15th Triennial World Congress, Barcelona, Spain

OPTIMIZED DYNAMIC CALIBRATION OF A SCARA ROBOT [1]

M. Indri * G. Calafiore * G. Legnani ** F. Jatta ** A. Visioli ***

* *Dip. di Automatica e Informatica, Politecnico di Torino, Italy*
** *Dip. di Ingegneria Meccanica, Università degli Studi di Brescia, Italy*
*** *Dip. di Elettronica per l'Automazione, Università degli Studi di Brescia, Italy*

Abstract: This paper presents experimental results for the dynamic calibration of the SCARA robot. In particular, optimized techniques are applied to determine optimal excitation trajectories for the identification experiment, and the resulting performances are compared with those obtained using standard robot "working" trajectories. Experiments show that the theoretically optimal trajectories provide an actual practical improvement on the quality of the resulting parameter estimates and torque reconstruction. *Copyright © 2002 IFAC*

Keywords: Robot calibration, optimal trajectory, parameter estimation, trajectory planning.

1. INTRODUCTION

The determination of a good robot dynamic model, and consequently of the parameters that define it, is fundamental for the application of various advanced control schemes, as well as for simulation purposes. Several papers and books appeared in literature, dealing with robot dynamic calibration: see e.g. (Kozlowski, 1998), (Neuman and Khosla, 1985), (Swevers *et al.*, 1997) and (Gautier and Khalil, 1990) about the use of different dynamic models for calibration, and the determination of the minimum set of identifiable parameters.

The estimation of the identifiable parameters is usually achieved by application of a Least-Squares criterion to the motion or the energy equations of the robot. However, the quality of the provided estimate is strongly influenced by the trajectory executed by the robot during the acquisition of the experimental data used for the calibration.

Starting from (Armstrong, 1987), perhaps one of the earliest papers about this topic, different solutions have been proposed to find trajectories that well excite the robot dynamic model used for calibration, by finding an optimal sequence of joint position-velocity and/or acceleration points, to be subsequently interpolated as in (Armstrong, 1987), (Caccavale and Chiacchio, 1994), and (Gautier and Khalil, 1992), or by looking for an optimal trajectory within a given parameterized family, as in (Swevers *et al.*, 1997) and (Calafiore *et al.*, 2001).

All these methods are based on different optimization criteria, which should theoretically guarantee some characteristics of the computed estimates, e.g. the minimization of the uncertainty bounds or of the estimate bias due to unmodeled dynamics errors. In some cases, special simple trajectories can be also adopted, as in (Visioli and Legnani, 2000).

This paper deals with the dynamic calibration of a two-dof SCARA robot: a good calibration is fundamental in this case, since no knowledge of its

[1] supported by *Ministero dell'Università e della Ricerca Scientifica e Tecnologica.*

inertial parameters is available, and previous tests have shown the presence of significant friction torques on both joints, especially at low velocities. This robot is then used to test *in practice* the actual importance of the search of an optimal (in some sense) trajectory for calibration, and to show if and how the use of different optimization criteria can provide parameters estimates, which allow a better reconstruction of the joint torques in a wide variety of different trajectories, and eventually a re-definition of the dynamic model itself. Particular attention will be devoted to the friction model, whose order could be possibly re-discussed during the calibration procedure itself. The tests are performed using two different groups of trajectories: the trajectories of the first group belong to the set of usual robot motions, whereas the second ones are harmonic functions, chosen following the optimization procedure presented in (Calafiore *et al.*, 2001).

The paper is organized as follows: the model of the robot is described in Section 2, while the methodology used for the calibration is illustrated in Section 3; the experimental results are reported and discussed in Section 4, and some conclusions are drown in Section 5.

2. THE ICOMATIC SCARA03 ROBOT

2.1 *General description*

An image of the considered robot, which is a SCARA manipulator having three degrees of freedom (dof), is represented in Figure 1.

