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A NEW FEEDBACK LINEARIZATION METHOD FOR MIMO PLANTS

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Abstract: A new method of the feedback linearization procedure of MIMO systems is developed. Some control variables are “sacrificed” and made equal to a linear combination of new states. Subsequently the feedback linearization procedure is carried out using the remaining control variables. The formal *Lemma* is stated and proved. A class of non-linear systems is defined, for which the derived procedure works well. The developed procedure works on plants, which are not feedback linearizable in a standard way. An example of power plant station was examined. *Copyright © 2002 IFAC*

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1. INTRODUCTION

The feedback linearization technique is one of modern tools, which allows synthesising a control law for smooth, continuous, non-linear systems. The basic grounds of this theory are well established (Isidori 1995). Sufficient and necessary conditions for feedback linearizability were given (Jakubczyk and Respondek 1980), (Boothby 1986). In this classic attitude the non-linear system is transformed to the linear one with new signals selected as control variables. Any control law can be adopted for these control variables. A standard solution in this case is to choose a static state feedback.

This paper is intended to broaden the class of MIMO feedback linearizable systems. The plants are usually not feedback linearizable because, the control signals are too “close” to the output functions (in the sense of relative degree). One of solutions in this case is to use a dynamic extension. But it means that the order of the plant is to be increased. On the other hand one may think of “sacrificing” of some control signals. They might be set to a constant value or simply remain unused. This may lead to the loss of controllability or at least to the deterioration of control quality. But there is still another possibility. Some of control variables might be adopted as a linear combination of new states before linearization. This idea is examined in detail in this paper. It occurred that there exist classes of non-linear systems, which are linearizable in this way, while they are not linearizable by the standard procedure. The price, which is to be paid for this possibility, is that not all control laws can be adopted for given control variables. For some of them it must be linear combination of the states. It means that it is a static state feedback – a common choice among control engineers. However, this drawback is not too restrictive.

In paper (Bolek and Sasiadek 2001) a similar problem was examined. But the class of linearizable systems was restricted to two input systems. And the “sacrificed” control law could be a linear combination of only two new states. The class of non-linear systems derived in this paper allows that the “sacrificed” control law can be a linear combination of all new states.

In paragraph 2 the idea of a new method is outlined. In the next paragraph the formal *Lemma* is stated and proved. A class of non-linear systems is defined, for which the derived procedure works well. In paragraph 4, this method is applied to the model of a power plant.

2. PRELIMINARIES

This section is intended to outline the main idea, which constitutes the new method of feedback linearization.

Let's consider a non-linear smooth MIMO system (1).

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \tilde{\mathbf{G}}(\mathbf{x})\tilde{\mathbf{u}} + \overline{\mathbf{B}}\overline{\mathbf{u}} \tag{1}$$

$\mathbf{x}$ – n-dimensional state vector, $\tilde{\mathbf{u}}$ $\overline{\mathbf{u}}$ - $\tilde{m}$ – and $\overline{m}$ - dimensional control signals.

New state variables $\Psi = \Psi(\mathbf{x})$ are chosen during the linearization process. The main point of procedure developed in this paper is that, before the linearization, the one set of control inputs is adopted as a linear combination of new states (2) (maybe unknown at this moment).

$$\overline{\mathbf{u}} = \overline{\mathbf{K}}\Psi \tag{2}$$

Now one should try to linearize (1) with (2). New state variables Ψ are usually chosen as consecutive time derivatives. This may cause that some of new variables will be given by implicit relation. However in some cases, those relations can be solved and the new variables can be obtained in explicit form.

In paper (Bolek and Sasiadek 2001) a certain structure (3) of a plant was found, where such attitude is suitable.

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1, x_2) + g_{21}(x_1, x_2)u_2 \\
\dot{x}_2 &= f_2(x_1, x_2, x_3) + g_{22}(x_1, x_2, x_3)u_2 \\
&\cdots \\
\dot{x}_{r-1} &= f_{r-1}(x_1, x_2, \ldots, x_r) + g_{2,r-1}(x_1, x_2, \ldots, x_r)u_2 \\
\dot{x}_r &= f_r(x_1, x_2, \ldots, x_r) + g_{2,r}(x_1, x_2, \ldots, x_r)u_2 + g_{1r}(\mathbf{x}) \cdot u_1 \\
\dot{\tilde{\mathbf{x}}} &= \mathbf{A}\tilde{\mathbf{x}} + \mathbf{b}u_2
\end{aligned} \tag{3}$$

where $\tilde{\mathbf{x}} = \begin{bmatrix} x_{r+1} \\ \vdots \\ x_n \end{bmatrix}$.

