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DELAY ADAPTATION FOR A CHAOS CONTROL WITH HALF-PERIOD DELAYED FEEDBACK

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Abstract: A robust adaptation scheme for the delay of a continuous-time chaos control with half-period delayed feedback is presented. The phase-difference between the measured signal and the delayed measurement control is detected to adjust the delay to its optimum. The method when applied to the Lorenz and the Duffing oscillator shows high robustness. *Copyright © 2002 IFAC*

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1. INTRODUCTION

Within many physical systems, chaotic behaviour is undesirable. Nevertheless, regulation of physical systems by changing some parameters to a large extent is often also undesirable. Thus, the control of chaotic behaviour by small parameter variations has been of common practical and theoretical interest for many years. In practical examples, such as laser systems (Ott, *et. al., 1994*) or aircraft engine combustion systems (Chen, 1999), it has been important to control chaotic oscillations to improve system efficiency. Several texts (Boccaletti, *et. al.*, 2000) are available reviewing methods for control of chaotic dynamics. Methods for stabilisation of intrinsic unstable periodic orbits within chaotic systems via small parameter perturbation can be divided into two classes, continuous and discrete control methods. Discrete control methods such as the OGY-technique (Ott, *et. al., 1990*) generally reside on an on-line Poincare cut analysis and a respective identification of a model for control, which, for noisy measurements in particular, can be very complex. Continuous-time control methods do not suffer from this problem since they are generally not model-based and do not demand an on-line data analysis. One particular continuous control technique, developed by Pyragas (1992) involving a delayed feedback, has attracted longstanding interest and has proven to be very useful in application to practical systems (Chen, 1999; Celka, 1994; Holyst, *et.al.*, 2000; Kittel, 1995; Pyragas, 1992; Pyragas, *et. al.*, 1993; Pyragas, 1995; Schoell, *et. al.*, 1993; Schoell, *et. al.*, 1994). This control method uses a delayed feedback employing a suitably amplified difference of an output measurement of the chaotic system and, respectively, the delayed measurement for control. The control signal vanishes in the post-transient behaviour for the stabilized orbit. Thus, the delay time has to be the exact value of the period of the unstable intrinsic orbit. However, Nakajima (1997) and Just, *el. al.*, (1997) proved a practical limitation of this continuous control method: a hyperbolic unstable periodic orbit with an odd number of real characteristic multipliers greater than unity can never be stabilised by the delay control method introduced by Pyragas (1992). A modification of Pyragas' (1992) control method suggested by Nakajima (1998) has resolved this problem and was successfully demonstrated for the Duffing equation and the chaotic Lorenz system. Provided the solution $\underline{x}_{per}(t)$ of the unstable periodic orbit with period T_{per} is symmetric, $\underline{x}_{per}(t) = -\underline{x}_{per}(t - T_{per}/2)$, then the control signal is chosen to be proportional to $\underline{x}(t) + \underline{x}(t - T_{con})$ involving a half-period delay $T_{con} = T_{per}/2$ for the delayed state vector $\underline{x}(t - T_{con})$. For a stabilised orbit, this signal becomes zero provided the employed delay is exactly adjusted. Nevertheless, this delay is not known a-priori. Recently, a continuous-time methodology (Herrmann, 2001) has been suggested which complements and improves currently used techniques for delay adaptation (Chen, *et. al.*, 1999, Pyragas, *et. al.*, 1993; Kittel, et. al., 1995; Yu, 1999). These iterative schemes rely on the analysis of time-sampled data from output measurements of the chaotic system. Hence, these techniques can provide good results if the measurements are not noisy. The continuous-time adaptation method introduced by Herrmann (2001) complements these existing schemes by using a guess for the period length of the orbit and evades the issue of discrete-time, on-line data analysis during the actual control process; it is therefore more robust to measurement noise. This technique employs a non-linear filter known from phase-locked loops (Blake, 1993) and incorporates a proportional-integral control (PI-control). Initially, the good guess of the period length can be taken from an output data analysis by identifying the unstable orbits and the respective period length for the unperturbed chaotic system (Auerbach, 1987; Lathrop, 1989) or subsequently from discrete control and delay adaptation results (Ott, *et. al., 1990*, Chen, *et. al.*, 1999, Pyragas, *et. al.*, 1993; Kittel, et. al., 1995; Yu, 1999). However, Herrmann (2001) has demonstrated that a highly accurate guess for the period duration is not necessary. To show the wide applicability of the suggested approach, the continuous technique has been modified suitably for the adaptation of Nakajima's (1998) half-period delay feedback. Hence, this paper has the following structure: In Section 2, Nakajima's (1998) control method is recalled. Section 3 introduces the idea of the non-linear filter and proves that the control scheme is robust to control parameter variation. In Section 4, the scheme in application to the Duffing oscillator and the Lorenz system is documented.

