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OPTIMAL CONTROL OF AIRLINE BOOKING PROCESS

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Abstract: This paper deals with the problem of optimization of airline booking levels for flights with several classes of passenger service and the scientific determination of reservation policies. The dynamic and static models of airline data for determining optimal reservation policies for airline booking process are presented. These models make it possible to maximize the unconditional expected gain of the flight or to minimize the unconditional expected value of loss and penalty. The numerical examples are given. *Copyright © 2002 IFAC*

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1. INTRODUCTION

A fundamental problem in the operation of airlines, hotels and other enterprises where a highly perishable product is to be sold at a definite time is the question of reservations (Nechval, 1982, 1984). In the literature, the following problem has been considered. It is known that fully reserved airline flights frequently depart with significant number of unused seats because of "no-shows" (passengers who fail to arrive for flight without notice) and passengers who cancel seats just before departure (in the knowledge that a refund of their fare will be made to them), i.e. when the airline cannot get replacements for them at such short notice. Many airlines seek to compensate for such passenger losses by a policy of deliberate overbooking. The airlines have recognized the necessity of this practice, stating that through "carefully controlled overbooking" they can reduce the number of empty seats and at the same time serve the public interest by accommodating more passengers. The loss due to overbooking must be balanced against the loss due to unused seats. Several airline booking models have already appeared in the literature. Beckmann (1958) published a model (in terms of G distributions) that balances the lost revenue of empty seats with the costs to the airline of passengers denied boarding (as a result of alternate transportation, airport dinners, hotel charges, lost goodwill, etc.). This model yields the 'booking level' or upper bound for reservations that minimizes expected costs. The model of Kosten (1960) has the same objective, but it provides for the interspersion of reservations and cancellations (which Beckmann ignores), and thus yields a booking level that depends upon the number of days yet to transpire before flight. Kosten's continuous time approach leads to a set of simultaneous differential equations that must be solved numerically, and the solution would present serious computational difficulties in realistic cases. Thompson (1961) developed a model to provide booking levels that constrain the probability of denied boardings. This model is quite different from Beckmann's and Kosten's in that costs and passenger reservation demands are omitted and only the cancellation patterns of any fixed number of reserved passengers are described. In Rothstein (1971), the procedure of reservations is viewed as a Markovian sequential decision process. Rothstein (1985) gave a survey of the application of Operations Research to airline overbooking. His article describes significant contributions and implementations of

Operations Research. In Alstrup, *et al.* (1986), an overbooking model for a fixed nonstop flight with two types of passengers is presented. The model treats the airline booking process as a Markovian nonhomogeneous sequential decision process. This model is solved by two-dimensional stochastic dynamic programming. In Ignaccolo and Inturri (2000), a fuzzy approach to the overbooking problem in air transportation is considered.

In the present paper, we consider the problem of optimization of airline booking levels for flights with several classes of passenger service. This problem is one of the most difficult problems of air transport logistics. On the one hand, one must have reasonable assurance that the requirements of customers for reservations will be met under most circumstances. On the other hand, one is confronted with the limitation of the capacity of the cabin, as well as with a host of other less important constraints. The problem is normally solved by the application of judgment based on past experience. The question arises whether or not it is possible to construct a simple mathematical theory of the above problem, which will allow one better to use the available data based upon airline statistics. Two models (dynamic model and static one) of airline data are proposed here. In the dynamic model, the problem is formulated as a sequential decision process. We present an optimal adaptive reservation policy under certainty, which is used at each stage (day) prior to departure time for non-stop flights with several classes of passenger service. The essence of determining the optimal adaptive reservation policy is maximization of the expected gain of the flight, which is carried out at each stage prior to departure time using the available data. The term (adaptive reservation policy) is used in this paper to mean a decision rule, based on the available data, for determining whether to accept a given reservation request made at a particular time for some future date. An optimal non-adaptive reservation policy under certainty is based on the static model. The models of airline data proposed here contain a simple and natural treatment of the airline reservation process and may be appropriate in practice.

