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DISTURBANCE ATTENUATION OF AN EXPERIMENTAL pH NEUTRALIZATION SYSTEM

L. G. S. Longhi[1] and P. R. Barrera[2]

[1]*Department of Chemical Engineering, Pontifícia Universidade Católica do Rio Grande do Sul, Rua Henrique Dias, 236/24, 90035-100, Porto Alegre, RS, Brazil*
FAX +55-51-3316-3277, llonghi@pucrs.br
[2]*Department of Chemical Engineering, Federal University of Rio Grande do Sul, Porto Alegre, RS, Brazil*
barrera@enq.ufrgs.br

Abstract: The classical control theory is based on the design of linear controllers for systems described by linear models. However, there exist some situations where it is not recommended, or even possible, to use a fixed linear controller to control a process. One of those situations arises when the magnitude of the process gain experiences a dramatic variation within the operating range of interest. A classic example of a chemical process where this situation occurs is the pH control around the neutralization point in a continuous stirred tank. In this work, the disturbance attenuation problem for a strong acid – strong base pH control system is addressed. To solve this problem, a nonlinear $\mathcal{H}_\infty$ control law is derived based on a nonlinear model previously developed. The attainment of that control law is done with the help of recent mathematical results from the authors concerning the solution of Hamilton-Jacobi inequalities. The nonlinear controller is implemented on an experimental reactor and its performance is compared with a PID control law tuned according to the classical minimum error integral criteria. The obtained results show that the nonlinear $\mathcal{H}_\infty$ control theory can be a good alternative to solve this difficult SISO control problem.

Keywords: Nonlinear control, pH control, H-infinity control, Control applications, Process control.

1. INTRODUCTION

The theory of classic control is based on the design of linear controllers for systems described by linear models or nonlinear models linearized around an operating point. However, there exist some situations where it is not recommended to use such a linear controller. One of those situations arises when the magnitude of the process gain experiences a dramatic variation within the operating range of interest. In this situation, the use of a fixed linear controller can lead to a poor performance of the closed loop system and even to its loss of stability. A classical example of a chemical process where this situation happens is the pH control around the neutralization point in a continuous stirred tank. In this control problem, the titration curve – which represents the system's input-output map - presents a highly nonlinear behavior in response to addition of acid or base. This behavior is amplified even more if the reagents are strong acid and/or base.

In the present work, the objective is to control the pH of an experimental system within this difficult range of operation. The problem can be stated as to maintain the pH in the neutralization point manipulating a strong base stream in response to disturbances on the strong acid flow. To solve this disturbance attenuation problem, a nonlinear $\mathcal{H}_\infty$ controller is designed and implemented in a bench-scale plant. This synthesis approach is possible due to recent mathematical results from the authors concerning the solution of Hamilton-Jacobi inequalities (Longhi, *et al.*, 2001). It must be emphasized that this control law synthesis is not based on any kind of linearization procedure, such as multi-linear models (Gálan, *et al.*, 2000), gain scheduling or adaptive (Su, *et al.*, 1998) schemes, nor in a change of the control objective to fit a known solution method (Li and Zhang, 1999).

2. DISTURBANCE ATTENUATION PROBLEM

Consider the IA (Input-Affine) nonlinear system description of equation (1).

$$\dot{x} = f(x) + g(x)u + k(x)w$$
$$z = \begin{bmatrix} h(x) \\ u \end{bmatrix} \tag{1}$$

where $x \in M$ ($M \subseteq \Re^n$) is the vector of the system's state variables defined on a neighborhood of the origin, $w \in \Re^q$ is the vector of exogenous inputs, $u \in \Re^m$ is the vector of control inputs and $z \in \Re^s$ is the vector of exogenous outputs which characterizes the control objective. The mappings f(x), g(x), k(x) and g(x) are assumed to be nonlinear smooth functions and, for simplicity, f(0) = h(0) = 0.