2. NAKAJIMA'S HALF-PERIOD DELAY FEEDBACK

As for Pyragas' control method, Nakajima's (1998) half-period delay feedback assumes a chaotically behaving differential system:

$$\frac{d\underline{x}(t)}{dt} = f(\underline{x}(t), \underline{a}), \ \underline{x} \in \Re^n, \ \underline{a} \in \Re^m, \quad (1)$$
$$\underline{x} = [x_1 \quad x_2 \quad \cdots \quad x_n]^T,$$
$$\underline{a} = [a_1 \quad a_2 \quad \cdots \quad a_m]^T,$$

which can be perturbed by a parameter vector $\underline{a} \in \Re^m$ continuously within an interval $a_i \in [a_{0i} - F_{0i}, a_{0i} + F_{0i}]$ around a nominal value $\underline{a}_0 \in \Re^m$ where $F_{0i} > 0$, $i = 1,2...m$. Further, it is assumed that the period length, T_{per}, is well defined for a particular unstable periodic orbit and that the trajectory $\underline{x}_{per}(t)$ of the unstable periodic orbit is symmetric:

$$\underline{x}_{per}(t) = -\underline{x}_{per}(t - T_{per}/2). \quad (2)$$

Nakajima's (1998) approach for a feedback control is to use a state $x_i(t)$ of (1) and to induce via $\underline{a}$ the following signal:

$$\underline{a}(t) = \underline{a}_0 + SAT(F(t)), \ \ SAT(F(t)) = \begin{bmatrix} sat(F_1(t)) \\ sat(F_2(t)) \\ \vdots \\ sat(F_m(t)) \end{bmatrix} \quad (3)$$

$$sat(F_i(t)) = \begin{cases} F_{0i} & \text{for } F_i(t) > F_{0i} \\ F_i(t) & \text{elsewhere} \\ -F_{0i} & \text{for } F_i(t) < F_{0i} \end{cases};$$

$$F(t) = K \cdot (x_i(t - T_{con}) + x_i(t)), \ K \in \Re^{m \times n},$$

$$T_{con} = \frac{T_{per}}{2}.$$

The perturbation around the value a_0 has not only been limited by a saturation function to keep the dynamical characteristics as close as possible to the original system's behaviour but also to prevent practically the problem of multi-stabilities (Pyragas, 1992). It has been found for suitable examples that for $T_{con} = T_{per}/2$ and for appropriate gain K the feedback of (3) is able to stabilize the respective periodic orbit. In this case, the perturbation vanishes and

$$\lim_{t\to\infty}(\underline{a}(t)) = \underline{a}_0, \ \lim_{t\to\infty}(F(t)) = 0. \quad (4)$$

It has been verified in application to practically valid examples that the gain K usually lies within a compact sub-set of $\Re^{m \times n}$ for which the feedback law is operating and relation (4) is satisfied. Nevertheless, the other important control parameter, the delay, T_{con}, has to be accurately adjusted to the uniquely defined $\frac{T_{per}}{2}$ so that it is possible to stabilize a true periodic orbit of the chaotic attractor. Thus, the next section suggests a method which allows the adjustment of T_{con} via output feedback employing a robust non-linear filter technique using a reasonable initial guess for T_{con}.