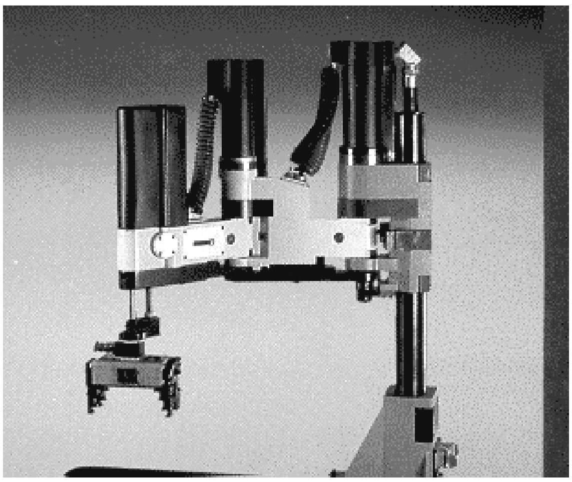

Fig. 1. The Icomatic SCARA03 robot.

Two DC motors actuate the first two joints moving the gripper in the x-y plane. The quote of the gripper is actuated by a third DC motor by means of a speed reducer and by a pinion-rack transmission. The vertical motion (direction Z) is decoupled with respect to X and Y and it is not considered in this paper.

The mechanical transmissions of the first two joints include two Harmonic Drive speed reducers having a transmission ratio $\tau_1 = \tau_2$=1/100. The position feedback is obtained by digital encoders (2000 steps/revolution), and the velocity is reconstructed by numeric differentiation and low pass Butterworth filter (100 Hz, damping factor 1.0). Two frames (see Figure 2), are attached to the links, to define the joint coordinates q_1 and q_2.

The length of both links is 0.33 m; the rotation ranges of the joints are $-35° \div 215°$ for the first joint, and $-125° \div 125°$ for the second one. The maximum joint velocity is 3.77 rad/s for both joints, while the maximum gripper horizontal acceleration is 9 m/s^2.

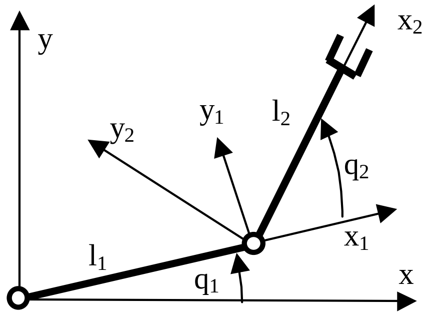

Fig. 2. Scheme of the robot.

The controller is a standard PC, using a Pentium processor and QNX4 real time operating system. The servo loop sampling time is 1 ms. The drives are configured in torque mode, and the desired torque is evaluated by a standard decentralized PID controller.

During normal operation the following data are collected and stored: the desired and the measured motor rotations, the estimated motor velocities, the desired and the measured motor torques, determined from the motor current measures.

2.2 *Robot dynamic model*

Using standard methodologies, a dynamic model of the robot is constructed in the form

$$U = WP, \quad W = W\left(Q, \dot{Q}, \ddot{Q}\right), \quad U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{1}$$

where U is the vector of the motor torques, P is a vector of m constant parameters to be identified, W is a $2 \times m$ matrix depending on the joint position, velocity and acceleration, and Q is the joint vector coordinate, defined as $Q = [q_1,\ q_2]^T$.

Vector P includes the dynamic parameters of the robot, plus some parameters describing the energy loss in the mechanical transmissions (friction).

The dynamic parameters are defined in order to obtain a complete and minimum model ((Gautier and Khalil, 1990)). The so determined parameters, collected in the dynamic parameters vector P_d, are

$$P_d = \begin{bmatrix} I_{1z} + m_2 l_1^2 + J_{m1}/\tau_1^2 \\ m_2 s_{2x} \\ m_2 s_{2y} \\ I_{2z} \\ J_{m2}/\tau_2^2 \end{bmatrix}, \qquad (2)$$

where l_i is the length of link i, I_{iz} is the inertia moment of link i with respect to joint axis i, m_2 is the mass of link 2, s_{2x} and s_{2y} are the coordinates of the center of mass of link 2 with respect to the frame of the second link, J_{mi} are the sum of the inertia of the motor i plus that of the corresponding speed reducer.