The disturbance attenuation problem via state feedback is concerned with the construction of a feedback controller, u(x), satisfying two objectives: (1) To asymptotically stabilize the resulting closed-loop plant, and (2) To minimize the influence of the exogenous inputs, w, on the objective variable, z. If the influence from w(t) on z(t) is measured as the finite $\mathcal{L}_2$-gain between these variables, this disturbance attenuation problem can be solved by using the results from the nonlinear $\mathcal{H}_\infty$ control theory (Isidori and Astolfi, 1992). In fact, for linear systems, the $\mathcal{L}_2$-gain has the same meaning of the $\mathcal{H}_\infty$ norm for the system operator. Here, the $\mathcal{L}_2$-gain is defined as in Van der Schaft (1992).

Definition 1 (Finite $\mathcal{L}_2$-gain). Given any $\gamma > 0$, the mapping from w(t) to z(t) is said to have finite $\mathcal{L}_2$-gain less than or equal to γ if, under the zero initial condition $x(0) = 0$,

$$\int_0^T \|z(t)\|^2 dt \le \gamma^2 \int_0^T \|w(t)\|^2 dt \tag{2}$$

for all $T \ge 0$ and all $w(.) \in \mathcal{L}_2(0,T)$, where $\| \bullet \|$ denotes the Euclidean norm. □

However, as the minimization of the $\mathcal{L}_2$-gain can lead to a controller with a very small validity region (Yazdanpanah, *et al.*, 1999) or very near to the stability frontier (Keel and Bhatacharyya, 1997), it is usual in the literature to consider only suboptimal solutions to the disturbance attenuation problem. In this case, the minimum of γ is replaced by the gain attenuation at some acceptable level. The suboptimal solution to the nonlinear $\mathcal{H}_\infty$ control problem for a system described by equation (1) can be given by theorem 1 (Van der Schaft, 1999).

Theorem 1 (Local sub-optimal solution to the nonlinear $\mathcal{H}_\infty$ control problem via state feedback for IA systems). Consider the nonlinear system of equation (1) and a real parameter $\gamma > 0$. Suppose that exists a smooth positive definite solution, $V(x) > 0$, to the HJI (Hamilton-Jacobi-Isaacs) inequality given by equation (3),

$$H_*(x) = \frac{\partial V}{\partial x}(x) f(x) + \frac{1}{2} h^T(x) h(x)$$
$$- \frac{1}{2}\frac{\partial V}{\partial x}(x)\left(g(x)g^T(x) - \frac{1}{\gamma^2} k(x)k^T(x)\right)\frac{\partial^T V}{\partial x}(x) < 0 \tag{3}$$

then, the closed-loop system with the feedback of equation (4),

$$u_*(x) = -g^T(x)\frac{\partial^T V}{\partial x}(x) \tag{4}$$

is asymptotically stable at the origin and has locally a $\mathcal{L}_2$-gain (from w to z) less or equal to γ. Moreover, the worst-case disturbance is given by equation (5).

$$w_*(x) = \frac{1}{\gamma^2} k^T(x)\frac{\partial^T V}{\partial x}(x) \tag{5}$$

□

It must be noted that theorem 1 does not give a method to solve the problem nor the size of the local state-space region where its solution works. In fact, these are the main drawbacks to apply the results from the nonlinear $\mathcal{H}_\infty$ theory to real systems. Before to go to the developed solution, it is necessary to define what is the validity region for the nonlinear $\mathcal{H}_\infty$ controller. This definition was based on (Yazdanpanah, *et al.*, 1999).

Definition 2 (Nonlinear $\mathcal{H}_\infty$ controller validity region). The region of the state space of equation (1) that, subject to the nonlinear state feedback law from theorem 1, simultaneously satisfies the HJI inequality and guarantees asymptotic stability of the worst-case disturbance of equation (5) in closed-loop system, is referred to as the validity region corresponding to the controller of equation (4). Any region that is a subset of this state-space region is referred to as an estimate of the validity region. □

In this work, the solution for the nonlinear $\mathcal{H}_\infty$ control problem is found by solving the optimization problem 1 (Longhi, *et al.*, 2000). The solution of this optimization problem furnishes a control law and an estimate of its validity region. Its formulation, based on preliminary results concerning the positivity of multivariable scalar functions (Longhi, *et al.*, 2001), requires the definition 3.

