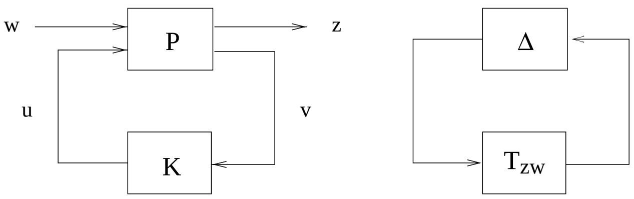
3. CONTROLLER DESIGN

The feedback controller design problem can be formulated as an H_∞ optimization problem, which can be posed under the general configuration shown in Fig.5a. In this figure, P is the generalized plant, K is the controller, u are the control signals, v the measured variables, w the exogenous signals and z are the so-called error variables.

Being T_{zw} the closed-loop transfer function matrix from w to z (see Fig.5b), it will be given by the linear fractional transformation $z = T_{zw}w = F_l(P,K)w$, where:

$$F_l(P,K) = P_{zw} + P_{zu}K(I - P_{vu}K)^{-1}P_{vw}$$

The well known *Small Gain Theorem* (Zames, 1966) says that a sufficient condition for stability of the closed-loop system is $\|T_{zw}\|_\infty \|\Delta\|_\infty < 1$, where $\|\cdot\|_\infty$ denotes the infinity norm and Δ is the system uncertainty. When this norm is applied to a transfer function matrix, T, it has the form $\|T\|_\infty \equiv \sup_\omega \bar{\sigma}(T(jw))$ where $\bar{\sigma}$ denotes the maximum singular value.



(a) General control configuration (b) Stability robustness

Fig. 5. The H_∞ optimization problem

It is known that the H_∞ control approach leads to a closed-loop system with good robustness properties. The closed-loop specifications in the case presented in this paper are:

(1) To guarantee stability for each operation point of the plant.
(2) To get an appropriate performance for all these operation points.

The problem has been solved by employing the classical S/T mixed sensitivity formulation in a one degree-of-freedom setting (Skogestad and Postlethwaite, 1996). This technique consists of *finding a stabilizing controller which minimizes*:

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} W_S S \\ W_T T \end{bmatrix} \right\|_\infty$$

where $S(z)$ is the sensitivity transfer function, $T(z)$ is the complementary sensitivity transfer function and W_S, and W_T are their respective weighting functions. These weighting filters allow to specify the range of frequencies of main importance for the corresponding closed-loop transfer function to be specified:

$$\begin{aligned} S(z) &= (I + G(z)K(z))^{-1} \\ T(z) &= G(z)K(z)(I + G(z)K(z))^{-1} \end{aligned}$$

This mixed sensitivity model was chosen because shaping T is desirable for tracking problems, noise attenuation and for robust stability with respect to multiplicative uncertainties. On the other hand, shaping S will allow to be controlled the performance of the system.

The way in which this minimization problem is put into the standard control configuration is shown in Fig. 6.

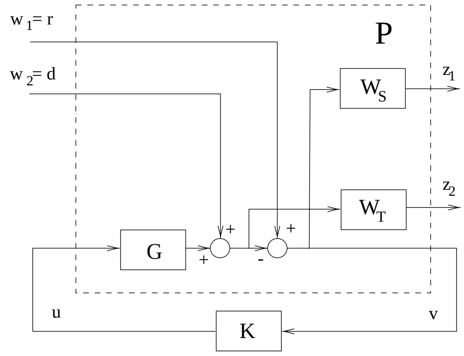


Fig. 6. S/T mixed sensitivity model.

To calculate the weighting functions, some properties of the system should be computed. The main steps are described in the following points.

3.1 *Scaling the Plant*

Scaling the plant is important as it makes the process of analyzing and designing controllers easier. To carry out the scaling it is necessary to determine the expected magnitude of the maximum changes of control signals over each input and the maximum variations allowed of the outputs. In this application the following values have been chosen as maximum variations:

$$\begin{aligned} \triangle F_{comb\ max} &= 0.05Kg/s \\ \triangle Prod_{\ max} &= 1.5Kg/s \\ \triangle T_{o\ max} &= 9.6^oC \\ \triangle Bh_{o\ max} &= 0.02806 \end{aligned}$$