3. A ROBUST SCHEME FOR ONLINE ADJUSTMENT OF THE DELAY

It is assumed that Nakajima's controller ensures, for a reasonable guess of T_{con} close to T_{per}, that the signal $x_i(t)$ becomes periodic with T_{per} similar to the intrinsic unstable periodic orbit:

$$x_i(t) = \alpha_0 + \sum_{n=1}^{\infty} \alpha_n \cdot \sin\left(\frac{2\cdot\pi}{T_{k\,per}} \cdot n \cdot t + \varphi_n\right) \quad (5)$$

Hence, the first harmonic of the signals $x_i(t)$ and $-x_i(t - T_{con})$ have a phase difference of

$$\frac{\left({T_{per}}/{2} - T_{con}\right)\cdot 2\pi}{T_{per}}. \tag{6}$$

This phase difference is an indicator how much in error the delay time T_{con} with respect to the demanded value of $T_{per}/2$ is. Thus, it is the aim to determine this phase difference practically for it to be used within the control scheme to adjust T_{con}. A non-linear filter, used for phase-locked loops within analogue communication systems, can extract this phase difference using another delayed signal $x_i(t - T_{con}/2)$. For delay adaptation, the first harmonics of $x_i(t - T_{con}/2)$ and $x_i(t) + x_i(t - T_{con})$ are obtained via two band-pass filters $BP_{T_1}(\cdot)$ and $BP_{T_2}(\cdot)$ with centre frequencies of $\frac{2\pi}{T_1}$, $\frac{2\pi}{T_2}$ radians and a passband width $\Delta\omega > 0$ which allows to suppress any other harmonic of $x_i(t - T_{con}/2)$ and $x_i(t) + x_i(t - T_{con})$:

$$0 < \frac{2\pi}{T_1} - \frac{\Delta\omega}{2}, \quad \frac{2\pi}{T_1} + \frac{\Delta\omega}{2} < \frac{4\pi}{T_{k\,per}}, \tag{7}$$
$$0 < \frac{2\pi}{T_2} - \frac{\Delta\omega}{2}, \quad \frac{2\pi}{T_2} + \frac{\Delta\omega}{2} < \frac{4\pi}{T_{k\,per}}$$

Practically, bandpass filters introduce a phase error and an amplitude damping. For T_1 and T_2 close enough to T_{per}, it follows:

$$BP_{T_1}\left(x_i(t - T_{con}/2)\right) = \hat{a}_1 \cdot \sin\left(\frac{2\cdot\pi}{T_{per}}(t - T_{con}/2) + \varphi_1\right) \tag{8}$$

$$BP_{T_2}(x_i(t) + x_i(t - T_{con})) \tag{9}$$
$$= \hat{a}_2 \cdot \left(\sin\left(\frac{2\cdot\pi}{T_{per}}t + \varphi_2\right) + \sin\left(\frac{2\cdot\pi}{T_{per}}(t - T_{con}) + \varphi_2\right)\right)$$

where $\hat{a}_1, \hat{a}_2$ are the resulting amplitudes and φ_1, φ_2 are the phase shifts of the first harmonic and due to the filters $BP_{T_1}(\cdot)$ and $BP_{T_2}(\cdot)$. Multiplying both filtered signals, it follows:

$$BP_{T_1}\left(x_i(t - T_{con}/2)\right) BP_{T_2}(x_i(t) + x_i(t - T_{con})) \tag{10}$$
$$= 2\cdot\hat{a}_1\hat{a}_2 \cdot \sin\left(\frac{2\pi}{T_{per}}t - \frac{\pi}{T_{per}}T_{con} + \varphi_2\right)$$
$$\cos\left(\frac{\pi}{T_{per}}T_{con}\right)\sin\left(\frac{2\cdot\pi}{T_{per}}(t - T_{con}/2) + \varphi_1\right)$$
$$= -\hat{a}_1\hat{a}_2 \sin\left(\frac{\pi(T_{con} - T_{per}/2)}{T_{per}}\right)$$
$$\left(\cos(\varphi_2 - \varphi_1) - \cos\left(\frac{4\pi}{T_{per}}t - \frac{2\pi}{T_{per}}T_{con} + \varphi_2 + \varphi_1\right)\right)$$