Definition 3 (Real local region). The real local region of a multivariable scalar function y(x) is the set composed by the subsets of the real field where each element of x can assume values such that y(x) is real and $y \neq 0$ unless $x = 0$. □

Optimization problem 1 (Size maximization of the validity region for the nonlinear $\mathcal{H}_\infty$ controller). Choose the form of function V(x) and substitute in $H_*(x)$. Write these two functions as quadratic form representations: $V(x) = \Theta(x)^T P_V \Theta(x)$ and $H_*(x) = \Phi(x)^T P_H \Phi(x)$, where P_V and P_H are symmetric real matrices obtained directly from the coefficients of V(x) and $H_*(x)$, respectively. Write the time derivative of V(x) as the quadratic form representation: $\dot{V}(x) = \Psi(x)^T.P_{Vd}.\Psi(x)$, where P_{Vd} is a symmetric real matrix obtained directly from the coefficients of $\dot{V}(x)$. Let α_i, β_i and δ_i be the parameters which define the positivity region of V(x) and the negativity regions of $\dot{V}(x)$ and $H_*(x)$, respectively. Choose the parameters of V(x), Θ(x), Φ(x) and Ψ(x) in a way to maximize the region defined by V(x) = C subject to the constraints $P_V > 0$, $P_H < 0$, $P_{Vd} < 0$ and $\gamma_{min} < \gamma < \gamma_{max}$. The parameter C $\in \Re^+$ is obtained as the minimum value of V(x) intersecting the positivity region of V(x) and the negativity regions of $\dot{V}(x)$ and $H_*(x)$. The solution, V(x), solves locally the problem within the validity region defined by V(x) < C for the γ level attenuation. □

Roughly speaking, the optimization problem 1 tries to find a solution to the HJI inequality that maximizes the size of the validity region associated with that solution. This is quite different from the usual approach in the nonlinear $\mathcal{H}_\infty$ control theory where the main objective is to maximize the level of attenuation regardless to the fact that the resulting controller has a practical validity region or not.

Despite the fact that optimization problem 1 can be considered a very general approach to solve the nonlinear $\mathcal{H}_\infty$ control problem via state feedback, usually it is a very complex problem, many times intractable. To reduce its dimension, some simplifications can be done. One expected problem occurs when it is desired to use non-ellipsoidal (non-quadratic) forms to represent V(x). In these cases, it could be very tedious to find an equation for the area of V(x) = C. Furthermore, the resulting equation can have a very complex form, inadequate for using in an optimization problem. So, aiming the simplification of the problem, it is recommended to use, when possible, quadratic forms to represent V(x).

Moreover, if the situations 1 and/or 2, below, occur, the problem's size can be considerably reduced:

1. If V(x) is globally positive definite, then the parameters α_i are eliminated from the problem.
2. If the condition of equation 6 is satisfied, then the signal of $\dot{V}(x)$ does not need to be evaluated.

$$\left(g(x)^T g(x) - \frac{1}{\gamma^2} k(x)^T k(x) \right) \geq 0 \qquad (6)$$

In the last case, the HJI inequality satisfaction implies the negativity of $\dot{V}(x)$ in the same state space region. This situation occurs when the description of the IA system is known and the lower bound of γ is defined as the minimum necessary to assure that the inequality (6) holds.

3. NEUTRALIZATION SYSTEM MODELING

The experimental apparatus is located at the Food Technology and Science Institute of Federal University of Rio Grande do Sul. It is composed by a 2.5 liters stirred tank, pH and flow sensors, and a remote computer responsible for the on-line computation of the control actions, see figure 1. The acid stream is composed by HCl 0.1 M and the basic stream by NaOH 0.1 M, where M denotes the molar concentration, [gmol L^{-1}].

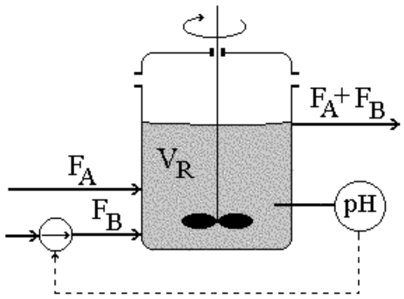

Fig. 1. Sketch of the continuous pH neutralization process.