2. DYNAMIC MODEL OF AIRLINE DATA

The dynamic model of airline data, as presented here, applies to a non-stop flight with several classes of passenger service. Time t=0 corresponds to departure time, t=1 is the index of the day of departure (stage 1), t=2 is the index of the day prior to departure (stage 2), etc. A booking policy will be determined for the span from departure time back to stage T. Prior to day T all reservations are to be accepted. T is determined from statistical and/or practical considerations. Moreover there will be an integer n_T so large that the following assumption can be made plausibly: the probability of having more than n_T reservations booked prior to day T is zero.

Following are the probability distributions, variables, and parameters of relevance for present purposes when we deal with the mth class of passenger service, m=1(1)M: $p_{m,T}(x)$ is the probability that x passengers will already be booked on the morning of day T, x=0, 1, … , $n_{m,T}$; $p_{m,t}(y)$ is the probability of y reservation demands during day t, t≥1, y=0, 1, … ,$h_{m,t}$ ($h_{m,t}$ is the maximum number of demands with positive probability on day t, so that $p_{m,t}(y)=0$, $y>h_{m,t}$); $p_{m,t}(z;x)$ is the probability of z cancellations during day t out of x passengers already booked on the morning of day t, z=0, 1, … ,x. This probability is assumed independent of when the x passengers were booked, which is the Markovian property. (The validity of this type of assumption was established for a very large data sample of passenger records by researches at Latvian Airlines.) No-shows (passengers with valid reservations who do not appear by flight time) are to be included among the cancellations on day 1; $p_m(s)$ is the probability of having s standby passengers (passengers without reservations who wait at the airport); $c_{1(m)}$ is the full fare; $c_{2(m)}$ is the cost per denied boarding. We assume that the cost of passengers denied boarding is estimable and proportional to the number of such passengers; u_m is the number of seats in flight for the mth class of passenger service (airline booking level for passengers of the mth category); w_m is the compactness factor giving some measure of the airplane space need for performing a passenger service of the mth class; U is the capacity of the flight; $r_{m,t}(x)$ is the number of additional reservations to accept on day t when x reservations are already recorded, a decision variable for passengers of the mth category. A set of values of $r_{m,t}(x)$ for all relevant x and t is an overbooking policy for passengers of the mth category. The maximum number of passengers of the mth category who can be booked already by the morning of day T has been defined as $n_{m,T}$. If that many were booked and no cancellations occurred on day T, and if the maximum number of reservation requests $h_{m,T}$ were made and accepted on day T, then the number of passengers of the mth category booked on the morning of day T-1 would be $n_{m,T-1}=n_{m,T}+h_{m,T}$. Similarly, by recursion, the maximum state of the system for the mth class of passenger service for each day t up to t=0 can be defined: $n_{m,t}=n_{m,t+1}+h_{m,t+1}$, t=T-1, T-2, … ,0. An overbooking policy is specified, therefore, when $r_{m,t}(x)$ is specified for x=0, … , $n_{m,t}$; t=1, 2, … ,T; m=1(1)M.

2.1 Optimal Overbooking Policy Based on the Dynamic Model

The revenue from fares minus the costs of passengers with reservations denied boarding because of overbooking is a random variable, the gain, whose probability distribution depends upon the particular overbooking policy, chosen. Let $G_{m,t}(u_m;x)$ be the maximum expected gain achievable through booking level u_m and any overbooking policy for passengers of the mth category, given that x passengers are already booked on the morning of day $t \geq 1$, and $G_{m,0}(u_m;x)$ be the expected gain when x passengers of the mth category with reservations, plus any standbys, arrive for flight. An optimal overbooking policy for passengers of the mth category is defined as one which produces $G_{m,t}(u_m;x)$ for all x and t, and it may be computed with the aid of dynamic programming as in the Bellman-Howard process (Howard, 1960). One begins by computing

$$G_{m,0}(u_m;x)$$

$$= \begin{cases} c_{1(m)}u_m - c_{2(m)}(x-u_m), & \text{if } x > u_m, \\ c_{1(m)}\left[\sum_{s=0}^{u_m-x}(x+s)p_m(s) + u_m \sum_{s=u_m-x+1}^{\infty} p_m(s)\right], & \\ & \text{if } x \leq u_m, \end{cases}$$