On the basis of some previous experimental tests, a third order model has been considered to represent friction on joint i, including motors, speed reducers, and joints friction terms

$$u_{f,i} = a_{0i}\mathrm{sign}(\dot{q}_i) + a_{1i}\dot{q}_i + a_{2i}\mathrm{sign}(\dot{q}_i)\dot{q}_i^2 + a_{3i}\dot{q}_i^3. \qquad (3)$$

The friction parameters vector P_f is then defined as

$$P_f = [a_{01}, a_{11}, a_{21}, a_{31}, a_{02}, a_{12}, a_{22}, a_{32}]^T. \qquad (4)$$

A more detailed friction model should include a term of losses, proportional to the torque transmitted by the speed reducer from the motors to the links. This term is equal to u/η or $u\eta$ depending on the direction of the power flow (where η is the reducer efficiency). The inclusion of this term would make equation (1) non linear with respect to the parameters. For this reason, and after verifying that this term was small compared to that of equation (3), it has been neglected.

Finally two parameters represent a torque offset in the joints. They could represent errors in the acquisition hardware or asymmetry of the joint friction with respect to the velocity

$$P_o = [p_{o1},\, p_{o2}]^T. \qquad (5)$$

Summarizing, P is a 15-element vector, given by

$$P = \begin{bmatrix} P_d^T & P_f^T & P_o^T \end{bmatrix}^T,$$

and matrix W can be expressed as follows, with $c_2 = \cos q_2$, $s_2 = \sin q_2$

$$W = [W_{i,j}], \quad i = 1,2; j = 1,\ldots,15$$

with

$$\begin{aligned}
&W_{1,1} = \tau_1 \ddot{q}_1, \\
&W_{1,2} = \tau_1 l_1 \left(c_2 \left(2\ddot{q}_1 + \ddot{q}_2\right) - s_2 \dot{q}_2 \left(2\dot{q}_1 + \dot{q}_2\right)\right), \\
&W_{1,3} = \tau_1 l_1 \left(-s_2 \left(2\ddot{q}_1 + \ddot{q}_2\right) - c_2 \dot{q}_2 \left(2\dot{q}_1 + \dot{q}_2\right)\right), \\
&W_{1,4} = \tau_1 \left(\ddot{q}_1 + \ddot{q}_2\right), \quad W_{1,6} = \mathrm{sign}\left(\dot{q}_1\right), \\
&W_{1,7} = \dot{q}_1, \quad W_{1,8} = \mathrm{sign}\left(\dot{q}_1\right)\dot{q}_1^2, \quad W_{1,9} = \dot{q}_1^3, \\
&W_{1,14} = 1, \quad W_{2,2} = \tau_2 l_1 \left(c_2 \ddot{q}_1 + s_2 \dot{q}_1^2\right), \\
&W_{2,3} = \tau_2 l_1 \left(-s_2 \ddot{q}_1 + c_2 \dot{q}_1^2\right), \\
&W_{2,4} = \tau_2 \left(\ddot{q}_1 + \ddot{q}_2\right), \quad W_{2,5} = \tau_2 \ddot{q}_2, \\
&W_{2,10} = \mathrm{sign}\left(\dot{q}_2\right), \quad W_{2,11} = \dot{q}_2, \\
&W_{2,12} = \mathrm{sign}\left(\dot{q}_2\right)\dot{q}_2^2, \quad W_{2,13} = \dot{q}_2^3, \quad W_{2,15} = 1,
\end{aligned}$$

all the other elements of W being zero.

3. THE ROBOT CALIBRATION PROCEDURE

Relation (1), which is linear with respect to P, can be used to estimate the parameters vector P, collecting the values of U, Q, $\dot{Q}$, $\ddot{Q}$, at n time instants, from t_1 to t_n, during the execution of a task, thus obtaining an equation of the form

$$y = HP + v \qquad (6)$$

with $y := [U(t_1) \ \ldots U(t_n)]^T$, and

$$H := \begin{bmatrix} W\left(Q(t_1), \dot{Q}(t_1), \ddot{Q}(t_1)\right) \\ \cdots \\ W\left(Q(t_n), \dot{Q}(t_n), \ddot{Q}(t_n)\right) \end{bmatrix},$$

where v is assumed to be the zero mean measurement noise vector, uncorrelated from P, having autocorrelation matrix R_v, and H is the regression matrix, assumed to be deterministic.