The following lemma was proved:

Let h be a smooth, nontrivial function of the first state variable $h = h(x_1)$ and

$$u_2(x_1, x_2) = \frac{k_1 h(x_1) + k_2 f_1(x_1, x_2)\frac{\partial h}{\partial x_1}}{1 - k_2 g_{21}(x_1, x_2)\frac{\partial h}{\partial x_1}} \tag{4}$$

with some coefficients k_1, k_2.

If

(i) $k_2 g_{21}(x_1, x_2)\frac{\partial h}{\partial x_1} \neq 0$ around $\mathbf{x}_0$

(ii) with u_2 as in (4) holds

$$\frac{\partial f_1}{\partial x_2} + \frac{\partial(g_{21} \cdot u_2)}{\partial x_2} \neq 0 \quad \frac{\partial f_2}{\partial x_3} + \frac{\partial(g_{22} \cdot u_2)}{\partial x_3} \neq 0 \quad \cdots$$

$$\cdots \quad \frac{\partial f_{r-1}}{\partial x_r} + \frac{\partial(g_{2,r-1} \cdot u_2)}{\partial x_r} \neq 0$$

(iii) $g_{1r}(\mathbf{x}) \neq 0$ around $\mathbf{x}_0$

then the system (3) can be transformed around $\mathbf{x}_0$ to the linear form:

$$\dot{\mathbf{z}} = \begin{bmatrix} 0 & 1 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & \ldots & 0 & 0 \end{bmatrix} \cdot \mathbf{z} + \begin{bmatrix} 0 \\ 0 \\ \ldots \\ 0 \\ 1 \end{bmatrix} v$$
$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{b}\begin{bmatrix} k_1 & k_2 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{5}$$

where $z_i = L_{\mathbf{f}^*}^{i-1} h$, $f_i^* = f_i + g_{2i} u_2$ for $i = 1,\ldots,r$;
$v = L_{\mathbf{f}^*}^{r} h + u_1 L_{\mathbf{g}_1} L_{\mathbf{f}^*}^{r-1} h$ - is a new control variable.

As it can be seen from (5), the possibility of control signal u_2 choice is restricted to only two variables.

In this paper a class of non-linear systems is found, where there is no such restriction.

3. MAIN RESULT

A special structure (7) of non-linear, smooth system (6) is considered.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x}) \cdot \mathbf{u} \tag{6}$$

$\mathbf{x} \in \mathbf{R}^n$, $\mathbf{u} \in \mathbf{R}^m$.

$$\begin{aligned}
\dot{\overline{\mathbf{x}}} &= \mathbf{A}\overline{\mathbf{x}} + \mathbf{H}\psi_0(\mathbf{x}) + \overline{\mathbf{B}}\overline{\mathbf{u}} \\
\dot{\tilde{\mathbf{x}}} &= \tilde{\mathbf{f}}(\mathbf{x}) + \tilde{\mathbf{G}}(\mathbf{x})\tilde{\mathbf{u}} + \tilde{\mathbf{B}}\overline{\mathbf{u}}
\end{aligned} \tag{7}$$

$$\bar{\mathbf{x}} = \begin{bmatrix} x_1 \\ \cdots \\ x_{\bar{n}} \end{bmatrix}, \ \tilde{\mathbf{x}} = \begin{bmatrix} x_{\bar{n}+1} \\ \cdots \\ x_{\bar{n}+\tilde{n}} \end{bmatrix} \qquad \bar{n} + \tilde{n} = n$$

$$\bar{\mathbf{u}} = \begin{bmatrix} u_1 \\ \cdots \\ u_{\bar{m}} \end{bmatrix}, \ \tilde{\mathbf{u}} = \begin{bmatrix} u_{\bar{m}+1} \\ \cdots \\ u_{\bar{m}+\tilde{m}} \end{bmatrix} \qquad \bar{m} + \tilde{m} = m$$