$$\forall m=1(1)M. \tag{1}$$

Next let $G_{m,t}(r,u_m;x)$ be the expected gain from passengers of the mth category when: (i) the booking level is equal to u_m and the state on the morning of day t is x passengers, (ii) the policy prescribes that up to r additional reservations may be accepted ($r \geq 0$), and (iii) an optimal overbooking policy is followed thereafter. It follows that

$$G_{m,t}(r,u_m;x)$$

$$= \sum_{y=0}^{r} p_{m,t}(y) \sum_{z=0}^{x} G_{m,t-1}(u_m; x-z+y) p_{m,t}(z;x)$$
$$+ \left[\sum_{y=r+1}^{h_{m,t}} p_{m,t}(y)\right] \sum_{z=0}^{x} G_{m,t-1}(u_m; x-z+r) p_{m,t}(z;x). \tag{2}$$

We have ignored the possibility that a passenger may make a reservation and cancel it on the same day, but without loss of generality. If the statistics indicate that this phenomenon is significant for some application, then subdivide some of the days into smaller time periods.

The "principle of optimality" of dynamic programming as formulated by Bellman (1957) asserts that

$$G_{m,t}(u_m;x) = \max_{0 \leq r \leq h_{m,t}} G_{m,t}(r,u_m;x). \tag{3}$$

For t=1 and any fixed x, compute $G_{m,1}(r,u_m;x)$ for r=0,1, …., $h_{m,1}$ and find the maximizing value, $r_{m,1}(x)$, for each $m \in \{1, \ldots ,M\}$. When this integer is substituted for r in (2) the resulting value is $G_{m,1}(u_m;x)$. In case several values of r maximize $G_{m,1}(r,u_m;x)$, select the largest arbitrarily. (Consider only the possibilities r=0, 1, … , $h_{m,1}$, since $h_{m,1}$ is the maximum number of demands with positive probability on day 1.) Thus recursively compute $r_{m,1}(x)$ and $G_{m,1}(u_m;x)$ for x=0, 1, …. ,$n_{m,1}$, and then $r_{m,2}(x)$ and $G_{m,2}(u_m;x)$ for x=0, 1, …. ,$n_{m,2}$, etc., until $r_{m,T}(x)$ and $G_{m,T}(u_m;x)$ have been determined for x=0, 1, … , $n_{m,T}$ and all m=1(1)M. The optimal overbooking policy for passengers of the mth category (with airline booking level u_m) maximizes the expected gain conditional upon having x passengers already booked on the morning of day t, for every conceivable x and t. The unconditional expected gain from such a policy is

$$G_m(u_m) = E\{G_{m,T}(u_m;x)\}$$
$$= \sum_{x=0}^{n_{m,T}} p_{m,T}(x) G_{m,T}(u_m;x), \quad \forall m = 1(1)M. \tag{4}$$

2.2 Optimal Booking Levels
Based on the Dynamic Model

Let us assume first that an overbooked passenger of the ith category is not willing to be transferred to an empty seat for passenger of the jth category ($i, j \in \{1, \ldots ,M\}$, $i \neq j$). Then the problem is to find the numerical maximum of the functional

$$G(u_1, \ldots, u_M) = \sum_{m=1}^{M} G_m(u_m) \tag{5}$$

subject to constraints

$$\sum_{m=1}^{M} w_m u_m \leq U, \quad u_m \geq 0, \quad \forall m = 1(1)M. \tag{6}$$

This problem can be treated by the functional equation method of dynamic programming. Let $\mathbf{u}=(u_1, \ldots ,u_M)$ be a vector of airline booking levels such that

$$\mathbf{u} \in \mathscr{U} = \left\{ \mathbf{u} : \sum_{m=1}^{M} w_m u_m \leq U; \quad u_m \geq 0, \quad \forall m = 1(1)M, \right\} \tag{7}$$

and let

$$G_{1,\ldots,M}(U) = \max_{\mathbf{u} \in \mathscr{U}} G(u_1, \ldots, u_M). \tag{8}$$