P can then be estimated by using a recursive formulation of the LS algorithm: let $H_i := W(t_i)$ and y_i represent the i-th row of H and the i-th torque measurement, respectively; let $\hat{P}_i$ and $S_{P,i}$ be defined as the estimate of the parameter vector P, given the measurements up to i, and the relative covariance, respectively; the estimate can then be recursively updated using the following recursions

$$\begin{aligned}
\hat{P}_{i+1} &= \hat{P}_i + K_{i+1}(y_{i+1} - H_{i+1}\hat{P}_i), \quad \hat{P}_0 = \bar{P}, \\
K_{i+1} &= S_{P,i} H_{i+1}^T (R_v + H_{i+1} S_{P,i} H_{i+1}^T)^{-1}, \\
S_{P,i+1} &= S_{P,i} - K_{i+1} H_{i+1} S_{P,i}, \quad S_{P,0} = R_P,
\end{aligned}$$

where $\bar{P}$ is some a priori information about P (considered as initial condition for the recursive estimation), having covariance matrix R_P. If no a priori knowledge is available, as in our case, then $R_P = \infty$, and it follows that the amplitude of the estimation error depends on $\Pi = H^T R_v^{-1} H$.

The execution of a particular robot trajectory during calibration determines H, and consequently Π. Different "measures" on Π have been defined in literature to (try to) predict in some sense the quality of the computed estimate of P. The most frequently used (see (Ljung, 1987)) are the condition number of Π, $J_k := \mathrm{cond}(\Pi)$ (to be minimized), and the determinant of Π or some scalar measure depending on it: for instance, the so-called *D-optimality* criterion makes use of index $J_d := \log\det(\Pi)$ (to be maximized).

In a previous paper (Calafiore *et al.*, 2001) by some of the coauthors, the optimal trajectory search was performed using a procedure based on genetic algorithms, and the so-determined trajectories were utilized to identify the dynamic parameters of a robot, achieving results that were considered quite satisfying. However, no comparison was made with the estimates obtained using some other kind of trajectories. It is not clear then how important is the use of such trajectories for calibration, and which kind of optimality criteria must be conveniently chosen in practice, taking into account the particular matters related to the considered robot (for example, the uncertainties about the order of the friction model and about the importance of the introduced offset parameters). These topics are analyzed in the remainder of this paper, after the definition of the adopted trajectories.

3.1 *Adopted trajectories*

Two kinds of trajectories have been selected to perform the experiments:

- standard working trajectories planned in the robot working space (type 1);
- trajectories planned in joint space in order to optimize some criteria (type 2).

The first group contains the three trajectories represented in Figure 3 and labelled as Line1, Line2 and Circle (L1, L2 and CI). The parameters of the trajectories were selected in order to use a consistent part of the working area but avoiding the singular configuration ($\sin q_2 = 0$), and the law of motion of the gripper was chosen in order to "excite" all the model parameters, simply reaching high speed and accelerations. The trajectories L1, L2 and CI were performed respectively in 4.5, 2.5, 5.25 seconds.

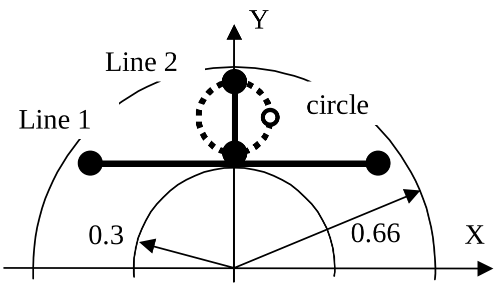

Fig. 3. The adopted trajectories (type 1) in the robot working area.

