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A MULTI-OBJECTIVE FILTERING APPROACH FOR FAULT DIAGNOSIS WITH GUARANTEED SENSITIVITY PERFORMANCE.

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Abstract: The paper addresses the problem of model-based fault detection. Synthesis techniques for the design of fault detection filters are developed. Fault sensitivity objectives and time-domain constraints are handled in the design procedure, while guaranteeing robustness to noise, disturbances and modeling errors. The design method involves linear matrix inequality (LMI) optimization techniques and the generalized structured singular value μ_g. The approach is applied to a 3-Tanks system and experimental results demonstrate the potential of the proposed method.

Keywords: Fault diagnosis, fault sensitivity performances, multi-objective filter design, linear matrix inequalities, generalized structured singular value.

1. INTRODUCTION

Fault Detection and Isolation (FDI) is an essential part in intelligent control of dynamic systems and has been a very active research field during the last twenty years (see (Chen and Patton, 1999; Frank *et al.*, 2000) for surveys). FDI schemes may be thought of two stages: generation of a fault accentuated signal (residual) and evaluation of this residual, i.e. decision making. The main objective of the FDI process is the achievement of a low missed-alarm and a low false-alarm rates. So robustness to all unknown inputs and modeling errors and sensitivity to faults are required in both residual generation and residual evaluation stages.

Recently, $\mathscr{H}_\infty$-filtering techniques have been developed, both in FDI scenario (Edelmayer *et al.*, 1997; Mangoubi, 1998) and robust estimation framework (Shaked and Theodor, 1992; Huaizhong and Minyue, 1997). The main idea consists in designing a filter so that the filtering error remains robust against disturbance and modeling errors, within a prespecified $\mathscr{H}_\infty$ attenuation level. In (Chen and Patton, 1999) the problem of maximizing sensitivity to additive faults while guaranteeing robustness constraints, is considered within an $\mathscr{H}_\infty$ / $\mathscr{H}_-$ setting. The method is based on a observer-based FDI scheme and involves LMI (Linear Matrix Inequality) optimization techniques. Maximizing the sensitivity to parametric faults is also discussed in (Stoustrup and Niemann, 1999). It is shown that the problem can be formulated into a μ optimization problem. Unfortunately, the solution often leads to a high order filter, because of the use of scaling matrices within the optimization procedure. Furthermore, whereas the developed approaches handle frequency-domain objectives, the design procedures do not account for time-domain specifications. As an improvement, an observer-based approach has been described in (Chen and Patton, 1999). The approach is based on a FDI observer combined with a residual weighting matrix. Different indices are set up for the design of the observer gain and the residual weighting factor. The solution involves eigenstructure assignment with a genetic algorithm. However, the developed method does not account for a large class of uncertainties (e.g. nonlinear parametric uncertainty

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and neglected dynamics). To overcome this problem, a FDI filter design method is proposed in (Henry *et al.*, 2001). The main idea is to restrict the filter dynamics to some specific regions, while guaranteeing simultaneously sensitivity to additive faults and robustness to a large class of uncertainty. The design procedure involves robust regional poles assignment within an LMI setting and the generalized structured singular value μ_g.

The goal of this paper is to present an improvement of the method proposed in (Henry *et al.*, 2001). More precisely, we are interesting to design a fault detection filter which takes into account multiple FDI objectives. The considered objectives are robustness against external disturbances, normal variations of system parameters and neglected dynamics, fault sensitivity objectives and time-domain constraints on the residual (e.g. peak amplitudes, settling time).
The two important advantages of the approach developped in this paper, are:

- First, it provides a framework where a large class of uncertainty surrounding the system model and a large set of possible additive faults modes can be included,
- Second, a constraint regarding the peak amplitude of the residual is taking into account within the design procedure. This feature becomes very important from a decision making point of view, as the residual is generally processed by an evaluation test to make a final decision about the fault.

The design procedure can be summarized as follows. First, a FDI filter is designed which satisfies the robustness constraints and a set of time-domain specifications on the residual. It is shown that the problem can be formulated into a LMI optimization problem. It is obvious that both sensitivity to faults and insensitivity to disturbances cannot be achieved if they manifest themselves at same frequencies. Faults having very similar frequency characteristics as those of uncertainties might not be detected. Second, the generalized structured singular value μ_g is used to check robust fault sensitivity. The procedure is iterative and stops when all achieved performances indeed objectives are judged satisfactory.