The process model considered in this work uses the change of coordinate proposed by (Narayanan et al., 1998): $\eta = [H^+] - [OH^-]$, where [A] denotes the molar concentration of the chemical specie A. The relation between the variable, η, and the original variable, pH ($pH = -\log_{10}[H^+]$), is given by equation (7). The advantage of using η instead of pH is the attainment of a concise IA model, as it can be seen in equation (8).

$$\eta = 10^{-pH} - \frac{K_W}{10^{-pH}} \Leftrightarrow pH = -\log_{10}\left(\frac{\eta + \sqrt{\eta^2 + 4K_W}}{2} \right) \qquad (7)$$

$$\frac{1}{V_R}\frac{d\eta}{dt} = F_A C_{A0} - F_B C_{B0} - (F_A + F_B)\eta \qquad (8)$$

where V_R is the reactor volume, F_A is the acid stream flow, F_B is the basic stream flow, C_{A0} is the acid concentration in the acid stream, C_{B0} is the base concentration in the basic stream and $K_W = 10^{-14}$ is the equilibrium constant for the water dissociation.

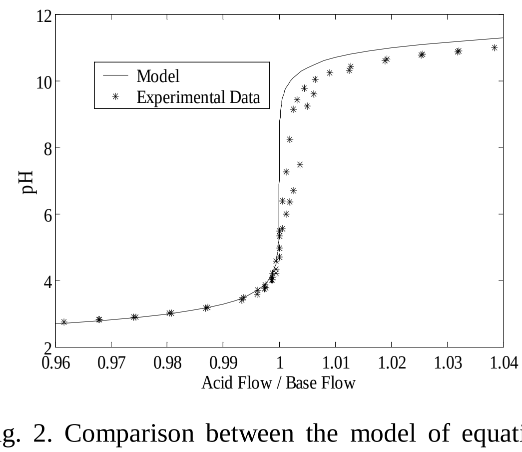

Fig. 2. Comparison between the model of equation (8) and the experimental data.

Figure 2 presents the comparison between the model of equation (8) and the experimental data. The highly nonlinear behavior of the pH system can be easily seen in this figure. The modeling error in the basic region - due to the acid characteristic of the available water used in the experiments - can also be seen in the figure.

4. NONLINEAR CONTROLLER SYNTHESIS

In order to obtain a description whose the steady state of interest is the origin, the following new variables were defined: $\theta_A = F_A / V_R$, $\theta_B = F_B / V_R$, $x_1 = \eta - \eta_{SS}$, $w = \theta_A - \theta_{ASS}$ and $u = \theta_B - \theta_{BSS}$.

Furthermore, to assure the off-set suppression, an additional state was incorporated to the original control variable: $v(x) = u(x) + T_i . x_2$ (where $\dot{x}_2 = dx_2/dt = x_1$). Now, the control system can be adequately represented by equation (9) and the objective variable $z = \begin{bmatrix} x_1 \\ u \end{bmatrix}$.

$$\dot{x} = \begin{pmatrix} -(C_{B0}+x_1).T_i.x_2 \\ x_1 \end{pmatrix} + \begin{pmatrix} -(C_{B0}+x_1) \\ 0 \end{pmatrix} u + \begin{pmatrix} C_{A0}-x_1 \\ 0 \end{pmatrix} w \quad (9)$$

If a simple Lyapunov function $V(x) = a x_1^2 + b x_2^2$ is considered, the HJI inequality (10) must be solved for V(x) > 0.

$$H_*(x) = \left(\frac{a^2}{\gamma^2} - a^2\right) x_1^4 + \left(-0.2\frac{a^2}{\gamma^2} - 0.2a^2\right) x_1^3 - 2aT_i x_1^2 x_2 + \left(-0.01a^2 + 0.01\frac{a^2}{\gamma^2} + 1\right) x_1^2 + \left(-0.2aT_i + 2b\right) x_1 x_2 < 0 \quad (10)$$

