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A SIMPLE ANTI-WINDUP STRATEGY FOR STATE CONSTRAINED LINEAR CONTROL

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Abstract: A new Anti-Windup Bumpless Transfer (AWBT) scheme for state constrained linear systems is presented. The AWBT strategy is based on the design of a special limiting circuit in the framework of standard AWBT control. The limiting circuit involves a two step calculation: first, the effect of the current control action over future state constraints is predicted; next, if any violation of the constraints is detected, an allowed control action is back-calculated. The resulting control action is such that the offending constrained states are taken to their saturation limits. Connections between the proposed state constraint AWBT scheme and Model Predictive Control (MPC) are also investigated. The state constraint AWBT and MPC strategies are shown to be equivalent in a non-trivial region of the state space.

Keywords: windup, constraints, predictive control, quadratic control, optimality.

1. INTRODUCTION

All real world control systems are subject to either or both input and state constraints. These can represent physical constraints (hard constraints) or limitations imposed on the relevant variables (soft constraints) in order to meet certain specified requirements *e.g.*, safety regulations. If one ignores the constraints when designing a control strategy, then significant degradation in the resulting closed loop performance may result.

Different methods can be adopted to deal with this difficulty. A possible classification includes the *cautious*, *evolutionary* and *tactical* approach (Goodwin, 2001). The *cautious* approach is the simplest way of tackling the problem, and involves reducing the demand on the control performance until the constraints are avoided under all expected operational regimes. However, this approach may lead to conservative designs since it does not take advantage of the available control authority, possibly compromising efficiency and productivity. The *evolutionary* approach consists in designing, first, a linear controller without considering the constraints, and then modifying the controller implementation in some way (typically by introducing nonlinearities) in order to compensate for the effect of the constraints on the closed loop performance. Traditional anti-windup schemes fall into this category (Åström and Rundqwist, 1989; Kothare *et al.*, 1994; Kapoor *et al.*, 1998). A clear advantage of these anti-windup strategies is that they are, in general, simple and easy to implement. Hence they have gained a strong appeal in practice. In the *tactical* approach the constraints are included from the beginning in the control design. This potentially leads to improved performance but at the expense of increased complexity of the control strategy. One well known example of this approach is Model Predictive Control (MPC) (Garcia *et al.*, 1989; Morari and Lee, 1997; Mayne *et al.*, 2000).

The scheme described in the current paper fits under the anti-windup classification. However, we depart from the traditional goal in this area of handling input amplitude or slew rate constraints and, instead, extend the ideas to deal with state constraints. A proposed scheme to deal with state constrained systems from an anti-windup perspective has been presented by Park (1999), where an additional compensator is added to the controller in order to reduce the error between the plant output and the output of a suitable process model when state constraints are active. The strategy relies on the assumption that the constraints in the plant are hard constraints.

Here we present a general purpose anti-windup strategy for state constrained systems which can effectively deal with both soft and hard state constraints. The controller is based on a relatively standard structure for AWBT schemes but incorporates a special design for the associated limiting circuit. For simplicity we consider the single input-single output case. However, extension to more general cases are possible and will briefly be addressed in Section 5. We also investigate the connection between the proposed AWBT strategy and MPC, describing the conditions under which both strategies are identical. The analysis extends the result recently presented by De Doná and Goodwin (2000) to state constrained systems.

2. REVIEW OF ANTI-WINDUP STRATEGIES FOR INPUT CONSTRAINTS

Input saturation is arguably the most common type of constraint encountered in practice. Consequently, the problem of dealing with input constraints has received attention since the early stages of the development of control applications (see, for example, Fertik and Ross (1967) and Lozier (1956)). AWBT schemes are generally based on various modifications to an otherwise linear control. These special embellishments of the controller compensate for the detrimental effects of the constraints on the plant. The corresponding modifications to the controller can be generated in many different ways. Thus, a variety of AWBT schemes have appeared in the literature (Bernstein and Michel, 1995). A general framework to describe and classify AWBT schemes can be found in Kothare *et al.* (1994). The main idea behind most of these anti-windup schemes is a mechanism for "informing" the controller states that the constraints are active so that appropriate modifications to the future control actions can be taken.