Notations

The notations are fairly standard. In dealing with vectors, the Euclidean norm is always used and is written without a subscript; for example $||x||$. Similarly, in the matrix case, $||M||$ is used to denote the induced vector norm, i.e. $||M|| = \overline{\sigma}(M)$,where $\overline{\sigma}(M)$ denotes the maximum singular value of M ($\underline{\sigma}(M)$ represents the minimum singular value of M). $M < 0$ ($M > 0$) indicates that the matrix M is negative (positive) definite. $M \otimes N$ denotes the Kronecker product of matrices M by N. Signals, for example $w(t)$ or w, are assumed to be of bounded energy, and their norm is denoted by $||w||_2$, i.e. $||w||_2 = \left(\int_{-\infty}^{\infty} ||w(t)||^2 dt\right)^{1/2} < \infty$. Linear models, for example, $P(s)$ or simply P, are assumed to be in $R\mathscr{H}_\infty$, real rational functions with $||P(s)||_\infty = \sup_\omega \overline{\sigma}(P(j\omega)) < \infty$.
Uncertain systems are represented by the linear fractional representation of the matrix Δ posed on P, which is referred to as the star product $(\Delta * P)$. Δ is modeled, without lost of generality, by a block diagonal structure and represents multiple perturbations. $\underline{\Delta}$ describes the set of all perturbations of a prescribed structure as $\underline{\Delta} = \{\text{block diag}(\delta_1^r I_{k_1}, \ldots, \delta_{m_r}^r I_{k_{m_r}}, \delta_1^c I_{k_{m_r+1}}, \delta_{m_c}^c I_{k_{m_r+m_c}}, \Delta_1^C, \ldots, \Delta_{m_C}^C), \delta_i^r \in \mathbf{R}, \delta_i^c \in \mathbf{C}, \Delta_i^C \in \mathbf{C}\}$ where $\delta_i^r I_{k_i}, i = 1, \ldots, m_r$, $\delta_j^c I_{k_{m_r+j}}, j = 1, \ldots, m_c$ and $\Delta_l^C, l = 1, \ldots, m_C$ are known respectively as the "repeated real scalar" blocks, the "repeated complex scalar" blocks and the "full complex" blocks.
Consider the feedback interconnection defined by $w = \tilde{M}v$ and $v = \tilde{\Delta}w$, where $\tilde{M}$ and $\tilde{\Delta}$ are partitioned according to $\underline{\tilde{\Delta}} = diag(\underline{\Delta}_J, \underline{\Delta}_K)$, $\tilde{M} = \begin{pmatrix} \tilde{M}_{JJ} & \tilde{M}_{JK} \\ \tilde{M}_{KJ} & \tilde{M}_{KK} \end{pmatrix}$, $||\Delta_J||_\infty \leq 1$, $||\Delta_K||_\infty \geq 1$ and where $\underline{\tilde{\Delta}}$ is defined like $\underline{\Delta}$. The positive real-valued function μ_g is defined by (Newlin and Smith, 1998)

$$\mu_{g\underline{\tilde{\Delta}}}(\tilde{M}) \stackrel{\triangle}{=} \max_{||v||=1} \left\{ \gamma : \begin{array}{l} ||v_j||\gamma \leq ||w_j||, \forall j \in J \\ ||v_k|| \geq ||w_k||\gamma, \forall k \in K \end{array} \right\}$$

$\tilde{M} \in \text{dom}(\mu_g)$ iff $\tilde{M}_{KK} v_K = 0 \Rightarrow v_K = 0$.

2. PROBLEM SETTING

Consider a general representation of the system as depicted on figure 1. y is the measured output, $z = Mx$ is a subset of states to be estimated. For full state estimation $M = I$. We assume that all internal plant uncertainty (i.e. parametric uncertainties and neglected dynamics) is represented by Δ so that

$$\Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1 \quad (1)$$

F, whose output is $\hat{z}$, an estimation of z, is the detection filter to be designed. The error signal defined as

$$e = z - \hat{z} \quad (2)$$

is taken as the residual signal. d and f represent respectively exogenous disturbances and additive faults. The signals η and ε are internal to the model. As the control input u does not affect the filter dynamics, u will be ignored from now on.