Applying the scaling method described in (Skogestad and Postlethwaite, 1996), the following scaled plant is obtained at the nominal operation point:

$$\hat{G}(z) = \frac{1}{den(z)} \begin{pmatrix} n_{11}(z) & n_{12}(z) \\ n_{21}(z) & n_{22}(z) \end{pmatrix}$$

where

$$n_{11}(z) = 0.01758z^8 - 0.0700z^7 + 0.1176z^6 - 0.1004z^5 + $$
$$+ 0.0373z^4 + 0.0041z^3 - 0.0061z^2 - 0.0011z + 0.0012$$
$$n_{21}(z) = -0.0012z^8 + 0.0080z^7 - 0.0352z^6 + 0.0993z^5 - $$
$$- 0.1797z^4 + 0.2109z^3 - 0.1593z^2 + 0.0715z - 0.0144$$
$$n_{12}(z) = -0.0407z^8 + 0.1678z^7 - 0.3073z^6 + 0.3122z^5 - $$
$$- 0.1810z^4 + 0.0550z^3 - 0.0072z^2 + 0.0023z - 0.0014$$

$$n_{22}(z) = 0.0060z^8 - 0.0346z^7 + 0.1150z^6 - 0.2516z^5 +$$
$$+ 0.3783z^4 - 0.3805z^3 + 0.2440z^2 - 0.0933z + 0.0170$$
$$den(z) = z^9 - 5.883z^8 + 15.721z^7 - 25.0005z^6 + 25.963z^5 -$$
$$- 18.1238z^4 + 8.3969z^3 - 2.4345z^2 + 0.3825z - 0.0217$$

This plant will be adopted like the nominal model for the design of the controller.

3.2 *Estimating uncertainties*

Several models have been obtained around the nominal operating point to estimate an unstructured uncertainty in its multiplicative output form:

$$\hat{E}_o(s) = (\hat{G}^*(s) - \hat{G}(s)) \cdot \hat{G}(s)^{-1}$$

where $\hat{G}(s)$ is the nominal plant and $\hat{G}^*(s)$ represent each one of the models obtained around the operating point. A plot of the singular values of $\hat{E}_o(j\omega)$ for every non-nominal model is shown in Fig.7.

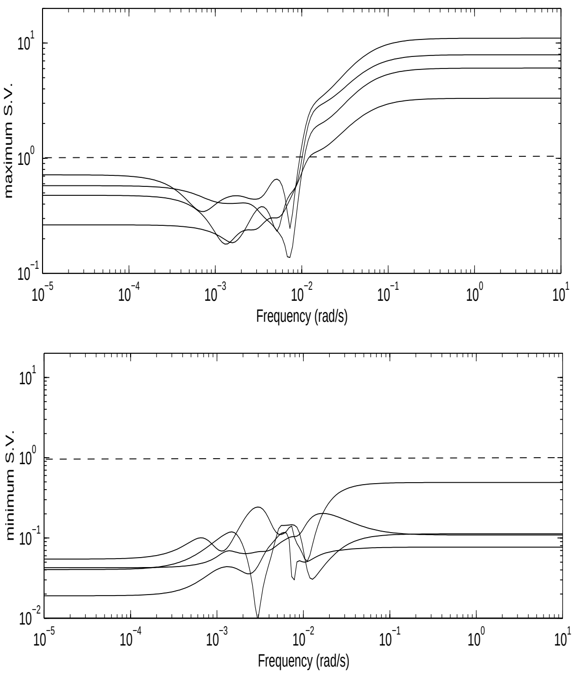


Fig. 7. Multiplicative output uncertainty.

3.3 *Selecting the Weighting Functions*

To obtain a controller via the S/T mixed sensitivity problem, two weighting matrices of transfer functions must be selected, one for shaping the sensitivity function $S(s)$ and the other one to shape the complementary sensitivity function $T(s)$. The above results have been taken into account to carry out this selection.

The weighting $W_T(s)$ has been selected as a square, bidimensional, diagonal matrix. The elements of its diagonal have been designed as a high pass filter such as:

$$|W_{T11}(j\omega)| > \overline{\sigma}(E_o(j\omega)) \qquad \forall\, \omega, \qquad \forall\, E_o$$
$$|W_{T22}(j\omega)| > \underline{\sigma}(E_o(j\omega)) \qquad \forall\, \omega, \qquad \forall\, E_o$$

The weighting $W_S(s)$ has been selected as a square, bidimensional, diagonal matrix with each of its diagonal elements with the form (Ortega, 2001):

$$W_{Sii}(s) = \frac{\alpha_i\ s + \omega_{Si}}{s + \beta_i \omega_{Si}} \qquad i = 1, 2$$

$$\alpha_i \geq 0.5 \qquad \beta_i \ll 1 \qquad \omega_{Si} = 10^{(\kappa_i - 1)} \omega_{T_{11}}$$

They are low pass filters where ω_{Si} represents their bandwidth frequencies. These frequencies were initially chosen as the values obtained from $\kappa_i = 0$. Therefore, $\omega_{Si} = 10^{-1}\omega_{T_{11}}$ where $\omega_{T_{11}}$ is the crossover frequency of $W_{T11}(s)$ function previously designed. Usually, this election provides slow responses and it is necessary to increase these values.