As the LaSalle's detectability condition is satisfied, it is sufficient to find a solution for the non-strict inequality $H_*(x) \leq 0$ to solve the disturbance attenuation problem. In order to cancel the quadratic cross product (x_1x_2) from inequality (10), it is assumed that T_i=10 b/a. Now, the HJI inequality can be rewritten as:

$$H_*(x) = a^2\lambda x_1^4 - 0.2a^2(\lambda+2)x_1^3 - 20b.x_1^2 x_2 + \left(1 + 0.01a^2\lambda\right)x_1^2 < 0 \quad (11)$$

where $\lambda = \left(\frac{1}{\gamma^2} - 1\right)$.

A necessary condition for a local solution to inequality (11) is $(1 + 0.01a^2\lambda) \leq 0$. This situation only occurs if $\lambda < 0$, which has the same meaning of $\gamma > 1$. Then, if the lower bound of γ is fixed as equal to 1 in the formulation of the optimization problem 1, the condition of equation (6) is automatically satisfied, and the searching for a local solution to inequality (11) becomes easier.

To solve the optimization problem 1, the quadratic form representation of equation (12) was used.

$$H_*(x) = \begin{bmatrix} x_1 \\ x_2 \\ x_1^2 \\ \sqrt{\beta_1 + x_1}\, x_1 \\ \sqrt{\beta_2 + x_2}\, x_1 \end{bmatrix}^T . P_{H_*} . \begin{bmatrix} x_1 \\ x_2 \\ x_1^2 \\ \sqrt{\beta_1 + x_1}\, x_1 \\ \sqrt{\beta_2 + x_2}\, x_1 \end{bmatrix} \quad (12)$$

where the matrix P_{H*} is givev by:

$$P_{H*} = \begin{bmatrix} 1 + 0.01a^2\lambda + 0.2a^2(\lambda+2)\beta_1 + 20b\beta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a^2\lambda & 0 & 0 \\ 0 & 0 & 0 & -0.2a^2(\lambda+2) & 0 \\ 0 & 0 & 0 & 0 & -20b \end{bmatrix}$$

So, the optimization problem 1 can be stated as:

$$[a, b, T_i, \beta_1, \beta_2, \gamma] = \arg \min_{\substack{a,b>0 \\ P_{H*} \leq 0 \\ \beta_1, \beta_2 > 0 \\ \gamma_{min} < \gamma < \gamma_{max} \\ \beta 1 = f(a,b),\ \beta_2 = f(a,b)}} (a.b) \quad (13)$$

In order to simplify the delimitation of the controller validity region, it was assumed that $\beta_1 = \beta_2$. In addition, it was arbitrarily chosen that the maximum and minimum level of attenuation (γ) are 1.0 and 1.5, respectively. The solution for the problem (13) is given by:

$$\begin{cases} a = b = 2837.1 \\ \beta_1 = \beta_2 = 1.874.10^{-2} \\ \gamma = 1.5 \\ T_i = 1.0 \end{cases} \quad (14)$$

and, consequently, the controller validity region is given by equation (15).

$$V(x) = 2837.1(x_1^2 + x_2^2) < 1 \quad (15)$$

which is nearly equivalent to the pH region given by: 12 < pH < 2.

Finally, one can construct the suboptimal nonlinear $\mathcal{H}_\infty$ controller as a function of the original control variables η:

$$F_B(\eta) = 1.510^3\left(283.71(\eta - \eta_{SP}) + 2837.1(\eta - \eta_{SP})^2 + 10\, x_2\right)$$
$$\dot{x}_2 = (\eta - \eta_{SP}) \quad (16)$$

Since the disturbance attenuation problem solved by the nonlinear $\mathcal{H}_\infty$ control theory only considers vanishing perturbations, for the more realistic case of persistent disturbances, it is hoped that the controller validity region be sufficiently large to support strong disturbances on w(t).