The FDI filtering problem can be formulated as follows: Assume that the transfer between the fault f and the system output y has no transmission zeros $\forall \Delta \in \underline{\Delta}$, i.e. detectability of the considered faults is guaranteed, see (Chen and Patton, 1999) for more details. The goal is to design a dynamical stable filter F

$$F : \begin{cases} \dot{x}_F = A_F x_F + B_F y \\ \hat{z} = C_F x_F \end{cases} \quad (3)$$

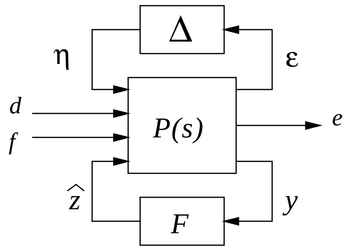

Fig. 1. The structure of fault detection filtering problem.

so that the following specifications are satisfied:

(S.1) In fault free situation, $||T_{ed}||_\infty \leq \beta_1$, for all perturbation model Δ.

(S.2) The poles of the filter F lie in a region $\mathscr{R}$ of the left-half complex plane, for all perturbation model Δ.

(S.3) The peak residual amplitude should be kept below a certain level β_3 in fault free situation.

(S.4) $\underline{\sigma}\left(T_{ef}(j\omega)\right) \geq \beta_2$ in a prespecified frequency range Ω, for all perturbation model Δ.

3. LMI SOLUTION OF THE MULTI-OBJECTIVE ESTIMATION PROBLEM.

Consider the estimation problem in the fault free case (i.e. $f = 0$) with robustness objectives and time-domain constraints (i.e. specifications (S.1) to (S.3)).

The robust estimation problem can be addressed within an $\mathscr{H}_\infty$ setting (Appleby, 1990). If there exists a solution to a Riccati equation, then a filter F exists that satisfies (S.1). The author has extended the method within a μ-framework, which is particularly appealing as it accounts for block structured perturbations. However, no systematic method was developed to account for estimation error dynamics specifications. As an improvement, the problem of $\mathscr{H}_\infty$ filtering design with regional filter poles constraints was investigated in (Palhares and Peres, 2000). However, the method is restricted to polytope type uncertainties. Here, we propose to solve the problem for any model perturbation type $\Delta \in \underline{\Delta}$, using LMI optimization techniques. In the interest of brevity, the focus of this section lies wholly with the main results and assumptions used.

A general state space representation of $P(s)$ (see figure 1) is

$$P : \begin{cases} \dot{x} = Ax + B_1\eta + B_2 d \\ \varepsilon = C_1 x + D_{11}\eta + D_{12} d \\ e = Mx - \hat{z} \\ y = C_2 x + D_{21}\eta + D_{22} d \end{cases} \tag{4}$$

Using equations (3) and (4), the filtering error dynamics is given by

$$\begin{cases} \dot{x}_e = \mathscr{A} x_e + \mathscr{B}\begin{pmatrix}\eta \\ d\end{pmatrix} \\ \varepsilon = \mathscr{C}_1 x_e + \mathscr{D}_1\begin{pmatrix}\eta \\ d\end{pmatrix} \\ e = \mathscr{C}_2 x_e \end{cases} \tag{5}$$

where $x_e = \begin{pmatrix} x \\ x_F \end{pmatrix}$. The state space matrices are defined according to

$$\begin{array}{c} \mathscr{A} = \begin{pmatrix} A & 0 \\ B_F C_2 & A_F \end{pmatrix}, \mathscr{B} = \begin{pmatrix} B_1 & B_2 \\ B_F D_{21} & B_F D_{22} \end{pmatrix} \\ \mathscr{C}_1 = \begin{pmatrix} C_1 & 0 \end{pmatrix}, \quad \mathscr{C}_2 = \begin{pmatrix} M & -C_F \end{pmatrix} \\ \mathscr{D}_1 = \begin{pmatrix} D_{11} & D_{12} \end{pmatrix} \end{array} \tag{6}$$

This state space representation defines the closed-loop transfer T_{ed}.
Now introduce two fictitious signals e_∞ and e_g so that:

$$e_\infty = w_\infty e, \quad ||w_\infty|| = 1/\beta_1 \tag{7}$$

$$e_g = w_g e, \quad ||w_g|| = 1/\beta_3 \tag{8}$$

w_∞ and w_g are also two weights which are designed in order to achieve the specifications (S.1) and (S.3). Including w_∞ and w_g into T_{ed} leads to the set up depicted in figure 2.