3.4 *Building up the Generalized Plant and Obtaining the Controller*

Finally, after calculating the transfer functions matrices, the generalized plant can be built. This plant will be given to an implemented H_∞ synthesis algorithm (for example (Balas *et al.*, 1995)) which yields the H_∞ controller. At this point, it is important to remember that the controller has been calculated for a scaled model. Thereby, an inverse-scale process must be performed on the controller before implementing it in the real plant.

4. SIMULATION RESULTS

A nonlinear simulator based on mass and energy balances has been developed (Holgado, 1999) based on the modelling language *Dymola* (Elmqvist *et al.*, 1996). The parameters of this model were adjusted until getting an accurate behavior with respects to the real plant. Thereby, this simulator constitutes a testbed which reflects the most dynamic aspects of the real plant.

Several tests were carried out on the nonlinear model plant, readjusting the weighting functions until obtaining a good performance.

The weighting matrix to shape the complementary sensitivity functions consisted of the following transfer functions:

$$W_{T11}(s) = \frac{75s + 0.225}{s + 0.3}$$

$$W_{T22}(s) = \frac{20s + 0.06}{s + 0.3}$$

and the magnitude of their Bode diagrams are shown in Fig. 8.

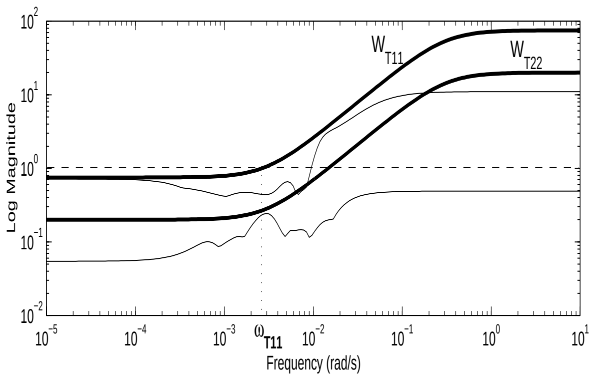


Fig. 8. Magnitude of the W_T weighting transfer functions.

The value of the frequency $\omega_{T_{11}}$ must be obtained from the previous design of $W_{T11}(s)$. In this case this frequency is about $0.003 rad/s$. This value is necessary to calculate the initial weighting matrix $W_S(s)$, which was designed with the following values:

$$\alpha_i = 0.5 \quad \beta_i = 10^{-4} \quad \kappa_i = 0 \quad \omega_{T_{11}} = 0.003$$

The above selection yields the following weighting functions:

$$W_{S11}(s) = \frac{0.5s + 3\ 10^{-4}}{s + 3\ 10^{-8}}$$

$$W_{S22}(s) = \frac{0.5s + 3\ 10^{-4}}{s + 3\ 10^{-8}}$$

which were employed to synthesize an initial controller. Singular values of the sensitivity transfer matrix and their respective weights for this initial design are shown in Fig. 9 .

The response of the system with this first controller is presented in Fig. 10. It can be observed how the moisture response (Bh_o) is very slow and presents some undesirable oscillations.

These results can be improved if parameters α_2 and κ_2 are modified. Increasing κ_2 yields faster moisture response (Bh_o), while increasing α_2 is desirable to reduce the amplitude of the oscillations. In this way, the following parameter values were proposed:

$$\alpha_1 = 0.5 \qquad \kappa_1 = 0$$

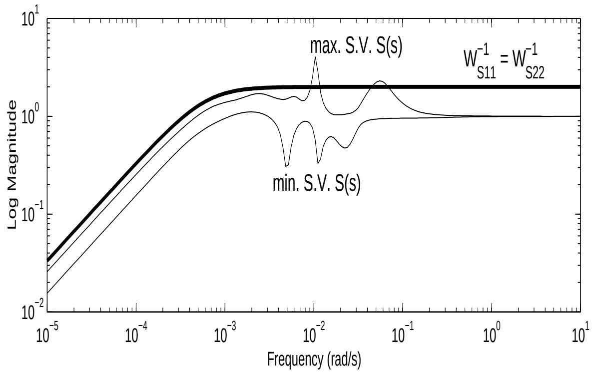


Fig. 9. Sensitivity singular value and W_S^{-1}. ($\alpha_i = 0.5$, $\beta_i = 10^{-4}$, $\kappa_i = 0$, $\omega_{T_{11}} = 0.003$)