The constant matrices has following dimensions:

$$\mathbf{A}(\bar{n}\times\bar{n}),\ \mathbf{H}(\bar{n}\times\tilde{m}),\ \bar{\mathbf{B}}(\bar{n}\times\bar{m}),\ \tilde{\mathbf{B}}(\tilde{n}\times\bar{m}),$$

$$\psi_0(\mathbf{x}) = \begin{bmatrix} \psi_{01}(\mathbf{x}) \\ \cdots \\ \psi_{0\tilde{m}}(\mathbf{x}) \end{bmatrix}, \ \tilde{\mathbf{f}}(\mathbf{x}) = \begin{bmatrix} \tilde{\mathbf{f}}_1(\mathbf{x}) \\ \cdots \\ \tilde{\mathbf{f}}_{\tilde{n}}(\mathbf{x}) \end{bmatrix}$$

$$\tilde{\mathbf{G}}(\mathbf{x}) = \begin{bmatrix} \tilde{\mathbf{g}}_{11}(\mathbf{x}) & \cdots & \tilde{\mathbf{g}}_{1\tilde{m}}(\mathbf{x}) \\ \cdots & \cdots & \cdots \\ \tilde{\mathbf{g}}_{\tilde{n}1}(\mathbf{x}) & \cdots & \tilde{\mathbf{g}}_{\tilde{n}\tilde{m}}(\mathbf{x}) \end{bmatrix}$$

In this case the notations as shown in (6) are as follows:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{A}\bar{\mathbf{x}} + \mathbf{H}\psi_0(\mathbf{x}) \\ \tilde{\mathbf{f}}(\mathbf{x}) \end{bmatrix}, \ \mathbf{G}(\mathbf{x}) = \begin{bmatrix} \bar{\mathbf{B}} & \mathbf{0} \\ \tilde{\mathbf{B}} & \tilde{\mathbf{G}}(\mathbf{x}) \end{bmatrix}$$

The notation $\mathbf{B} = \begin{bmatrix} \bar{\mathbf{B}} \\ \tilde{\mathbf{B}} \end{bmatrix}$ will be also used.

LEMMA

IF

A1 The relative degree vector between $\psi_0(\mathbf{x})$ and $\tilde{\mathbf{u}}$ is equal to $\Big[\underbrace{2 \quad 2 \quad \cdots \quad 2}_{\tilde{m} \ \text{components}}\Big]$

A2 The matrix $\bar{\mathbf{K}}_3(\bar{m}\times\tilde{m})$ is chosen such that $\frac{\partial\psi_0(\mathbf{x})}{\partial\mathbf{x}}\cdot\mathbf{B}\cdot\bar{\mathbf{K}}_3 \neq \mathbf{I}_{\tilde{m}}$, where $\mathbf{I}$ - identity matrix.

A3 $\tilde{n} = 2\tilde{m}$

A4 $\frac{\partial\psi_1(\mathbf{x})}{\partial\tilde{\mathbf{x}}}\tilde{\mathbf{G}}(\mathbf{x})$ is non-singular in the considered area.

THEN

the system (7) is feedback linearizable in new co-ordinates (9) with the feedback (10).

$$\Psi(\mathbf{x}) = \begin{bmatrix} \bar{\mathbf{x}} \\ \psi_0(\mathbf{x}) \\ \psi_1(\mathbf{x}) \end{bmatrix}, \text{ where } \psi_1(\mathbf{x}) = \begin{bmatrix} \psi_{11}(\mathbf{x}) \\ \cdots \\ \psi_{1\tilde{m}}(\mathbf{x}) \end{bmatrix}. \tag{8}$$