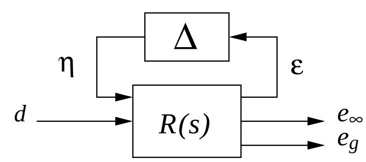

Fig. 2. The structure of multi-objective filter design.

3.1 *LMI formulation for $\mathscr{H}_\infty$ specifications.*

Consider the transfer $R_\infty(s) = L_\infty R(s) K_\infty$, where the matrices L_∞ and K_∞ are defined to select the channels $\begin{pmatrix}\eta \\ d\end{pmatrix} \to \begin{pmatrix}\varepsilon \\ e_\infty\end{pmatrix}$, and let us denote $(\mathscr{A}_\infty, \mathscr{B}_\infty, \mathscr{C}_\infty, \mathscr{D}_\infty)$ the associated state space representation. Note that, by construction, $\mathscr{A}_\infty \equiv \mathscr{A}$ where $\mathscr{A}$ is defined as in (6).

By virtue of the Bounded Real Lemma (Boyd *et al.*, 1994), R_∞ is stable (and so F is a stable filter) and $||R_\infty||_\infty \leq \gamma$, for all Δ satisfying relation (1) if and only if there exists a symmetric matrix $X_\infty > 0$ so that

$$\begin{pmatrix} \mathscr{A}_\infty^T X_\infty + X_\infty \mathscr{A}_\infty & X_\infty \mathscr{B}_\infty & \mathscr{C}_\infty^T \\ \mathscr{B}_\infty^T X_\infty & -\gamma I & \mathscr{D}_\infty^T \\ \mathscr{C}_\infty & \mathscr{D}_\infty & -\gamma I \end{pmatrix} < 0 \tag{9}$$

This formulation does not account for block structured perturbations. Consequently, as it has been already noted in (Scherer *et al.*, 1997), the resulting estimator might be too conservative. The structured singular value μ (Doyle, 1982) can be used to test the degree of conservatism of the resulting filter F. If necessary, the weight w_∞ can be tuned to obtain less conservative solution.

3.2 *LMI formulation for residual peak amplitude objective.*

In addition to the $\mathscr{H}_\infty$-norm specification considered in the above section, it is desirable to keep the peak amplitude of the estimation error signal $e(t)$ below a certain level (see specification (S.3)). If the disturbances d are quantified by their energy, this leads to considering the so-called generalized $\mathscr{H}_2$-norm, denoted $||T_{ed}||_g$, which measures the peak amplitude of the signal $e(t)$ over all unity-energy bounded disturbances.

Consider the transfer $R_g(s) = L_g R(s) K_g$, where the matrices L_g and K_g are defined to select the channel $d \to e_g$, and let us denote $(\mathscr{A}_g, \mathscr{B}_g, \mathscr{C}_g, \mathscr{D}_g)$ the associated state space representation. Note that, by construction, $\mathscr{A}_g \equiv \mathscr{A}$ where $\mathscr{A}$ is defined as in (6).

It is shown in (Scherer *et al.*, 1997) that a necessary condition for the norm bound $||R_g||_g^2 < \alpha$ to hold, is the existence of a symmetric matrix $X_g > 0$ satisfying

$$\begin{pmatrix} \mathscr{A}_g^T X_g + X_g \mathscr{A}_g & X_g \mathscr{B}_g \\ \mathscr{B}_g^T X_g & -I \end{pmatrix} < 0 \tag{10}$$

$$\begin{pmatrix} X_g & \mathscr{C}_g^T \\ \mathscr{C}_g & \alpha I \end{pmatrix} > 0, \quad \mathscr{D}_g = 0 \tag{11}$$

It is important to outline that, due to the definition of the matrices L_g and K_g, there is no guarantee that $||R_g||_g^2 < \alpha$ yields for all Δ satisfying (1).

3.3 *LMI region for robust pole placement.*

Consider an LMI region $\mathscr{R}$ formed by the intersection of N elementary LMI regions $\mathscr{R}_i$ (i.e. $\mathscr{R} = \mathscr{R}_1 \cap \ldots \cap \mathscr{R}_N$) characterized by their characteristic functions

$$\begin{aligned} f_{\mathscr{R}_i}(\chi) = L_i + \chi M_i + \chi^* M_i^T < 0 \\ M_i = M_{1i}^T M_{2i} \end{aligned} \tag{12}$$

where M_{1i} and M_{2i} have full column rank (such factorization can be easily obtained from the SVD of M_i).