For the SISO case a general implementation of an AWBT scheme for input constraints is depicted in Figure 1. To explain the symbols in Figure 1, we assume that the controller $C(q)$ (where q is the forward shift operator) is biproper and minimum phase. Recall that biproperness is not restrictive, since it can be accounted for by adding *fast* zeros to the controller numerator. In that case the controller can be decomposed as:

$$C(q)^{-1} = h_\infty + H(q) \tag{1}$$

where h_∞ is the high frequency gain of $C(q)^{-1}$ and $H(q)$ is a strictly proper transfer function. If the limiting circuit in Figure 1 is replaced by a unity gain, it can be seen that the transfer function that relates the plant

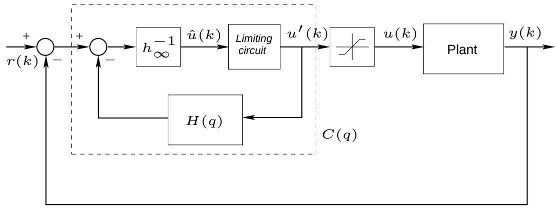

Fig. 1. General anti-windup scheme for input constraints.

input to the error signal is equal to the linear controller transfer function *i.e.*,

$$\frac{U(q)}{E(q)} = \frac{h_\infty^{-1}}{1 + h_\infty^{-1} H(q)} = \frac{1}{h_\infty + H(q)} = C(q) \tag{2}$$

The controller setup presented in Figure 1 has two main characteristics (actually, these are general properties of many AWBT schemes (Goodwin *et al.*, 2001)):

- The states of the controller are driven by the actual constrained control signal feeding the plant. This is achieved by forcing the limiting circuit of Figure 1 to contain the same nonlinearity as the plant input, so that $u'(k) = u(k)$.
- The feedback path $H(q)$ in the controller is stable.

3. ANTI-WINDUP STRATEGY FOR STATE CONSTRAINTS

We next turn to the issue of state constraints. State constraints can potentially cause the same type of difficulties encountered with input constraints. Specifically, whenever the control signal $u(k)$ forces any particular constrained variable into saturation, integrator windup of the controller can occur. We will design the state constrained anti-windup controller by modifying the limiting circuit shown in Figure 1 so that the predicted states never violate the imposed saturation limits. We adopt a discrete time framework. We begin by assuming that the full state vector is measured (we will later show how state estimates can be utilized in lieu of the true states). Thus, consider the discrete time linear system described in state space form by:

$$x(k+1) = A\,x(k) + B\,u(k) \tag{3}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}$, subject to the constraint:

$$z(k) = C\,x(k) \in \mathbb{Z} \triangleq [-\Delta, \Delta],\ \Delta > 0 \tag{4}$$

We restrict the development to the case of a single constraint for clarity. Extensions to the case of multiple constraints will be discussed in Section 5.

A key observation pertaining to the development of an AWBT scheme for state constraints is that the state constraint set $\mathbb{Z}$, implicitly defines a corresponding input constraint set $\mathbb{U}$. This property has been studied before in different contexts. A formal analysis in terms of the theory of maximal output admissible sets can be found in Gilbert and Tan (1991). The relationship between the constraint sets $\mathbb{Z}$ and $\mathbb{U}$ is described in the following Lemma:

Lemma 1. Consider the system (3) subject to the state constraint (4). Then there exists a nonlinear mapping $\varphi(z,x) : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$ which converts the state constraint set $\mathbb{Z}$ into an equivalent input constraint set $\mathbb{U}$. In addition, the set $\mathbb{U}$ is a time varying set which depends at time $t = k$ upon the current value of the plant state vector $x(k)$, that is $\mathbb{U} = \mathbb{U}(x)$.

Proof. The derivation of the nonlinear mapping $\varphi(z,x)$ relies upon the use of a one-step ahead predictor of $z(k)$. Using the system dynamics (3) and the relation $z(k) = C\,x(k)$ we have:

$$z(k+r) = CA^r\,x(k) + CA^{r-1}B\,u(k) \qquad (5)$$

where r is the relative degree of the transfer function $G(q)$ relating the plant input to the constrained variable $z(k)$. Equation (5) shows that the first time in which it is possible to affect the state $z(k)$, given a certain control move $u(k)$ at time $t = k$, is at time $t = k + r$ (this is a consequence of the fact that the first Markov parameters D, CB, CAB, $\ldots$ $CA^{r-2}B$, which appear in the time response of $z(k)$, are zero). Solving for $u(k)$ in (5), shows that the required mapping $\varphi(z,x) : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$ such that $\mathbb{U}(x) = \varphi(\mathbb{Z}, x)$ is given by:

$$u = \varphi(z,x) = (CA^{r-1}B)^{-1}(z - CA^r\,x) \qquad (6)$$

where, for clarity, the time dependence of the variables has been omitted. Using $\varphi(z,x)$ we have the following induced constraint set for the input:

$$\mathbb{U} = \mathbb{U}(x) \triangleq \left[\overline{\Delta}(x)^-, \overline{\Delta}(x)^+\right] \qquad (7)$$

where

$$\begin{aligned}\overline{\Delta}(x)^- &= (CA^{r-1}B)^{-1}(-\Delta - CA^r\,x)\\ \overline{\Delta}(x)^+ &= (CA^{r-1}B)^{-1}(\Delta - CA^r\,x)\end{aligned} \qquad (8)$$

It follows from the above development that $z \in \mathbb{Z} \iff u \in \mathbb{U}$. Moreover, from equation (8) we see that the induced input constraint set is time varying with the limits depending on the current value of the system state vector $x(k)$. $\square$

Lemma 1 shows that there is a way of translating a given state constraint set $\mathbb{Z}$ into an equivalent input constraint set $\mathbb{U}$. In other words, if $u(k)$ is constrained to the set $\mathbb{U}$, then $z(k)$ will be restricted to the desired set $\mathbb{Z}$. This result immediately suggests that a simple way of dealing with state constraints in the AWBT framework is to define the limiting circuit of Figure 1 based upon the definition of $\mathbb{U}(x)$ in (7)-(8). Thus, an appropriate limiting circuit can be thought of being the result of performing the following two stage procedure:

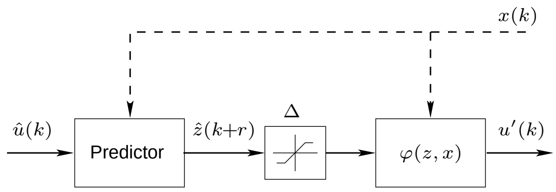

Fig. 2. Limiting circuit $u'(k) = sat_{\mathbb{U}(x)}(\hat{u}(k))$ for the state constraint anti-windup scheme.

- given the current state $x(k)$ and the control action $\hat{u}(k)$ demanded by the controller, a prediction $\hat{z}(k+r)$ of the constrained variable is performed, using expression (5).
- Any constraint violation is detected by applying the saturation function $sat_\Delta(\cdot)$ defined by
$$sat_\Delta(\cdot) \triangleq \text{sign}(\cdot)\min(|\ \cdot\ |, \Delta)\,,$$
to the prediction $\hat{z}(k+r)$. Based on the saturated value of $\hat{z}(k+r)$, an allowed control action $u'(k)$ is computed by back-calculating its value using the mapping $\varphi(\cdot, x)$ in (6).

We will use the notation $sat_{\mathbb{U}(x)}(\cdot)$ to refer to the limiting circuit described above. This circuit is illustrated in Figure 2. It is clear that if no violation of the constraint occurs, then $u'(k) = \hat{u}(k)$. On the contrary, when the constraint on $z(k)$ is active, $u'(k)$ is such that $z(k)$ is taken to the limit of the constraint set $\mathbb{Z}$. As a consequence, controller windup is effectively averted, since the controller dynamics contained in $H(q)$ will be driven by the correct input control signal to the plant which, in turns, carries the embedded information that the constraint on $z(k)$ is active. Notice that no distinction has been made between soft and hard constraints which shows that the proposed state constraint AWBT strategy can be used in both cases.

4. CONNECTIONS BETWEEN ANTI-WINDUP FOR STATE CONSTRAINTS AND MODEL PREDICTIVE CONTROL

The problem of dealing with state constraints can also be formulated as a Model Predictive Control problem (Goodwin *et al.*, 2001). Model Predictive Control is a control strategy which solves, at each iteration, a fixed horizon optimal control problem defined as:

$$\mathcal{P}_N(x):\ V_N^o = \min_U V_N(x,U) \qquad (9)$$

subject to the state constraint $z = C\,x \in \mathbb{Z}$, where $x = x(0)$ denotes the current state and $U = [u(k), u(k+1), \ldots, u(k+N-1)]$ is an admissible control sequence yet to be determined. Also $V_N(x,U)$ is a quadratic cost function defined as:

$$V_N(x,U) = \sum_{k=0}^{N-1} x(k)^T Q\,x(k) + R\,u(k)^2 + x(N)^T P x(N) \qquad (10)$$

where Q is a non-negative definite matrix, R is positive real and P is the unique positive definite solution of the algebraic Riccati equation:

$$P = A^T PA + Q - K^T \bar{R} K \quad (11)$$
$$\bar{R} = R + B^T PB. \quad (12)$$