Introducing, after this non-linear operation, a low pass filter $LP(\cdot)$ with cut-off frequency $\omega_{LP} << \frac{4\pi}{T_{per}}$, it is possible to extract the zeroth harmonic of the latter signal:

$$m_\varphi = LP\left(BP_{T_1}\left(x_i(t - T_{con}/2)\right)\cdot BP_{T_2}(x_i(t) + x_i(t - T_{con}))\right) \tag{11}$$
$$= -a_{Tp}\cdot\hat{a}_1\cdot\hat{a}_2 \sin\left(\frac{\pi(T_{con} - T_{per}/2)}{T_{per}}\right)\cos(\varphi_2 - \varphi_1)$$
$$\approx -a_{Tp}\cdot\hat{a}_1\cdot\hat{a}_2\cdot\cos(\varphi_2 - \varphi_1)\frac{\pi(T_{con} - T_{per}/2)}{T_{per}}$$
$$\text{for } \left|\frac{\pi(T_{con} - T_{per}/2)}{T_{per}}\right| << 1.$$

For small values of $(T_{con} - T_{per}/2)$, the signal m_φ (11) is proportional to the phase difference of (6). Note that the resulting phase error measure signal m_φ is robust to slightly mismatched bandpass filters $BP_{T_1}(\cdot)$ and $BP_{T_2}(\cdot)$ as long as $(T_{con} - T_{per}/2)$ and $\varphi_1 - \varphi_2$ are sufficiently small (Herrmann, 2001). In this case, the factor $a_{Tp}\cdot\hat{a}_1\cdot\hat{a}_2\cdot\cos(\varphi_2 - \varphi_1)$ introduces a non-linear damping effect but remains positive. Hence, m_φ (11) has the same sign as the phase difference of (6). Thus, the signal m_φ can now be used to adjust the delay T_{con} to its optimal value using a simple Proportional-Integral (PI)-control scheme.

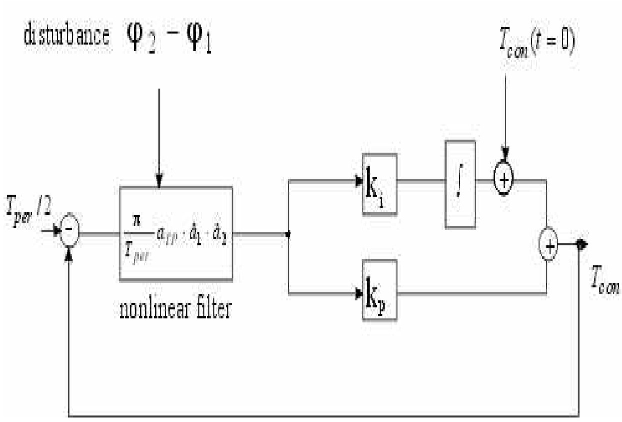

Fig. 1. Delay control system structure

Suppose the nonlinear filter for m_φ (11) is reasonably linear and fast so that it can be regarded as a static linear element amplifying the error $(T_{con} - T_{per}/2)$ with a gain ${\pi}/{T_{K\,per}}\, a_{TP}\cdot\hat{a}_1\cdot\hat{a}_2$. Under this condition, the characteristic equation of this simple PI-control loop (Figure 1) is a first order polynomial in s:

$$s+\frac{\frac{\pi}{T_{K\,per}}k_i\cdot a_{TP}\cdot\hat{a}_1\cdot\hat{a}_2}{\left(\frac{\pi}{T_{K\,per}}k_p\cdot a_{TP}\cdot\hat{a}_1\cdot\hat{a}_2+1\right)} \quad (12)$$

The PI-loop is stable for a negative root of the characteristic function in s, e.g. for positive values of the loop gains k_i and k_p.

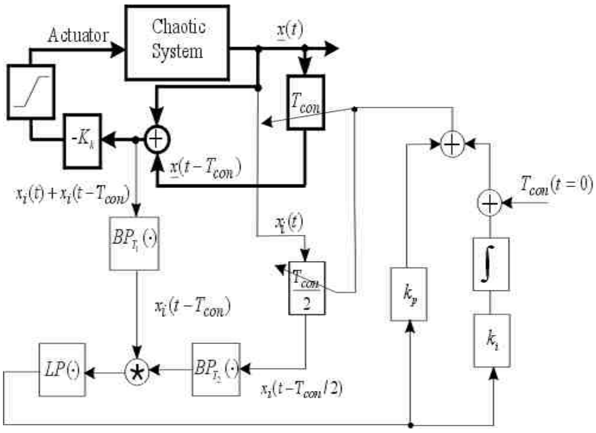

Fig. 2. Nakajima's modified controller

The merit of employing the integral element using k_i for control is that the PI-control will ultimately force $\left(T_{con}-T_{per}/2\right)$ to be 0. The modified scheme of Nakajima (1998) is presented with Figure 2. In the next section it will be seen that this control strategy is robust and easily applicable to chaotic systems.