In (Chilali *et al.*, 1999), it is shown that a sufficient condition for all eigenvalues of $\mathscr{A}$ (see equation (6)) lying in the region $\mathscr{R}$ for all Δ satisfying equation (1), is the existence, for each region $\mathscr{R}_i$, of a pair of matrices (X_i, P_i) so that

$$\begin{gathered} \begin{pmatrix} \mathscr{M}_{\mathscr{R}_i}(\mathscr{A}, X_i) & M_{1i}^T \otimes (X_i \mathscr{B}) & (M_{2i}^T P_i) \otimes \mathscr{C}^T \\ M_{1i} \otimes (\mathscr{B}^T X_i) & -P_i \otimes I & P_i \otimes \mathscr{D}^T \\ (P_i M_{2i}) \otimes \mathscr{C} & P_i \otimes \mathscr{D} & -P_i \otimes I \end{pmatrix} < 0 \\ P_i > 0, \quad X_i > 0, \quad i = 1, \ldots, N \end{gathered} \tag{13}$$

where

$$\begin{aligned} \mathscr{M}_{\mathscr{R}_i}(\mathscr{A}, X_i) := L_i \otimes X_i + M_i \otimes (X_i \mathscr{A}) + \ldots \\ \ldots + M_i^T \otimes (\mathscr{A}^T X_i), \qquad i = 1, \ldots, N \end{aligned} \tag{14}$$

$$\mathscr{C} = \begin{pmatrix} \mathscr{C}_1 \\ \mathscr{C}_2 \end{pmatrix}, \quad \mathscr{D} = \begin{pmatrix} \mathscr{D}_1 \\ 0 \end{pmatrix} \tag{15}$$

The matrices $\mathscr{A}, \mathscr{B}, \mathscr{C}_1, \mathscr{C}_2$ and $\mathscr{D}_1$ are also defined in (6).

Coming back to the expression of the matrix $\mathscr{A}$, it is clear that the set of its eigenvalues are equal to the set of the eigenvalues of A (see (4)) and A_F (see (3)). Thus, the inequalities (13) and (14) correspond to a robust regional poles assignment of the filter F.

Remark 1. LMI Feasibility: (Scherer *et al.*, 1997) have demonstrated that for feasibility problems, one must jointly solve all inequalities (9), (10), (11) and (13) by imposing

$$\begin{aligned} X = X_\infty = X_g = X_1 = \ldots = X_N \\ P_i = I, \quad i = 1, \ldots, N \end{aligned} \tag{16}$$

This leads to a more conservative filter F because of the requirement of a single matrix X satisfying all constraints. Furthermore, it is obvious that the inequalities are not affine in the filter realization matrices A_F, B_F, C_F (which are the solution we are looking for) and in X. To overcome this problem, (Scherer *et al.*, 1997) have defined changes of variables that allow all inequalities to be transformed into LMIs.

4. ROBUST FAULT SENSITIVITY PERFORMANCES.

Consider $f \neq 0$. The main problem is now to satisfy the robust fault sensitivity objectives (see specification (S.4)). To proceed, include the filter F into the model R_∞ defined in section 3.1. This leads to the set up described by the block diagram shown on figure 3, where $\Delta_e \in \mathbf{C}^{dim(d) \times dim(e)} : ||\Delta_e||_\infty \leq 1$ is an introduced fictitious uncertainty block. e_f is a fictitious estimation error signal, weighted by w_f:

$$e_f = w_f e, \quad ||w_f|| = 1/\beta_2 \tag{17}$$

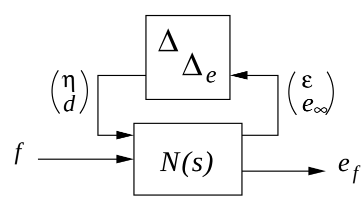

Fig. 3. The generic structure of robust detection performance problem.