5. EXPERIMENTAL RESULTS

To implement the controller of equation (16) in the experimental plant, it was needed to develop an interface to connect the plant to a remote computer. This interface was written using the Matlab/Simulink environment. In figure 3, the graphical interface used to control the plant is shown. The interface's inner program measures the pH and proceeds a nonlinear static transformation into the variable η. Then, this variable is feed in a PI controller, which is simply the controller of equation 16 without the quadratic term, together with the desired set-point. This slight modification of the control law is justified because several simulations had shown that this last term has a negligible contribution in the control law.

In the figure 4, the nonlinear $\mathcal{H}_\infty$ controller behavior is shown in response to some disturbances on the acid stream (see table 1). In the same figure, it is plotted the performance of an usual PI controller tuned according with the classical minimum integral error criteria. The PI tuning was constructed using the ITAE parameters for the disturbance attenuation case and an experimentally identified FOPDT (First Order Plus Dead Time) model (Seborg, *et al*., 1989) for the same operating point.

In the figure 5, the responses of the same both controllers is shown for set-point changes. It can be noted that the good properties of the nonlinear $\mathcal{H}_\infty$ controller are not maintained. This could be expected because the nonlinear controller was not designed to compass this kind of situation. In fact, as distant from the neutralization point, worst is the nonlinear $\mathcal{H}_\infty$ controller performance. A possible ad-hoc solution for this problem could be found by choosing different controller's settings for each operating condition, characterizing a gain scheduling procedure. However, this approach was not validated yet.

Table 1. Disturbance applied on the acid stream.

Time [s]	Acid Flow [ml/min]
0	0
0^+	2.40
500	3.52
1000	4.80
1300	1.28

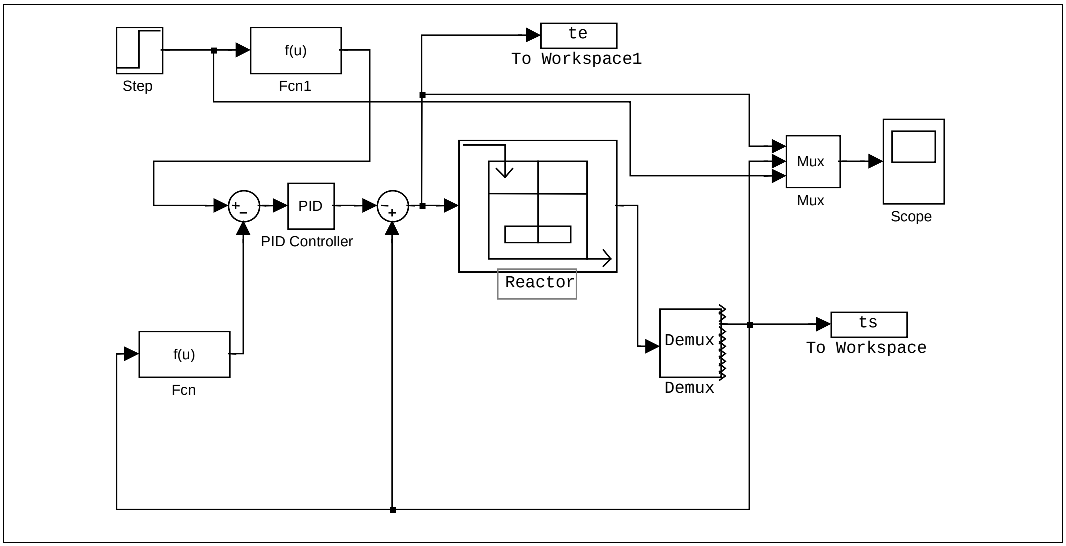

Fig. 3. The developed MATLAB/SIMULINK interface used to control the experimental system.