$$\psi_1(\mathbf{x}) = \left(\mathbf{I}_{\tilde{m}} - \frac{\partial\psi_0(\mathbf{x})}{\partial\mathbf{x}}\mathbf{B}\bar{\mathbf{K}}_3\right)^{-1}\cdot \\ \cdot\begin{bmatrix} \frac{\partial\psi_0(\mathbf{x})}{\partial\bar{\mathbf{x}}}\left(\mathbf{A}\bar{\mathbf{x}} + \mathbf{H}\psi_0(\mathbf{x}) + \bar{\mathbf{B}}\cdot\mathbf{K}_\psi\right) + \\ + \frac{\partial\psi_0(\mathbf{x})}{\partial\tilde{\mathbf{x}}}\left(\tilde{\mathbf{f}}(\mathbf{x}) + \tilde{\mathbf{B}}\cdot\mathbf{K}_\psi\right) \end{bmatrix} \tag{9}$$

$$\mathbf{K}_\psi = \left[\bar{\mathbf{K}}_1 \quad \bar{\mathbf{K}}_2\right]\cdot\begin{bmatrix} \bar{\mathbf{x}} \\ \psi_0(\mathbf{x}) \end{bmatrix}$$

with some matrices $\bar{\mathbf{K}}_1(\bar{m}\times\bar{n})$ $\bar{\mathbf{K}}_2(\bar{m}\times\tilde{m})$.

$\bar{\mathbf{u}} = \bar{\mathbf{K}}\cdot\Psi$, where $\bar{\mathbf{K}} = \left[\bar{\mathbf{K}}_1 \quad \bar{\mathbf{K}}_2 \quad \bar{\mathbf{K}}_3\right]$.

$$\tilde{\mathbf{u}} = \left[\frac{\partial\psi_1}{\partial\tilde{\mathbf{x}}}\tilde{\mathbf{G}}(\mathbf{x})\right]^{-1}\cdot \\ \cdot\begin{bmatrix} \mathbf{v} - \frac{\partial\psi_1}{\partial\bar{\mathbf{x}}}\left(\mathbf{A}\bar{\mathbf{x}} + \mathbf{H}\psi_0 + \bar{\mathbf{B}}\bar{\mathbf{K}}\Psi\right) + \\ - \frac{\partial\psi_1}{\partial\tilde{\mathbf{x}}}\left(\tilde{\mathbf{f}}(\mathbf{x}) + \tilde{\mathbf{B}}\bar{\mathbf{K}}\Psi\right) \end{bmatrix} \tag{10}$$

$\mathbf{v}$ - new $\tilde{m}$ -dimensional control variable

The system has the form (11) after linearization.

$$\begin{bmatrix} \dot{\bar{\mathbf{x}}} \\ \dot{\psi}_0 \\ \dot{\psi}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}\cdot\begin{bmatrix} \bar{\mathbf{x}} \\ \psi_0 \\ \psi_1 \end{bmatrix} + \begin{bmatrix} \bar{\mathbf{B}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}\cdot\begin{bmatrix} \bar{\mathbf{u}} \\ \mathbf{v} \end{bmatrix} \tag{11}$$

with $\bar{\mathbf{u}} = \bar{\mathbf{K}}\Psi$ and $\mathbf{v}$ freely assignable.

PROOF

The time derivatives of the new co-ordinates will be evaluated consecutively.

$$\dot{\bar{\mathbf{x}}} = \mathbf{A}\bar{\mathbf{x}} + \mathbf{H}\psi_0(\mathbf{x}) + \bar{\mathbf{B}}\bar{\mathbf{K}}\Psi \tag{12}$$

$$\dot{\psi}_0(\mathbf{x}) = \frac{\partial \psi_0(\mathbf{x})}{\partial \mathbf{x}} \cdot \dot{\mathbf{x}} =$$

$$= \frac{\partial \psi_0(\mathbf{x})}{\partial \overline{\mathbf{x}}}\left[\mathbf{A}\overline{\mathbf{x}} + \mathbf{H}\psi_0 + \overline{\mathbf{B}}\mathbf{K}_\psi\right] +$$

$$+ \frac{\partial \psi_0(\mathbf{x})}{\partial \overline{\mathbf{x}}}\left[\mathbf{A}\overline{\mathbf{x}} + \mathbf{H}\psi_0 + \overline{\mathbf{B}}\mathbf{K}_\psi\right] + \frac{\partial \psi_0(\mathbf{x})}{\partial \overline{\mathbf{x}}}\overline{\mathbf{B}}\overline{\mathbf{K}}_3\psi_1 +$$

$$+ \frac{\partial \psi_0(\mathbf{x})}{\partial \tilde{\mathbf{x}}}\tilde{\mathbf{B}}\overline{\mathbf{K}}_3\psi_1 + \psi_1 - \psi_1$$