The trajectories of the second group are of the form

$$q_i(t) = \alpha_{0i} + \sum_{j=1}^{n_a} \alpha_{j,i} \sin(\omega_{j,i} t), \tag{7}$$

for the i-th joint. Assigned the number of harmonics n_a, the parameters $\alpha_{j,i}$ and $\omega_{j,i}$ were chosen according to the methodology presented in (Calafiore *et al.*, 2001) in order to optimize the J_k and J_d indexes: the first one (OJK) is a four harmonics trajectory (with duration time $T = 20$ s, $\alpha_{01} = 1.57$, $\alpha_{11} = 0.2545$, $\alpha_{21} = -0.1091$, $\alpha_{31} = 0.1091$, $\alpha_{41} = -0.1091$, $\alpha_{02} = 0$, $\alpha_{12} = 0.5454$, $\alpha_{22} = -0.5454$, $\alpha_{32} = -0.4727$, $\alpha_{42} = -0.5454$, $\omega_{11} = 3.7082$, $\omega_{21} = 0.2472$, $\omega_{31} = 2.7194$, $\omega_{41} = 0.4944$, $\omega_{12} = 1.7305$, $\omega_{22} = 0.2472$, $\omega_{32} = 0.2472$, $\omega_{42} = 0.2472$), while the second one (OJD) has one harmonic term only (with $T = 25$ s, $\alpha_{01} = 1.57$, $\alpha_{11} = -2.18166$, $\alpha_{02} = 0$, $\alpha_{12} = 2.18166$, $\omega_{11} = 1.37509$, $\omega_{12} = 1.17865$). All the $\alpha_{j,i}$'s are expressed in radians, while the $\omega_{j,i}$'s are in rad/s.

	CI	L1	L2	OJK	OJD
J_d	55.51	58.08	39.99	51.69	75.83
J_k	$7.35\,10^6$	$4.09\,10^6$	$7.02\,10^7$	$2.45\,10^4$	$1.49\,10^6$
σ_1	0.036	0.037	0.061	0.035	0.040
σ_2	0.012	0.012	0.031	0.012	0.012
$\dot{q}_{1,m}$	1.47	1.87	0.87	1.29	3.04
$\dot{q}_{2,m}$	1.95	2.03	1.72	1.29	2.58
$\ddot{q}_{1,m}$	6.55	6.55	3.91	5.34	3.16
$\ddot{q}_{2,m}$	10.11	10.02	6.99	4.12	3.03

Table 1. Trajectories characteristics.

Each trajectory was performed six times to investigate the repeatability of the system and the measurement noise. Data were collected at 1 KHz, and 2500 equally spaced samples were considered for the robot dynamic calibration.

The main characteristics of all the trajectories are reported in Table 1, which shows: the performance indexes J_k and J_d computed on the collected 2500 samples on each trajectory, the standard deviation σ_i (in Nm) of the torque measurements for the i-th joint (computed on the samples of the six repetitions of each trajectory, as in (Calafiore *et al.*, 2001)), and the maximum values of the joint velocities and accelerations $\dot{q}_m$, $\ddot{q}_m$, in rad/s and rad/s^2, respectively.

The rms value of the joint torques during the execution of the different trajectories is reported in the first column of Table 2.

The trajectories of the second group cover a greater portion of the robot working space than those of the first group; in particular, they are characterized by a much greater excursion of the second joint ($-2.1 \div 2.1$ rad for OJK and $-2.18 \div 2.18$ rad for OJD, versus only $0.70 \div 2.06$ rad, $0.78 \div 2.02$ rad and $0.86 \div 2.02$ rad for CI, L1 and L2, respectively). This fact, however, does not indicate that such trajectories are necessarily more suitable for calibration than the other ones.

4. EXPERIMENTAL RESULTS

In this section, we discuss the experimental results obtained using the previous trajectories for identification.

From the theoretical point of view, the uncertainty around the estimated parameters should be smaller when trajectories with a large value of J_d are employed to collect data (OJD and L1, in our case), whereas trajectories with a small value of J_k (OJK, OJD and L1) should guarantee a reduction of the estimate bias due to unmodelled dynamics errors. Starting from these considerations, our expectations before performing the experimental tests can be summarized as follows.