By applying for m=2, … ,M Bellman's dynamic programming optimality principle, we obtain the basic recurrence relation

$$G_{1,\ldots,m}(u)$$

$$= \max_{(u_m, u^{(m-1)}) \in \mathscr{U}^{(m)}} \left\langle G_m(u_m) + G_{1,\dots,m-1}(u^{(m-1)}) \right\rangle,$$

$$u=0(1)U, \tag{9}$$

where

$$\mathscr{U}^{(m)} = \left\{ \begin{matrix} (u_m, u^{(m-1)}) : w_m u_m + u^{(m-1)} \le u; \\ u_m, u^{(m-1)} \ge 0 \end{matrix} \right\}, \tag{10}$$

$$u^{(1)}=w_1u_1,\ G_1(u^{(1)})=G_1(u_1). \tag{11}$$

Thus,

$$G_{1,\dots,M}(U)$$

$$= \max_{(u_M, u^{(M-1)}) \in \mathscr{U}^{(M)}} \left\langle G_M(u_M) + G_{1,\dots,M-1}(u^{(M-1)}) \right\rangle. \tag{12}$$

2.3 Numerical Example

Suppose we are making reservations for a flight to be made at some date in the future. Plane capacity is equal to U=10. There are two classes of passenger service: the first (or business) class, where fare $c_{1(1)}=1.1$, the cost per denied boarding $c_{2(1)}=1.1$, and the second (or tourist) class, where fare $c_{1(2)}=1$, the cost per denied boarding $c_{2(2)}=1$; $w_1=w_2=1$. Let T=10 be the number of days until departure. We let $p(y_{m,t})$ be the probability that $y_{m,t}$ customers for passenger service of the mth class (m=1, 2) will arrive during day t (t=1, … ,10). For simplicity, we assume that the arrival distribution is a binomial with parameters $p_{m,t}$ and $n_{m,t}$,

$$p(y_{m,t}) = \binom{n_{m,t}}{y_{m,t}} [p_{m,t}]^{y_{m,t}} [1-p_{m,t}]^{n_{m,t}-y_{m,t}},$$

$$y_{m,t}= 0, 1, \dots ,n_{m,t}, \tag{13}$$

where $p_{1,t}= p_{2,t}=0.2$, $n_{1,t}=n_{2,t}=5$ ($\forall t=1, \dots ,10$). In practice, a Poisson distribution might be more appropriate, but the binomial will serve our illustrative needs. Again for simplicity, we assume that each customer for passenger service of the mth class (m=1, 2) who has a reservation at the end of each day has the same probability ρ_m of canceling it during the night. This leads us directly to a binomial distribution for $p_{m,t}(z;x)$, the probability that z of the x reservations at the end of a day will cancel during the night,

$$p_{m.t}(z;x) = \binom{x}{z} [\rho_m]^z [1-\rho_m]^{x-z},$$

$$z=0, 1, \dots x, \quad x=0, 1, \dots ; \quad \forall t=1(1)T, \tag{14}$$

where $\rho_1=\rho_2=0.1$. It is assumed that the probability of having s standby passengers (passengers without reservations who wait at the airport), $p_m(s)$, is equal to 1 for s=0 and m=1, 2. The essence of determining the optimal overbooking policy is balancing the cost of over-reserving and the cost of empty seats. The balance depends, of course, on the uncertainties associated with the arrival of new customers and of cancellations. The optimal booking levels are given by $u_1^*=u_2^*=5$. Table 1 gives the optimal overbooking policy.

Table 1. Maximum levels of reservation

Passenger Service Class (m)	Day									
	1	2	3	4	5	6	7	8	9	10
m=1, 2	5	6	6	6	7	7	7	8	8	9

3. STATIC MODEL OF AIRLINE DATA

It is assumed that we deal with a non-stop flight with several classes of passenger service. Let us introduce the following notation for the m-th class of passenger service: x is the number of reservations accepted; y is the number of reservations demanded; z is the number of late cancellations and no-shows out of x passengers with reservations; s is the number of additional passengers (standby passengers) available; $F_{y(m)}(y)$ is the probability distribution function of y; $F_{z(m)}(z;x)$ is the conditional probability distribution function of z; $F_{s(m)}(s;x)$ is the conditional probability distribution function of s; $q_{1(m)}$ is the loss per unutilized seat; $q_{2(m)}$ is the penalty per excess passenger; r is the sales limit (variable of an overbooking policy, $r \ge u_m$).