4. APPLICATION TO THE DUFFING-OSCILLATOR AND THE LORENZ SYSTEM

Nakajima (1998) could show that an unstable one-periodic orbit of the *Lorenz system* with the following parameterization

$$\begin{aligned}\dot{x}(t)&=-10(x(t)-y(t))-sat(F(t))\\ \dot{y}(t)&=28x(t)-y(t)-x(t)z(t)\\ \dot{z}(t)&=x(t)y(t)-\frac{8}{3}z(t)\end{aligned} \quad (13)$$

can be stabilized employing a control $F(t)=k(x(t-T_{con})+x(t))$, $F_0=5$, and $K>4$. The period, $T_{per}\approx 1.559$, of the unstable orbit has not been exactly determined by Nakajima (1998) with the result that the control signal did not converge to 0 and the post-transient behaviour showed that the value for the delay, T_{con}, has to be optimized. Hence, the delay adaptation scheme has been tested for this system employing for Nakajami's control $K=7.5$, for the PI-control gains $k_i=0.001$ and $k_p=0$ and for the band-pass and the low-pass standard butterworth filters of third order, where the respective centre frequencies; the band width and the cut-off frequency have been selected

$$\frac{2\pi}{T_1}=\frac{2\pi}{T_2}=\frac{\pi}{T_{con}(t=0)},\Delta\omega=\frac{\pi}{T_{con}(t=0)} \quad (14)$$
$$\omega_{LP}=\frac{0.3\pi}{2T_{con}(t=0)}$$

via the initial value, $T_{con}(t=0)$, of the controller delay. Several initial values, $T_{con}(t=0)$, have been tested: Nakajima's control without delay adaptation was initiated at $t=40$ and the delay adaptation started at $t=60$. The results (Figure 3) show that the optimal value of the delay and subsequently the period of the unstable orbit is more accurately given by

$$T_{con}=\frac{T_{per}}{2}=\frac{1.5746}{2}. \quad (15)$$

From Figure 4 and 6, it can be seen that Nakajima's regulator in combination with the delay-adaptation scheme indeed stabilises a symmetric unstable periodic orbit of the Lorenz system as the value of $x(t)+x(t-T_{con})$ converges to numerically negligible values.

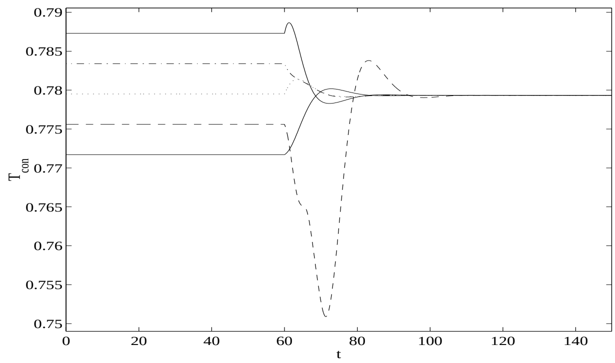

Fig. 3. Adaptation of delay $T_{con}(t)$ considering different initial $T_{con}(t=0)$ for the Lorenz system

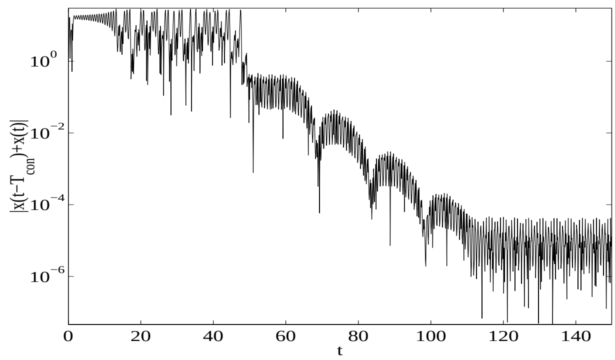

Fig. 4. Dynamics of $x(t)+x(t-T_{con})$ of the Lorenz system for $T_{con}(t=0)=0.7717$