- If the structure of the employed dynamic model is not correct (especially for friction), trajectories OJK, OJD and L1 should somehow reduce the effects of such errors.

	Torques [Nm]						
	rms	errors					
Traj. (j.)		CI	L1	L2	OJK	OJD	Aver.
CI (1)	0.535	0.055	0.066	0.190	0.065	0.060	0.087
CI (2)	0.223	0.023	0.030	0.096	0.041	0.031	0.044
L1 (1)	0.512	0.100	0.079	0.796	0.148	0.083	0.241
L1 (2)	0.230	0.061	0.040	0.084	0.054	0.043	0.056
L2 (1)	0.360	0.073	0.066	0.047	0.063	0.062	0.062
L2 (2)	0.199	0.040	0.031	0.029	0.035	0.031	0.033
OJK (1)	0.531	0.111	0.062	0.260	0.057	0.064	0.111
OJK (2)	0.135	0.048	0.029	0.126	0.021	0.022	0.049
OJD (1)	0.602	0.167	0.077	5.376	1.298	0.048	1.393
OJD (2)	0.173	0.082	0.030	0.153	0.115	0.018	0.080
Aver. (1)		0.101	0.070	1.334	0.326	0.063	0.379
Aver. (2)		0.051	0.032	0.097	0.053	0.029	0.052

Table 2. Torques: rms values and errors (complete model).

- The "best" estimates (from the point of view of the parameter estimation uncertainty) should be performed by trajectories OJD and L1.
- Since the J_d index of the OJD trajectory is much greater than the other ones, and its J_k is relatively small (only OJK has a better one, but with a poor value of J_d), this trajectory should provide the best results.

The results obtained using the various trajectories are compared to test their capability of reconstruction of the joint torques, when different robot motions are considered.

Results of the experimental analysis are presented in Table 2, which shows the rms joint torque values and the torque reconstruction errors obtained for each trajectory, using the parameters estimated on another trajectory. For instance, if the parameters estimated using data collected on L1 are used to reconstruct the torques when trajectory CI is executed by the robot, the rms joint torque errors are 0.066 Nm for joint 1 and 0.030 Nm for joint 2. The average values reported on the right of each row are indexes showing how much a trajectory is "difficult to be reconstructed"; in other words, a high value indicates that the parameters estimated on other trajectories are not suitable to reconstruct the torques on this trajectory. Similarly, low values at the bottom of each column indicate that the parameters estimated using that trajectory can be reliably used to predict the torques of the others.

Table 3 reports the same values, after a normalization on the torque errors resulting from the parameters estimated on the same trajectory for which the torque reconstruction is performed. This table shows that the best torque prediction on each trajectory is (obviously) always performed using the parameters estimated on it (all the other elements are > 1). The "easiest" trajectory to be reconstructed is L2 (the normalized average relative errors are 1.335 and 1.157 for the two joints, respectively), which has the worst indexes (and in fact the parameters estimated on it give the worst results in the reconstruction of the other trajectories). The most difficult to be reconstructed is indeed OJD (errors 29.054 and 4.399), which is the most "reliable" trajectory to predict the torques of the other ones (errors 1.116 and 1.113), as expected.

	CI	L1	L2	OJK	OJD	Aver.
CI (1)	1.000	1.210	3.472	1.192	1.096	1.594
CI (2)	1.000	1.289	4.185	1.793	1.367	1.927
L1 (1)	1.264	1.000	10.017	1.860	1.043	3.037
L1 (2)	1.534	1.000	2.117	1.365	1.084	1.420
L2 (1)	1.567	1.422	1.000	1.350	1.334	1.335
L2 (2)	1.406	1.085	1.000	1.214	1.081	1.157
OJK (1)	1.937	1.086	4.535	1.000	1.109	1.933
OJK (2)	2.262	1.344	5.903	1.000	1.033	2.308
OJD (1)	3.487	1.610	112.106	27.066	1.000	29.054
OJD (2)	4.561	1.640	8.445	6.348	1.000	4.399
Aver. (1)	1.851	1.266	26.226	6.494	1.116	7.390
Aver. (2)	2.153	1.272	4.330	2.344	1.113	2.242

Table 3. Comparison of torque errors (complete model) - normalized data.