In contrast to the robust performance problem considered in section 3, the robust fault detection sensitivity problem is essentially a robust minimum gain problem, over a prespecified frequency grid. This problem is also equivalent to

$$\begin{aligned} \underline{\sigma}\left(\overline{\Delta} * N\right) \geq 1, \quad \forall \omega \in \Omega, \quad \forall \Delta \\ \overline{\Delta} = \text{diag}(\Delta, \Delta_e) \end{aligned} \tag{18}$$

The solution is based on the recently developed generalized structured singular value (denoted μ_g), which was first introduced in (Newlin and Smith, 1998). μ_g can be see as a measure of the stability degree of closed-loop structures so-called "$M-\Delta$", like illustrated on figures 2 and 3. In contrast to the robust performance problem, some elements of the perturbation structure Δ are bounded from above and some are bounded from below (Henry *et al.*, 2002). Like the μ-function, μ_g is particularly appealing as it accounts for block structured perturbations Δ.

The following theorem, which is an adaptation of the theorem 5 in (Newlin and Smith, 1998) for our purpose, gives the solution of the robust sensitivity problem. The proof is omitted here, as it can be found in (Newlin and Smith, 1998).

Theorem 2. Let N be robustly stable, $\tilde{\Delta} = \begin{pmatrix} \overline{\Delta} & 0 \\ 0 & \hat{\Delta} \end{pmatrix}$ where $\overline{\Delta}$ is defined by (18) and $N \in \text{dom}(\mu_g)$. $\hat{\Delta} \in \mathbf{C}^{dim(f)\times dim(e)}$ is a fictitious uncertainty block. Then

$$\min_{\substack{\overline{\Delta}\in\underline{\Delta} \\ \overline{\sigma}(\overline{\Delta})\leq 1}} \underline{\sigma}(\overline{\Delta}*N) \geq 1 \quad \text{iff} \quad \mu_{g\tilde{\underline{\Delta}}}(N(j\omega)) < 1 \quad \forall\omega\in\Omega$$

The constraint $N \in \text{dom}(\mu_g)$ is equivalent to a non-trivial solution, i.e. the maximization part in the μ_g problem is finite. The robust stability condition of N is strictly equivalent to the robust stability condition of P, as the filter F does not affect the state of the system.

Using the above theorem, the robust sensitivity performance can be tested by calculating the μ_g function of N over the block structure $\tilde{\underline{\Delta}}$, at those frequencies where the energy of the fault is likely to be concentrated.

5. APPLICATION TO A THREE-TANKS SYSTEM

The experimental study is based on a pilot three-tank system. The plant consists of three cylinders connected serially with one another cylindrical pipes. The out-flowing liquid is collected in a reservoir, which supplies two pumps. The three water levels are also measured. For the purpose of simulating clogging and operating errors, the connecting pipes are equipped with manually adjustable ball valves, which allow the corresponding pipe to be closed. A simple Proportional Integral controller was implemented, as the control performance was not of prime interest in this work.

The FDI objective is to detect a leak affecting the first, the second and the third tank.

According to the strategy depicted in the previous sections, a detection filter F is designed. The method must be thought as follows:

• *Step 1:* The interconnection system model shown in figure 2 is constructed with $M = I_3$. The uncertainty block Δ is given by

$$\Delta = \text{diag}(\delta_1, \delta_2, \delta_3, \Delta_T) \tag{19}$$

$\delta_k \in \mathbf{R} : |\delta_k| \leq 1, k = 1,2,3$ represent parametric uncertainties related to outflow coefficient, and $\Delta_T \in \mathbf{C}^{3\times 3} : ||\Delta_T||_\infty \leq 1$ models actuators neglected dynamics. The weighting functions W_T associated with Δ_T is experimentally determined as

$$W_T(s) = 0.2\frac{1+3.35s}{1+s}I_3 \tag{20}$$

The exogenous disturbance d consists here in the measurement noise. Based on a power spectral analysis of the input/output data, a weighting function W_n for the measurement noise is determined as

$$W_n(s) = 0.087\frac{1+0.2s}{1+0.125s}I_3 \tag{21}$$

The interested reader can refer to (Henry *et al.*, 2002) for more details on modeling the hydraulic system.

• *Step 2:* The inequalities (9), (10) and (11) are solved in order to determine the filter matrices A_F, B_F and C_F.

• *Step 3:* Finally, according to theorem 2, the robust sensitivity performance is tested by evaluating the μ_g-function. If the level of achieved sensitivity is not considered satisfactory (i.e. if the μ_g-test fails), go to step 1 and reshape the weights w_∞, w_g and w_f (see relations (7), (8) and (17)). If both desired robustness level and detection performance are not achievable, the disturbance robustness requirement should be relaxed. A new filter F is then designed with an increased disturbance sensitivity bound. The procedure stops when a reasonable balance between fault sensitivity and robustness performance is achieved.