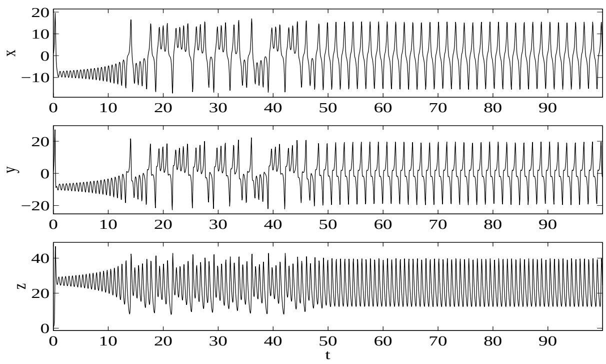

Fig. 5. Dynamics of $x(t), y(t), z(t)$ of the Lorenz system for $T_{con}(t=0) = 0.7717$

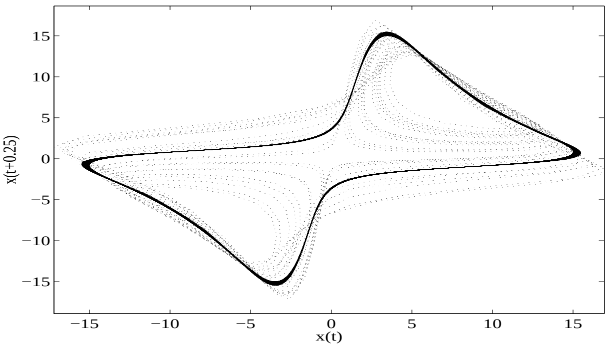

Fig. 6. Lorenz attractor (dashed) and stabilized periodic orbit (line) displayed via delay coordinates $(x(t), x(t+0.25))$

Robustness of the control scheme can be shown in case of additive white noise $\xi(t)$ introduced for the measurement $x(t)$:

$$\tilde{x}(t) = x(t) + \xi(t). \qquad (16)$$

The regulator is evaluated for a noise power of ${\sigma_\xi}^2 = 0.001$ showing that the delay adaptation (Figure 7, Figure 8) is still operating despite the high level of noise (Figure 6, Figure 9). The post-transient value of $T_{con}(t)$ is *in average* the half-period value $\frac{T_{per}}{2} = \frac{1.5746}{2}$ (15).

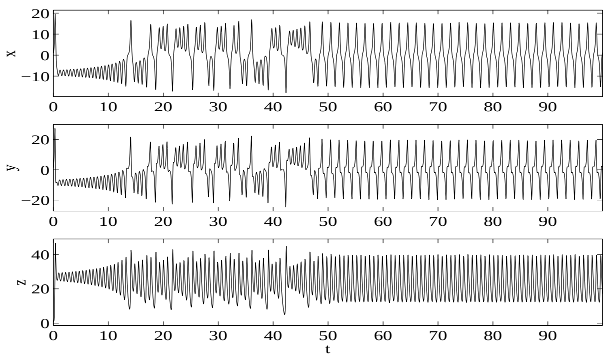

Fig. 7. Dynamics of $x(t), y(t), z(t)$ of the Lorenz system for $T_{con}(t=0) = 0.7717$, ${\sigma_\xi}^2 = 0.001$

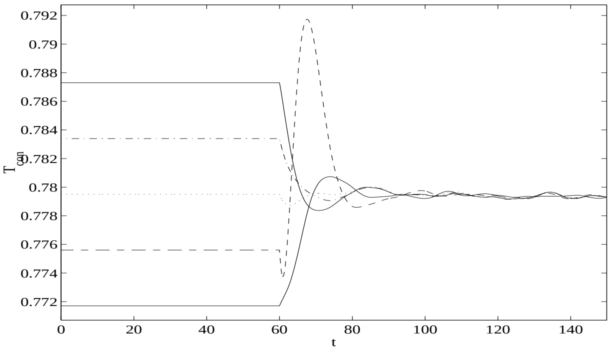

Fig. 8. Adaptation of $T_{con}(t)$ for the Lorenz system, ${\sigma_\xi}^2 = 0.001$ and different $T_{con}(t=0)$.