J_d seems to be the most significant index (at least in our case) to determine if a a trajectory can provide "good" parameter estimates, suitable for torque reconstruction during various robot motions. In fact, the average error on the torque prediction decreases when J_d increases. On the contrary, the value of the J_k index does not seem to have particular importance in our case with respect to the torque reconstruction.

It should also been noticed that the value of J_d tends to increase with joint velocity: OJD (the best performing trajectory) has the highest value of J_d and the highest value of the joint velocities.

A further analysis was performed comparing the friction torque-velocity relation (3) evaluated with the parameters estimated on the different trajectories. The predictions were very similar if the comparison was made, for each trajectory, in the range of velocity experienced during the acquisition. However the parameters estimated on the "slow" trajectories (e.g. L2) are unreliable to predict the torque for high velocities, as shown in Figure 4, where the estimated friction torque on the first joint is reported, together with five vertical lines in correspondence of the maximum velocity reached by this joint during the execution of each trajectory.

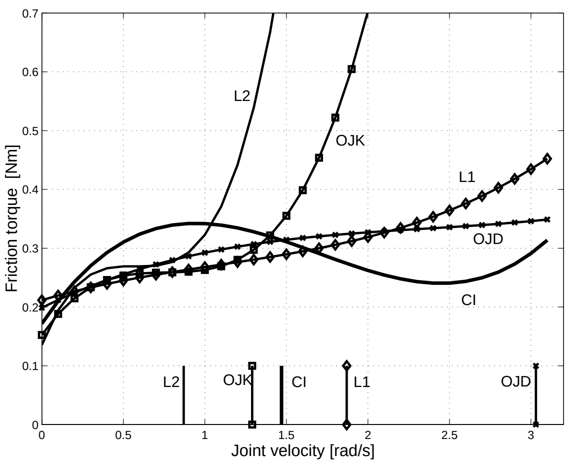

Fig. 4. Estimated friction torque on joint 1.

	Torques [Nm]						
	rms	errors					
Traj. (j.)		CI	L1	L2	OJK	OJD	Aver.
CI (1)	0.535	0.055	0.060	0.235	0.061	0.061	0.094
CI (2)	0.223	0.025	0.031	0.089	0.035	0.033	0.043
L1 (1)	0.512	0.100	0.082	0.257	0.106	0.084	0.126
L1 (2)	0.230	0.058	0.040	0.085	0.047	0.045	0.055
L2 (1)	0.360	0.072	0.063	0.049	0.063	0.066	0.063
L2 (2)	0.199	0.039	0.033	0.030	0.033	0.033	0.034
OJK (1)	0.531	0.110	0.069	0.249	0.065	0.068	0.112
OJK (2)	0.135	0.057	0.030	0.123	0.023	0.024	0.052
OJD (1)	0.602	0.334	0.054	1.107	0.229	0.051	0.355
OJD (2)	0.173	0.068	0.025	0.148	0.022	0.019	0.057
Aver. (1)		0.134	0.066	0.379	0.105	0.066	0.150
Aver. (2)		0.049	0.032	0.095	0.032	0.031	0.048

Table 4. Torques: rms values and errors (reduced model).

It was finally checked whether a more simple model could be more robust with respect to torque prediction, i.e. if it could be preferable to exclude from the model all the parameters that cannot be estimated with enough accuracy. A simplified model (10 parameters) was obtained setting to zero the third dynamic parameter (theoretically null due to geometrical symmetry), the two torque offsets, and the friction coefficients of the third order terms.