A standard result establishes that in the absence of constraints, the solution to the fixed horizon optimal control problem is given by the control sequence $u(k) = -Kx(k)$, $k = 0, 1, \ldots, N-1$, where:

$$K = \bar{R} B^T PA. \quad (13)$$

On the other hand, when constraints are imposed, the minimization problem (9) has to be solved numerically using standard optimization methods. The quadratic cost function (10) is characterized by a prediction horizon, a control horizon and a constraint horizon. The prediction horizon determines the number of steps ahead in which the state evolution is considered (in this case N). The control horizon corresponds to the number of changes allowed in the control sequence U (in this case also N). Finally, the constraint horizon is the time interval over which the constraints are imposed *i.e.*, the value N_c such that:

$$z(l) \in \mathbb{Z},\ l = k, k+1, \ldots, k + N_c \quad (14)$$

It is interesting to notice that when the constraint horizon is reduced to $N_c = r$, where r is the relative degree of the transfer function which relates $z(k)$ to $u(k)$, Lemma 1 establishes that this is equivalent to imposing an input constraint only on the first element of the control sequence U *i.e.*, $u(k)$. It is not difficult to see that, in this case, the MPC strategy is equivalent to the use of the saturation function $sat_{\mathbb{U}(x)}(\cdot)$ on the *unconstrained* optimal state feedback control law $u(k) = -Kx(k)$. In other words, $sat_{\mathbb{U}(x)}(-Kx(k))$ is optimal if the constraint horizon is $N_c = r$. Notice also that the control law $sat_{\mathbb{U}(x)}(u(k))$ can be thought of as being equivalent to an AWBT scheme in the state space framework; since if we implement the state space feedback using an observer, we will be feeding the observer dynamics with the real control signal applied to the plant. More will be said on this in Section 5.

A recent result presented by De Doná and Goodwin (2000) has shown that, for an arbitrarily large constraint horizon N_c, the clipped version of the unconstrained optimal control law is still optimal provided the states of the system are confined to the interior of a certain region S_N of the state space. This shows that, in a non-trivial region of the state space, the constraint horizon can be taken as 1 (for the case of input constraints) even though future controls may also reach saturation. It is tempting to conjecture that a similar result should hold for the AWBT state constraint scheme. However the result in De Doná and Goodwin (2000) does not immediately extend to the state constrained case, since the derivation of the result is based on the inherent assumption that the constraint is fixed and independent from the current state x. On the other hand, we have seen from Lemma 1 of Section 3, that this is not the case when dealing with state constrained systems. To overcome this restriction we will present below an extension of the result of De Doná and Goodwin (2000) applied to linear systems subject to a single state constraint $z \in \mathbb{Z}$. This is described in the following

Theorem 2. Given the fixed horizon optimal control problem $\mathcal{P}_N(x)$ defined in (9), where x denotes the initial state $x = x(0)$ of the system (3), then $\forall x \in S_N$ the minimum cost is:

$$V_N^0(x) = x^T Px + \bar{R} \sum_{k=1}^{N} \gamma_k (\widetilde{A}^{k-1} x)^2 \quad (15)$$

and for all $x \in S_N$ the optimal sequence U that attains this minimum is:

$$u^0(k, x) = sat_{\mathbb{U}(x)}(-Kx(k)),\ \forall k = 0, 1, \ldots, N-1 \quad (16)$$

where,

$$S_i \triangleq \{x | \phi_{nl}^k(x) \in Y_{i-k},\ k = 0, 1, \ldots i-2\} \quad (17)$$
and $S_0 \triangleq S_1 \triangleq \mathbb{R}^n$, $i = 2, 3, \ldots N$
$$Y_i = \cap_{j=1}^{i-1} X_j,\ i = 2, 3, \ldots N \quad (18)$$
and $Y_0 \triangleq Y_1 \triangleq \mathbb{R}^n$
$$X_i \triangleq \{x | \gamma_i (\widetilde{A}^{i-1} \phi_l(x)) = 0\} \quad (19)$$
$$\phi_l(x) = (A - BK)x \quad (20)$$
$$\phi_{nl}(x) = Ax + B sat_{\mathbb{U}(x)}(-Kx) \quad (21)$$