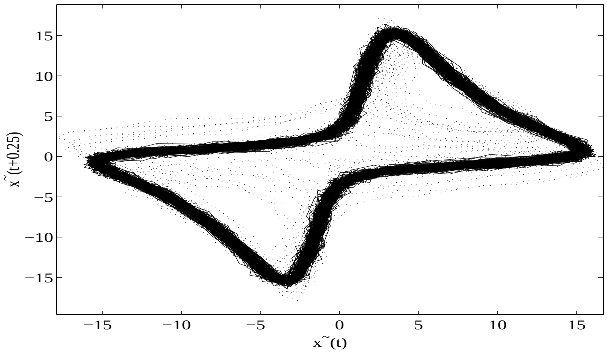

Fig. 9. Trajectory of $(\tilde{x}(t), \tilde{x}(t+0.25))$ derived from the *noisy* measurement (16) of the controlled Lorenz system $({\sigma_\xi}^2 = 0.001)$ (Transient=dashed, post-transient=line)

Note that the numerical absolute and relative accuracy for testing different numerical simulation methods was kept to a maximum of 10^{-6} while the maximal time step was 0.0005.

Similar tests have been conducted for the forced *Duffing oscillator:*

$$\begin{aligned} \dot{x}(t) &= y(t) - sat(F(t)) \\ \dot{y}(t) &= -0.25y(t) + x(t) - x^3(t) + 0.3\cos(t) \end{aligned} \qquad (17)$$

where $F(t) = K(x(t - T_{con}) + x(t))$, $F_0 = 1$, $K = 3$. and the low-pass and band-pass filter have been chosen as in (14) while for the PI-control the choice of $k_i = 3$ and $k_p = 0$ has been suitable. The optimal value for T_{con} is $\frac{T_{per}}{2} = \pi$. Numerical simulation shows that T_{con} adjusts to this value for different initial values (Figure 10) using a similar procedure for adaptation of T_{con}: the control scheme without delay adaptation is initiated at $t = 50$ followed by the start of the delay adaptation at $t = 150$. In practice, the adaptation of forced chaotic systems might be of interest for systems where the exact characteristic of the driving signal is not known.

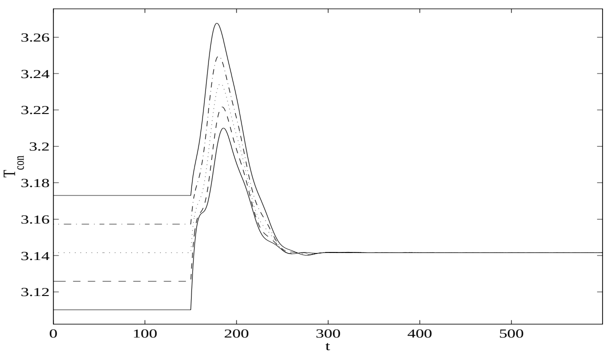

Fig. 10. Adaptation of $T_{con}(t)$ for the Duffing system

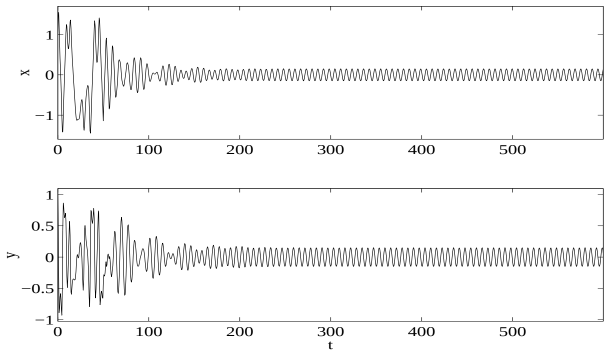

Fig. 11. Dynamics of $x(t), y(t)$ of the Duffing system, $T_{con}(t=0)=0.99\pi$

3. CONCLUSIONS

This paper has presented the application of a robust delay adaptation approach for Nakajima's half-period-delay chaos control methodology. For delay adaptation, the phase difference of output measurement and delayed output measurement is extracted via a non-linear filter known from phase-locked loop methods. The filter combined with a simple PI-control is readily incorporated into Nakajima's control. The design of the non-linear filter is based on a good guess of the actual period duration of the periodic orbit. The control scheme is robust to a large variation of this initial value used for control design. Further, the scheme is robust to output measurement noise since it prevents a complex sampled-data analysis of output measurement data. Thus, the scheme known from a practical extension of Pyragas' delay feedback has shown to be effective in application to a wider class of chaos control techniques.

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