Results presented in Table 4, compared with those of Table 3, show that, using the reduced model, the torque reconstruction on the same trajectory used for calibration is slightly worse compared with that provided by the complete model (for every trajectory). However, the results obtained reconstructing the torques applied to perform *other* trajectories are better: in particular, even if OJD remains the best performing trajectory, similar results are obtained in this case also by L1, while an acceptable (even if poor, anyway) reconstruction of the torque applied on the first joint is obtained this time also by L2 (the worst trajectory), with a reduction of the average error for this joint from 1.334 Nm to 0.379 Nm.

Finally, it should be remarked that a complete and correct analysis of the quality of the *parameter estimates* would be possible only if the true values of the parameters were known. However, we notice that the values estimated by the worst trajectories are sometimes quite different from those computed by the best ones and sometimes unfeasible: for example, the estimate of $P_d(4)$ should be always positive, as it represents the inertia moment I_{2z} of the second link. Table 5 shows the parameter values (each one expressed in its proper unit) estimated by the various trajectories in the reduced model case.

	Trajectories				
Parameters	CI	L1	L2	OJK	OJD
$P_d(1)$	16.307	16.442	12.308	16.594	16.165
$P_d(2)$	-0.547	1.855	1.352	2.404	2.415
$P_d(4)$	-0.592	0.443	-3.554	0.749	0.758
$P_d(5)$	4.963	4.443	6.440	3.947	4.014
a_{01}	0.185	0.195	0.144	0.175	0.210
a_{11}	0.304	0.120	0.427	0.181	0.093
a_{21}	-0.148	-0.024	-0.328	-0.085	-0.016
a_{02}	0.084	0.107	0.077	0.073	0.088
a_{12}	0.021	0.039	0.097	0.089	0.049
a_{22}	0.002	-0.007	-0.034	-0.023	-0.007

Table 5. Parameter estimates (reduced model).

5. CONCLUSIONS

The presented experiments confirm that, at least for the considered robot, the identification of the dynamic parameters takes advantage of the maximization of index J_d, and that the optimization procedure developed in (Calafiore *et al.*, 2001) works satisfactorily in practice. The obtained results also suggest that when some parameters cannot be identified properly, it is preferable to set them to their nominal values (as $P_d(3)$ and P_o, which are in our case nominally equal to zero), or to exclude them from the model (as the cubic friction terms in our case).

6. REFERENCES

Armstrong, B. (1987). On finding exciting trajectories for identification experiments involving systems with non-linear dynamics. In: *Proc. IEEE Int. Conf. On Robotics and Automation.* pp. 1131–1139.

Caccavale, F. and P. Chiacchio (1994). Identification of dynamic parameters and feedforward control for a conventional industrial manipulator. *Control Engineering Practice* **2**(6), 1039–1050.

Calafiore, G., M. Indri and B. Bona (2001). Robot dynamic calibration: Optimal excitation trajectories and experimental parameter estimation. *J. of Robotic Systems* **18**(2), 55–68.

Gautier, M. and W. Khalil (1990). Direct calculation of minimum set of inertial parameters of serial robots. *IEEE Trans. of Robotics and Automation* **6**(3), 368–373.

Gautier, M. and W. Khalil (1992). Exciting trajectories for the identification of base inertial parameters of robots. *The Int. J. of Robotics Research* **11**(4), 362–375.

Kozlowski, K.R. (1998). *Modelling and identification in robotics.* Springer-Verlag. London.

Ljung, L. (1987). *System Indentification, Theory for the User.* Englewood Cliffs: Prentice-Hall.

Neuman, C.P. and P.K. Khosla (1985). Identification of robot dynamics: an application of recursive estimation. In: *Proc. 4th Yale Workshop on Appl. Of Adaptive System Theory.* Yale University, New Haven. pp. 42–49.

Swevers, J., C. Ganseman, D.B. Tukel, J. De Schutter and H. Van Brussel (1997). Optimal robot excitation and identification. *IEEE Trans. on Robotics and Automation* **13**(5), 730–740.

Visioli, A. and G. Legnani (2000). Experiments on model identification and control of an industrial robot. In: *Proc. 6th IFAC Symp. on Robot Control (SYROCO 2000).* Vol. 1. Vienna, Austria. pp. 283–288.