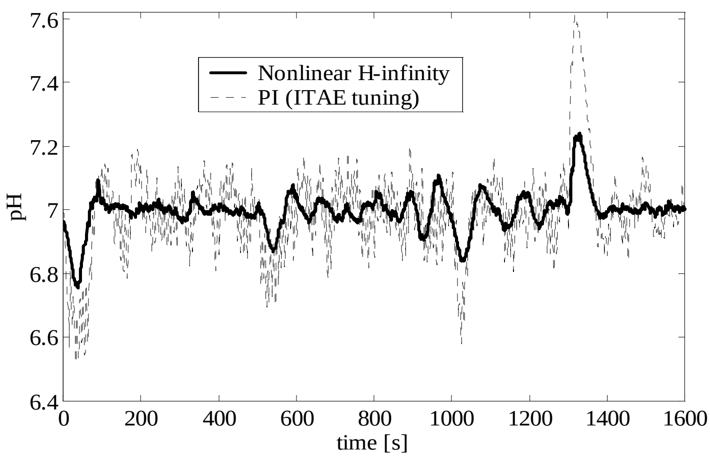

Fig. 4. Comparison between the nonlinear $\mathcal{H}_\infty$ controller and a well-tuned PI controller for the disturbance attenuation case.

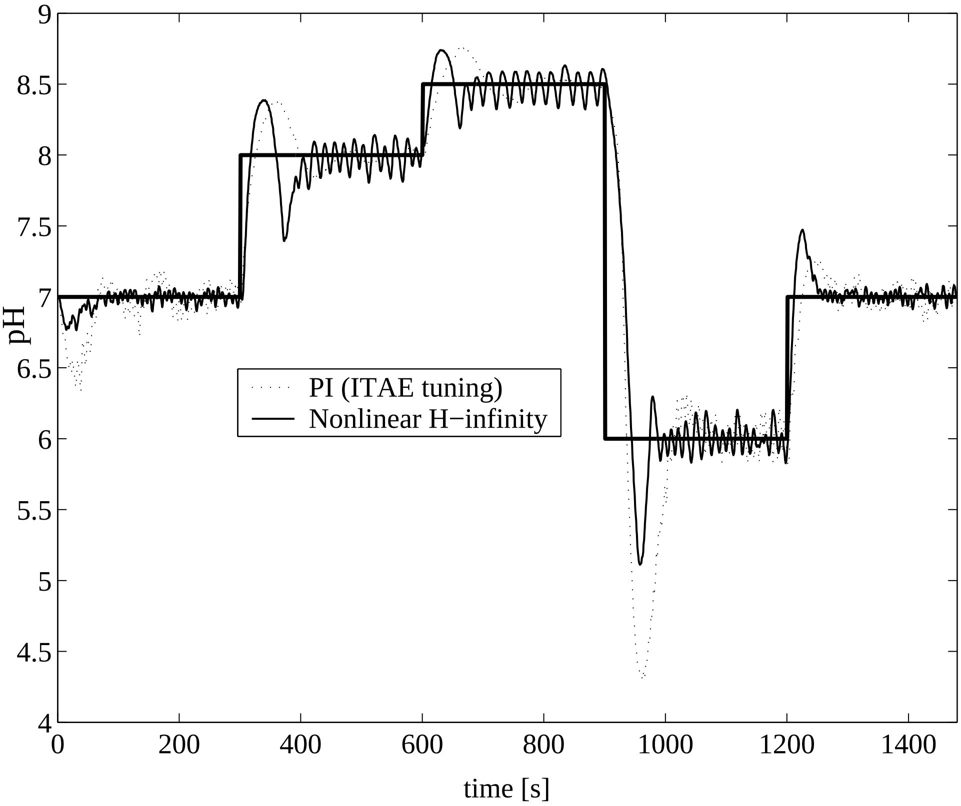

Fig. 5. Comparison between the nonlinear $\mathcal{H}_\infty$ controller and the PI controller for set-point changes. (The solid line steps represent the set-points)

6. CONCLUSIONS

In this work, the pH control of a continuous stirred tank was addressed. A nonlinear $\mathcal{H}_\infty$ controller was designed and implemented in a bench-scale plant. The controller synthesis, based on preliminary mathematical results from the authors, guarantees a validity region for the controller. The main disadvantage of the design method is that it is still not automated as an algorithm. For the author's knowledge, this is the first application of nonlinear $\mathcal{H}_\infty$ controller to an experimental chemical process. The performance of the nonlinear controller was also compared with a well-tuned PI controller.

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