After some simple manipulations one obtains (13).

$$\begin{aligned} \dot{\psi}_0 &= \psi_1 - \left(\mathbf{I} - \frac{\partial \psi_0(\mathbf{x})}{\partial \mathbf{x}}\mathbf{B}\overline{\mathbf{K}}_3\right)\psi_1 + \\ &+ \frac{\partial \psi_0(\mathbf{x})}{\partial \overline{\mathbf{x}}}\left[\mathbf{A}\overline{\mathbf{x}} + \mathbf{H}\psi_0 + \overline{\mathbf{B}}\mathbf{K}_\psi\right] + \\ &+ \frac{\partial \psi_0(\mathbf{x})}{\partial \overline{\mathbf{x}}}\left[\mathbf{A}\overline{\mathbf{x}} + \mathbf{H}\psi_0 + \overline{\mathbf{B}}\mathbf{K}_\psi\right] \end{aligned} \tag{13}$$

If (9) is substituted for the second ψ_1, one obtains (14).

$$\dot{\psi}_0 = \psi_1 \tag{14}$$

The derivative of ψ_1 is as follows (15).

$$\begin{aligned} \dot{\psi}_1 &= \frac{\partial \psi_1}{\partial \overline{\mathbf{x}}}\left[\mathbf{A}\overline{\mathbf{x}} + \mathbf{H}\psi_0 + \overline{\mathbf{B}}\overline{\mathbf{K}}\Psi\right] + \\ &+ \frac{\partial \psi_1}{\partial \tilde{\mathbf{x}}}\left[\tilde{\mathbf{f}}(\mathbf{x}) + \tilde{\mathbf{B}}\overline{\mathbf{K}}\Psi\right] + \frac{\partial \psi_1}{\partial \tilde{\mathbf{x}}}\tilde{\mathbf{G}}(\mathbf{x})\tilde{\mathbf{u}} \end{aligned} \tag{15}$$

If the relation (10) is inserted, then one obtains:

$$\dot{\psi}_2 = \mathbf{v}\,. \tag{16}$$

Finally, the equation (12), (14), (16) constitute the linearized system (11).

The assumption *A1* is quite restrictive. It is adopted in order to find solution for ψ_1 in a simple way. If relative orders would be greater than *2* then the implicit relation for ψ_1 include its partial derivatives. One could consider a system, that partial derivatives would not appear, but this idea needs some more investigations.

4. EXAMPLE

The developed procedure is tested on a power plant station model, which is described in more details in (de Mello *et al.* 1991) and (Bolek *et al.* 2000).

$$\begin{aligned} T_h \dot{x}_1 &= -x_1 + u_1 \\ T_{i1}\dot{e}_1 &= x_1 - P_z \\ T_{i2}\dot{e}_2 &= p_T - 1 \\ C_{sh}\,\dot{p}_T &= k\sqrt{\gamma p_D - p_T} - u_1 \\ C_D\,\dot{p}_D &= m_w - k\sqrt{\gamma p_D - p_T} \\ T_w \dot{m}_w &= -m_w + u_2 \end{aligned} \tag{17}$$

State variables are: x_1 - power produced by the turbine, e - error between nominal power P_z and current power, p_T - pressure before turbine, p_D - pressure in the drum, m_w - steam flow produced by the boiler. Control signals are: u_1 - steam flow to the turbine (related to the main valve opening), u_2 - the fuel flux delivered to the boiler. The other parameters are constant coefficients. The state variables are relative to the nominal values. They are always positive. The difference γp_D-p_T must be positive, because the steam flows from drum to the turbine and not in the opposite direction.

The plant (17) can be described in the standard non-linear form (6) with elements defined in (18), (19).