also,

$$\widetilde{A} \triangleq A - BL \quad (22)$$
$$L \triangleq (CA^{r-1}B)^{-1} CA^r \quad (23)$$
$$\gamma_i \triangleq \widetilde{K}x - sat_{\widetilde{\Delta}_i}(\widetilde{K}x) \quad (24)$$
$$\widetilde{K} \triangleq K - L \quad (25)$$
$$\widetilde{\Delta}_i \triangleq \left(1 + \sum_{k=0}^{i-2} \left|\widetilde{K}\widetilde{A}^k B\right|\right) \widetilde{\Delta} \quad (26)$$
$$\widetilde{\Delta} \triangleq \left|(CA^{r-1}B)^{-1} \Delta\right| \quad (27)$$

Proof outline. The theorem is proved by induction, based on dynamic programming arguments and mirroring the analysis in De Doná and Goodwin (2000). In the sequel u and x denote $u = u(i)$ and $x = x(i)$ respectively.

Starting from $i = N$, the value function associated to the optimal control problem $\mathcal{P}_N(x)$ is:

$$V_0^0(x) = x^T Px;\ \ \forall x \in S_0 \triangleq \mathbb{R}^n \quad (28)$$

Using the principle of optimality, the value function at the following step $i = N-1$ is given by:

$$\begin{aligned} V_1^0(x) &= \min_{u \in \mathbb{U}} \left\{x^T Qx + Ru^2 + V_0^0(Ax + Bu)\right\} \\ &= \min_{u \in \mathbb{U}} \left\{x^T Px + \bar{R}(u + Kx)^2\right\} \quad (29) \end{aligned}$$

where we have used (11) and (12). It is clear that the unconstrained optimal solution to (29) is given by

$u = -Kx$, $\forall x \in S_1 \triangleq \mathbb{R}^n$. From the convexity of the cost function, the constrained optimal control law is given by:

$$u^0(N-1,x) = sat_{\mathbb{U}(x)}(-Kx),\ \forall x \in S_1 \quad (30)$$

A key observation, at this stage, is that the limits of the saturation function $sat_{\mathbb{U}(x)}(\cdot)$ are not fixed but depend on the current value of x. However, the definition of the mapping $\varphi(z,x)$ in (6) suggests that it is possible to express the time varying constraint imposed to $u = \ -Kx$ as a fixed constraint imposed to a shifted version of u. Indeed, if we adopt the variable $\tilde{u}$ defined by

$$\tilde{u} = u + L\,x \quad (31)$$

where L is specified in (23); then we have the result:

$$u \in \mathbb{U}(x) \Leftrightarrow \tilde{u} \in \widetilde{\mathbb{U}} \triangleq [-\widetilde{\Delta}, \widetilde{\Delta}] \quad (32)$$

with $\widetilde{\Delta}$ defined in (27). This is readily seen from (6) and the definition of $\tilde{u}$ in (31). Based on this result, the optimal control law (30) can be written as:

$$u^0(N-1,x) = -Lx - sat_{\widetilde{\Delta}}(\widetilde{K}x) \quad (33)$$

which, if replaced in the optimal value function (29), gives the result:

$$V_1^0(x) = x^T Px + \bar{R}\gamma_1(x)^2,\ \forall x \in S_1 \triangleq \mathbb{R}^n \quad (34)$$

with $\gamma_1(x)$ defined in (24). With this observation, the remainder of the proof follows that of De Doná and Goodwin (2000), since the problem has been translated into an equivalent problem having fixed input constraints, now in terms of the shifted variable $\tilde{u}$. $\square$

This result solves the fixed horizon optimization problem. To extend the result to the receding horizon problem we need to ensure that the state trajectories are kept inside a positively invariant set. For the state constraint case this extra step parallels the result in De Doná and Goodwin (2000) and will thus not be repeated here.

5. EXTENSION TO MORE GENERAL PROBLEMS

We briefly discuss extensions to more general situations.