Experimental results.

The weights w_∞, w_g and w_f were computed so that $||R_\infty||_\infty$ and $||R_g||_g$ are minimized and $\underline{\sigma}(T_{ef})$ is maximized over the frequency range $[0.01rd/s, 0.05rd/s]$ (which is the frequency range where the energy of the fault is concentrated). Figure 4 illustrates synthesis results. The obtained $\mathscr{H}_\infty$ performance is approximately $\beta_1 = 0.08$ and the LMI optimization on α (see (10) and (11)) yields a generalized $\mathscr{H}_2$-norm lower than $\beta_3 \approx 0.12$. Furthermore, as it can be seen on figure 4, the fault sensitivity specifications are met (see the μ_g behavior). The achieved sensitivity performance is approximately $\beta_2 = 0.021$ for the first leak, $\beta_2 = 0.032$ for the second and $\beta_2 = 0.022$ for the third. As it can be seen on figures 5, the designed procedure succeeds as all faults are detected.

For comparison, a pure $\mathscr{H}_\infty$-based design was performed. The achieved optimal $\mathscr{H}_\infty$ performance was approximately the same. However, in contrast with the multi-objective approach, the fault sensitivity level achieved by the detection filter is much lower (the μ_g-test gives $\beta_2 \approx 0.0027$ for the first leak, $\beta_2 \approx 0.005$ for the second and $\beta_2 \approx 0.003$ for the third).

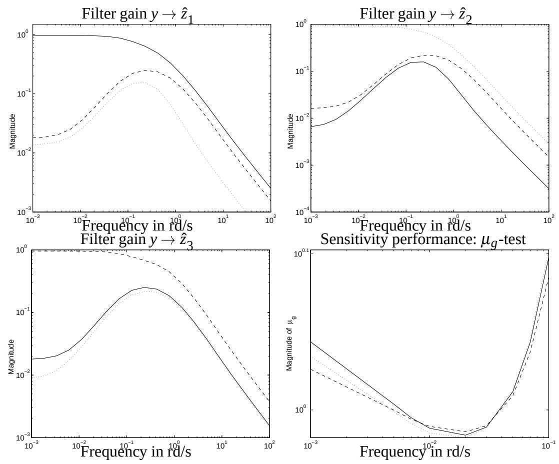

Fig. 4. Detection filter synthesis results.

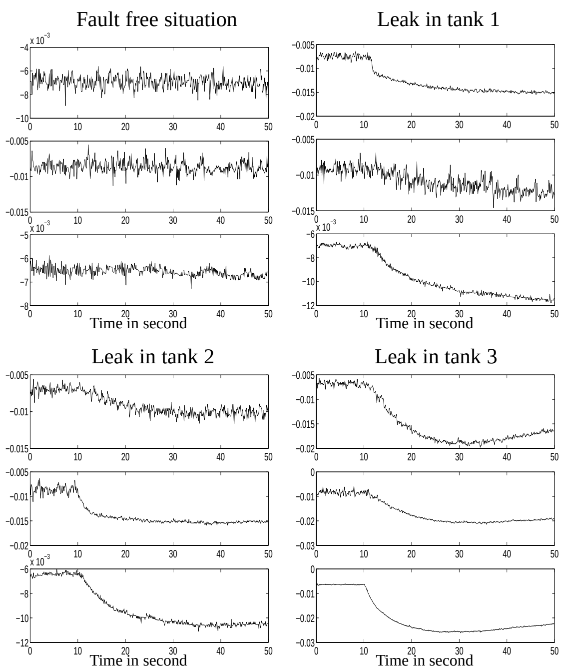

Fig. 5. Behavior of $e(t) = z(t) - \hat{z}(t)$.

6. CONCLUSIONS.

In this paper, we have presented an approach for designing FDI filters. The most important advantage of the proposed scheme is that it provides a framework where many FDI objectives (robustness and fault sensitivity objectives, frequency-domain and regional filter pole placement constraints) can be included. Moreover, the method allows to include time-domain constraints on the residual behavior (for instance the peak amplitude of the residual), which is particularly appealing from a decision making point of view, as the residual is generally processed by a threshold-based evaluation stage.

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