3.1 Optimal Overbooking Policy Based on the Static Model

We must now take into account the following: (1) if $x - z > u_m$ then are $x - z - u_m$ excess passengers and the penalty is $q_{2(m)}(x - z - u_m)$; (2) if $x - z + s < u_m$ there are $u_m + z - x - s$ seats left and the revenue lost is $q_{1(m)}(u_m+ z - x - s)$; (3) if $x - z \le u_m \le x - z + s$ no loss or penalty arises. Then, for a given number x of reservations accepted the expected value of loss and penalty, if an airline booking level for passengers of the mth category is equal to u_m and the sales limit is equal to r, is therefore

$$Q_m(r, u_m; x = y)$$

$$= q_{2(m)} \int_0^{y-u_m} (y - z - u_m) dF_{z(m)}(z;y) + q_{1(m)}$$

$$\times \int_{y-u_m}^{y} \int_0^{u_m+z-y} (u_m + z - y - s) dF_{s(m)}(s;y) dF_{z(m)}(z;y) \tag{15}$$

for $u_m \le y \le r$ (where x=y), and

$$Q_m(r, u_m; x = r)$$

$$= q_{2(m)} \int_0^{r-u_m} (r - z - u_m) dF_{z(m)}(z; r) + q_{1(m)}$$

$$\times \int_{r-u_m}^{r} \int_0^{u_m+z-r} (u_m + z - r - s) dF_{s(m)}(s; r) dF_{z(m)}(z; r) \tag{16}$$

for y > r (where x=r). The unconditional expected value of loss and penalty is therefore

$$Q_m(r, u_m) = \int_{u_m}^{r} Q_m(r, u_m; x = y) dF_{y(m)}(y)$$

$$+ \int_r^{\infty} Q_m(r, u_m; x = r) dF_{y(m)}(y). \tag{17}$$

This expression is to be minimized with respect to r, where $r \ge u_m$. Since the sales limit r will be differ little from the booking level u_m, it may be argued that the precise value of r does not matter in the above conditional probability distributions, as it will be very close to u_m. The probability distribution functions with u_m substituted for r will now be written $F_{s(m)}(s)$ and $F_{z(m)}(z)$. The condition of optimality for r is that the derivative of (15) with respect to r should be zero and it assumes then the simple form

$$q_{2(m)} \int_0^{r-u_m} dF_{z(m)}(z)$$

$$+ q_{1(m)} f_{z(m)}(r) \int_0^{u_m} (u_m - s) dF_{s(m)}(s)$$

$$- q_{1(m)} \int_{r-u_m}^{r} \int_0^{u_m+z-r} dF_{s(m)}(s) dF_{z(m)}(z) = 0. \tag{18}$$

Since the value of $f_{z(m)}(r)$, the probability density function, for r late cancellations and no-shows is quite small we have as a reasonable approximation

$$q_{2(m)} \int_0^{r-u_m} dF_{z(m)}(z)$$

$$= q_{1(m)} \int_{r-u_m}^{r} \int_0^{u_m+z-r} dF_{s(m)}(s) dF_{z(m)}(z), \tag{19}$$

which upon integration becomes

$$\frac{1}{F_{z(m)}(r - u_m)} \int_{r-u_m}^{u_m+r-u_m} F_{s(m)}(z - (r - u_m)) dF_{z(m)}(z)$$

$$= \frac{q_{2(m)}}{q_{1(m)}}. \tag{20}$$

(20) may be solved in terms of the proposed oversales $r - u_m$. We conclude that the optimal value of r does not depend on the demand distribution $F_{y(m)}(y)$.