Cases where the state is not directly measured.
In the above analysis we have assumed that the full state vector (including disturbances) are directly measured. This will rarely be the case in practice. However, it is straightforward in principle to replace the true state by a state estimate - say given via a Kalman filter. Indeed, it is the observer dynamics that contribute to the dynamics in $H(q)$ of Figure 1. These connections are further discussed in Goodwin *et al.* (2001)

Multiple constraints.
The results presented in Sections 3 and 4 can be directly extended to more general cases. For example, they can be applied to the case where the number of state constraints is equal to the degrees of freedom available in the input control signal. This allows one to deal with multiple constraints, provided the control signal is not a scalar. If the number of constraints exceeds the number of control signals then feasibility problems may arise . This is a well known problem associated with the control of constrained systems (Maciejowski, 2002). One way of getting around this difficulty is to impose a certain hierarchy among the constraints. In this case, it is possible to use the same hierarchy to modify the AWBT circuitry in order to automatically discard the constraints with less priority when in-feasibility arises.

6. SIMULATION EXAMPLE

Consider the marginally stable linear system (3) with

$$A = \begin{bmatrix} 1 & 0 \\ 0.4 & 1 \end{bmatrix},\ \ B = \begin{bmatrix} 0.4 \\ 0.08 \end{bmatrix},\ \ C = \begin{bmatrix} 1 & 0 \end{bmatrix},\ (35)$$

subject to the state constraint

$$|x_1(k)| = |Cx(k)| \leq \Delta = 1. \quad (36)$$

In the fixed horizon cost function (10) we consider $N = 10$, $Q = 10I_{2\times 2}$ and $R = 0.1$.

In Figure 3 we can observe the system state trajectory when applying the AWBT control law $sat_{\mathbb{U}(x)}(-Kx)$ (with K computed from (13)), compared to the state trajectory obtained using MPC for the initial condition $x_0 = [-0.2\ -1.6]^T$. The dashed lines show the state constraint set $\mathbb{Z}$, whilst the continuous lines represent the set Y_N, defined in (18). Figure 3 clearly shows that both state trajectories match exactly, which confirms the results of Theorem 2. Notice that $x(k) \in S_N$ $\forall k$, since the state trajectory evolves in the interior of the set Y_N. Notice also that the state constraint is active for more than one step; as a result, the state space region in which the AWBT strategy and MPC are equivalent is larger than the trivial region where the state $x_1(k)$ stays unsaturated.

In Figure 4 the control sequences obtained with both control strategies are compared, confirming that they are coincident. The dashed lines represent the input constraint set $\mathbb{U}(x)$ which, as described in Lemma 1, changes with time depending upon the current value of the system state $x(k)$. The control sequence stays saturated during the first two sample times which agrees with the fact that $x_1(k)$ saturates for $k = 1, 2$ and that the relative degree of the transfer function which relates $x_1(k)$ to $u(k)$ is $r = 1$.

7. CONCLUSIONS

This paper has presented an AWBT scheme for state constrained linear systems. The proposed strategy is

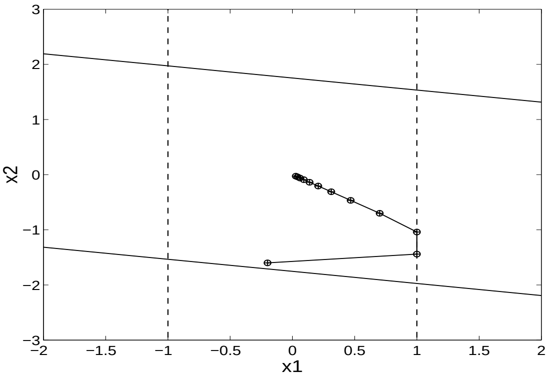

Fig. 3. State trajectory with $sat_{\mathbb{U}(x)}(-Kx)$ (circle) and with MPC (plus).

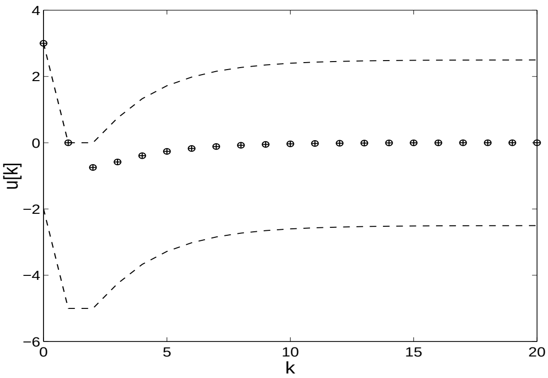

Fig. 4. Control sequence with $sat_{\mathbb{U}(x)}(-Kx)$ (circle) and with MPC (plus).

an extension of traditional AWBT schemes used for input constrained linear systems. The method has been shown to be equivalent to Model Predictive Control in a non-trivial region of the state space.

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