In this case these functions are defined as follows.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ e_1 \\ e_2 \\ p_T \\ p_D \\ m_w \end{bmatrix} \qquad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} -\frac{1}{T_h}x_1 \\ \frac{1}{T_{1i}}(x_1 - P_z) \\ \frac{1}{T_{12}}(p_T - 1) \\ \frac{k}{C_{sh}}\sqrt{\gamma p_D - p_T} \\ \frac{1}{C_D}\left(m_w - k\sqrt{\gamma p_D - p_T}\right) \\ -\frac{1}{T_w}m_w \end{bmatrix} \tag{18}$$

$$\mathbf{G}(\mathbf{x}) = [\mathbf{g}_1 \quad \mathbf{g}_2] = \begin{bmatrix} \frac{1}{T_h} & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{1}{C_{sh}} & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_w} \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{19}$$

The function **f** is smooth in the area of normal plant operation.

The process of feedback linearization is quite complex. In fact a special notation is used. The interested reader should refer to the (Isidori 1995). The necessary conditions for feedback linearizability are that distributions $G_0=span\{\mathbf{g}_1, \mathbf{g}_2\}$ and $G_1 = span\{\mathbf{g}_1, \mathbf{g}_2, ad_{\mathbf{f}}\mathbf{g}_1, ad_{\mathbf{f}}\mathbf{g}_2\}$ have constant dimension in the considered area of operation and they are involutive.

The distribution G_0 has a constant dimension, which is equal to *2*. It is also involutive because

$$\frac{\partial \mathbf{g}_1}{\partial \mathbf{x}} = \frac{\partial \mathbf{g}_2}{\partial \mathbf{x}} = \mathbf{0}.$$

The distribution G_1 has also a constant dimension, which is equal to *4*. But it is not involutive. The Lie bracket $[\mathbf{g}_1, ad_{\mathbf{f}}\mathbf{g}_1] \notin G_1$.

The plant (17) is not feedback linearizable in a standard sense and there is no need to check other conditions.

The plant (17) is in the form of (7).

$$\bar{\mathbf{x}} = \begin{bmatrix} x_1 \\ e_1 \\ e_2 \\ p_T \end{bmatrix},\ \tilde{\mathbf{x}} = \begin{bmatrix} p_D \\ m_w \end{bmatrix},\ \bar{\mathbf{u}} = u_1,\ \tilde{\mathbf{u}} = u_2$$

$n = 6$, $\bar{n} = 4$, $\tilde{n} = 2$, $\bar{m} = \tilde{m} = 1$

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{T_h} & 0 & 0 & 0 \\ \frac{1}{T_{i1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{T_{i2}} \\ 0 & 0 & 0 & 0 \end{bmatrix},\ \mathbf{H} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{C_{sh}} \end{bmatrix},\ \bar{\mathbf{B}} = \begin{bmatrix} \frac{1}{T_h} \\ 0 \\ 0 \\ -\frac{1}{C_{sh}} \end{bmatrix}$$

$$\tilde{\mathbf{f}}(\mathbf{x}) = \begin{bmatrix} \frac{1}{C_D}\left(m_w - k\sqrt{\gamma p_D - p_T}\right) \\ -\frac{1}{T_w} m_w \end{bmatrix},\ \tilde{\mathbf{G}}(\mathbf{x}) = \begin{bmatrix} 0 \\ \frac{1}{T_w} \end{bmatrix}$$

$$\psi_0(\mathbf{x}) = k\sqrt{\gamma p_D - p_T},\ \tilde{\mathbf{B}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The assumptions of *Lemma* are to be checked now.

A1 The relative degree between ψ_0 and u_2 is equal to *2*. The related Lie derivatives are:

$$L_{\mathbf{g}_2}\psi_0 = 0 \quad \text{and}$$

$$L_{\mathbf{g}_2} L_{\mathbf{f}}\psi_0 = \frac{k\gamma}{2C_D T_w \sqrt{\gamma p_D - p_T}} > 0$$

for positive $\gamma p_D - p_T$.

A2 The coefficient $\bar{k}_3$ has to be chosen in the way that $\frac{\partial \psi_0(\mathbf{x})}{\partial \mathbf{x}} \cdot \mathbf{B} \cdot \bar{k}_3 \neq 1$.