3.2 Optimal Booking Levels Based on the Static Model

Let

$$Q_m(u_m) = \min_{r \ge u_m} Q_m(r, u_m). \tag{21}$$

It is assumed that an overbooked passenger of the ith category is not willing to be transferred to an empty seat for passenger of the jth category (i, j $\in$ {1, … , M}, i≠j). Then the problem is to find the numerical minimum of the functional

$$Q(u_1, \ldots, u_M) = \sum_{m=1}^{M} Q_m(u_m) \tag{22}$$

subject to constraints

$$\sum_{m=1}^{M} w_m u_m \le U, \quad u_m \ge 0, \quad \forall m = 1(1)M. \tag{23}$$

This problem can be treated by the functional equation method of dynamic programming. Let **u**=(u_1, … ,u_M) be a vector of airline booking levels such that

$$\mathbf{u} \in \mathscr{U} = \left\{ \mathbf{u} : \sum_{m=1}^{M} w_m u_m \le U; \quad u_m \ge 0, \quad \forall m = 1(1)M \right\} \tag{24}$$

and let

$$Q_{1,\ldots,M}(U) = \min_{\mathbf{u} \in \mathscr{U}} Q(u_1, \ldots, u_M). \tag{25}$$

By applying for m=2, … ,M Bellman's dynamic programming optimality principle, we obtain the basic recurrence relation

$$Q_{1,\ldots,m}(u)$$

$$= \min_{(u_m, u^{(m-1)}) \in \mathscr{U}^{(m)}} \left\langle Q_m(u_m) + Q_{1,\ldots,m\text{-}1}(u^{(m-1)}) \right\rangle$$

$$u = 0(1)U, \tag{26}$$

where

$$\mathscr{U}^{(m)} = \left\{ \begin{array}{r} (u_m, u^{(m-1)}) : w_m u_m + u^{(m-1)} \le u; \\ u_m, u^{(m-1)} \ge 0 \end{array} \right\}, \quad (27)$$

$$u^{(1)}=w_1u_1,\ Q_1(u^{(1)})=Q_1(u_1). \quad (28)$$

Thus,

$$Q_{1,\dots,M}(U)$$

$$= \min_{(u_M, u^{(M-1)}) \in \mathscr{U}^{(M)}} \left\langle Q_M(u_M) + Q_{1,\dots,M\text{-}1}(u^{(M-1)}) \right\rangle. \quad (29)$$

3.3 Numerical Example

Let the capacity of the flight be U=50 (An 24). There are two classes of passenger service (M=2): the first (or business) class, where $q_{1(1)}=4$, $q_{2(1)}=10$, $w_1=2$, and the second (or tourist) class, where $q_{1(2)}=1$, $q_{2(2)}=10$; $w_2=1$. Let

$$dF_{y(m)}(y) = \theta_m^{-1} \exp(-y/\theta_m)dy, \quad m = 1, 2, \quad (30)$$

where $\theta_1=12$, $\theta_2=30$,

$$dF_{z(m)}(z) = \frac{(\beta_m z)^{u_m-1} e^{-\beta_m z}}{\Gamma(u_m)} \beta_m dz, \quad m = 1, 2, \quad (31)$$

where $\beta_1=\beta_2=10$,

$$dF_{s(m)}(s) = \frac{(\gamma_m s)^{u_m-1} e^{-\gamma_m s}}{\Gamma(u_m)} \gamma_m ds, \quad m = 1, 2, \quad (32)$$

where $\gamma_1=\gamma_2=10$. The optimal solution is then given as follows: for the first class, the optimal booking level is $u_1^*=16$, the margin of admissible oversales is equal to $r-u_1^*=1$; for the second class, the optimal booking level is $u_2^*=18$, the margin of admissible oversales is equal to $r-u_2^*=1$. In this case, $Q_{1,2}(U)=0.044$.

In practice the margin of admissible oversales is chosen more or less arbitrarily from among the small integers "on the basis of experience". However, a good policy in this respect cannot even be formulated without openly facing and properly weighing all implications. The lost revenue of a seat raises no difficulties, it is simply the price of a ticket; what an appropriate penalty for refusing a passenger should be must remain a value judgment to be rendered by the airline in the light of the "public relations" sensitivity.

4. CONCLUSIONS

The mathematical models described in this paper attempt to provide a consistent and valid approach to optimization of airline booking levels. Simulations and comparisons with existing simpler models from airline companies seem to indicate that the decision rules obtained from the above mentioned models form an efficient operational tool in the planning of an airline's booking policy. Even though the two models (static and dynamic) are very different they gave very similar effectivity measures. The dynamic model gave slightly better results, but it is relatively difficult to put this model into practice. The similarity in the results ought to underline the validity of our dynamic model.

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