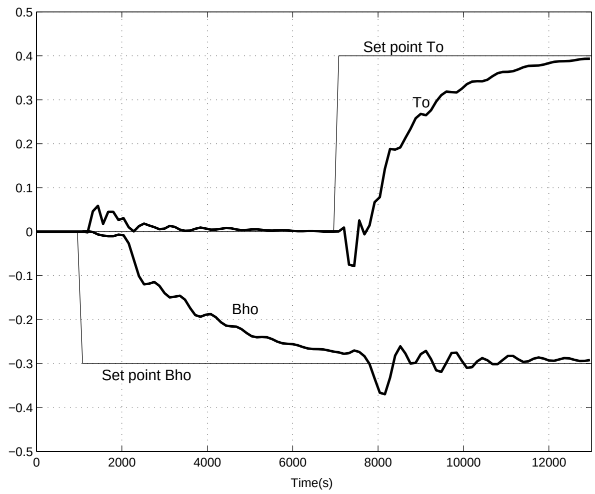


Fig. 10. Simulation results with the initial controller.

$$\alpha_2 = 0.7 \qquad \kappa_2 = 0.5$$

which supply the following weighting functions (see Fig.11):

$$W_{S11}(s) = \frac{0.5s + 3\ 10^{-4}}{s + 3\ 10^{-8}}$$

$$W_{S22}(s) = \frac{0.7s + 9.48\ 10^{-4}}{s + 9.48\ 10^{-8}}$$

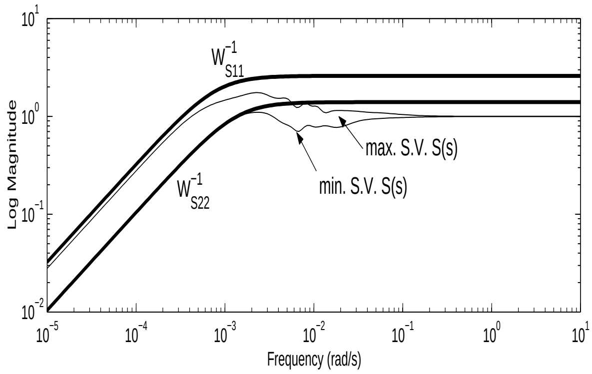


Fig. 11. Sensitivity singular value and W_S^{-1}. ($\alpha_1 = 0.5$, $\alpha_2 = 0.7$, $\kappa_1 = 0$, $\kappa_2 = 0.5$)

The results obtained with this second controller are shown in Fig.12, where it can be seen how the performance obtained is quite acceptable. Rising time for the moisture (Bh_o) is improved from 5000 *seconds* to 2000 *seconds* and its oscillations disappear. However, these improvements imply a deterioration in the behavior of the exhaust air temperature (T_o) when a change of moisture reference is required.

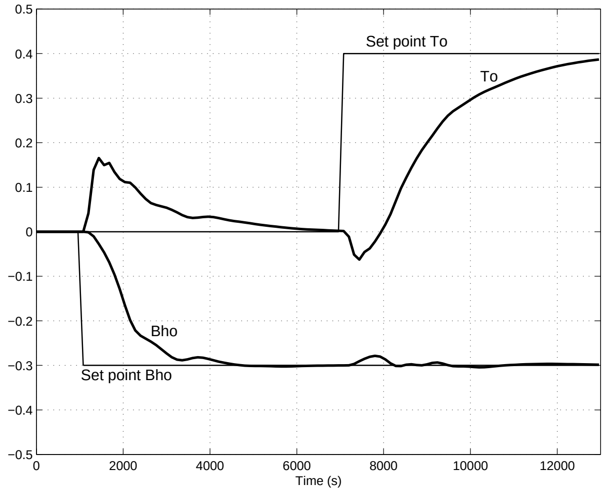


Fig. 12. Simulation results with the final controller.

The computed controllers were reduced in order to avoid numeric problems in the real distributed control system. Other tests have been performed at other operating points obtaining similar results, which validates the robustness of the controller.

5. CONCLUSIONS

A multivariable H_∞ controller for a rotary dryer has been designed. After scaling several linear identified plants at different operating points, and estimating an unstructured uncertainty with respect to a nominal plant, a classical S/T mixed sensitivity problem has been used to design a robust controller. The selection of the weighting matrix was carried out by adjusting the performance of the simulated closed loop response. Finally, simulation results have been presented, which show an appropriate performance of the proposed controller.

ACKNOWLEDGEMENTS

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