So, $k_3 \neq \frac{2C_{sh}}{k}\sqrt{\gamma p_D - p_T}$. If k_3 is negative (negative feedback for the linearized plant), then this condition is fulfilled.

A3 $\tilde{n} = 2 = 2\tilde{m}$ is obviously fulfilled.

A4 $\frac{\partial \psi_1(\mathbf{x})}{\partial \tilde{\mathbf{x}}} \tilde{\mathbf{G}}(\mathbf{x}) = \frac{\gamma k}{2C_D T_w \sqrt{\gamma p_D - p_T}} \neq 0$

All assumptions of *Lemma* are fulfilled. The plant (17) can be linearized in the described way.

The new co-ordinates are as is shown below

$$\Psi = \begin{bmatrix} x_1 \\ e_1 \\ e_2 \\ p_T \\ \psi_0 \\ \psi_1 \end{bmatrix},\ \psi_0(\mathbf{x}) = k\sqrt{\gamma p_D - p_T}.$$

The control u_1 is a linear combination of new states (20).

$$u_1 = \bar{k}_{11} \cdot x_1 + \bar{k}_{12} \cdot e_1 + \bar{k}_{13} \cdot e_2 + \bar{k}_{14} \cdot p_T + \bar{k}_2 \cdot k\sqrt{\gamma p_D - p_T} + \bar{k}_3 \psi_1 \qquad (20)$$

On the other hand ψ_1 is a time derivative of ψ_0.

$$\psi_1 = \dot{\psi}_0 = \frac{\partial \psi_0}{\partial \mathbf{x}} \cdot \mathbf{f}(\bar{\mathbf{x}}, \psi_0, \psi_1) \qquad (21)$$

The equation (21) is an implicit relation for ψ_1, but formula (8) gives a solution for it.

$$\psi_1 = k\frac{-kC_D\sqrt{\gamma p_D - p_T} + C_D \mathbf{K}_\psi + \gamma C_{sh}\left(m_w - k\sqrt{\gamma p_D - p_T}\right)}{C_D\left(2C_{sh}\sqrt{\gamma p_D - p_T} - k \cdot \bar{k}_3\right)}$$

where

$$\mathbf{K}_\psi = \begin{bmatrix} \bar{k}_{11} & \bar{k}_{12} & \bar{k}_{13} & \bar{k}_{14} & \bar{k}_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ e_1 \\ e_2 \\ p_T \\ k\sqrt{\gamma p_D - p_T} \end{bmatrix}$$

The u_2 is evaluated accordingly to (10). However, this formula is quite long and it is not shown in this paper.

Eventually, one obtains the plant in the form (11). Let static state feedback be chosen for the control variable $\mathbf{v}$.

$$\mathbf{v} = \tilde{\mathbf{K}}\Psi$$

The gain matrix $\mathbf{K} = \begin{bmatrix} \bar{\mathbf{K}} \\ \tilde{\mathbf{K}} \end{bmatrix}$ can be then computed e.g. a as solution to the Riccati equation for the linearized plant. Therefore, the adoption of control law (20) before linearization, is not too restrictive for control law structure. The restriction is only in that there in not possible to choose a control law for u_1 other than state feedback.

5. CONCLUSIONS

A new method of the feedback linearization procedure of MIMO systems was developed. Some control variables are “sacrificed” and made equal to a linear combination of new states. Subsequently the feedback linearization procedure is carried out using the remaining control variables. After linearization the previously adopted linear combination of states can not be changed. This restricts the set of possible control laws, which might be adopted after linearization. However, the linear combination of states (static state feedback) is a common choice among control engineers, and therefore this obstacle could not be considered as very restrictive.

The “sacrificed” control variable is made a linear combination of all states. This is a much broader possibility, than in the previous result, where it might have been a linear combination of only two states.

The developed procedure works in plants, which are not feedback linearizable in a standard way. An example of power plant station was examined. This plant does not fulfil necessary condition to be feedback linearizable, but it can be linearized using developed procedure.

The result shown in this paper is a preliminary one. The assumptions of *Lemma* are severe, but a more detailed inspection of (7) can yield an even broader class. However, so far, formal results were obtained only for system as given in (7). Nevertheless, this area has a great potential for further research work.

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