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S0ME REMARKS ON NONLINEAR FEEDBACK CONTROL OF A RIGID SPACECRAFT

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Abstract: This paper is dealing with the control of a rigid spacecraft. The nonlinear feedback controls, derived by using the approaches that have been recently developed, based on the port controlled hamiltonian (PCH) structure, are compared to the one derived by the methods based on the eigenstructure proposed by the author in previous work. The aim is to emphasize that, using the second approach appears as a particular case than using the first approach. Moreover, the results have been applied to derive the control laws of a launcher in the phase outside the atmosphere. It may be interesting to observe that the coefficients of the damping matrix, involved in PCH approach, have not the same effect on the system sensitivity. This paper somehow extends the related one presented by the author at the IFAC symposium in aerospace control. *Copyright© 2001 IFAC.*

Key words: Nonlinear Feedback control, rigid body, Port Controlled Hamiltonian Structure

1. INTRODUCTION

In this paper, it is proposed a comparison between the nonlinear feedback control of a rigid body by using the approaches that have been developed very recently (Ortega, *et al.*,1999a; 1999b; Van der Shaft, 2000) and those that can be derived by using the eigenstructure theory presented by the author in previous work (Siguerdidjane, 91, 94). The first approach involves the port controlled hamiltonian structure while the second involves the desired analytical solutions for the system under consideration. Each approach presents its advantages and drawbacks.
A similar procedure has been used to determine the feedback control laws of a launcher in the phase outside the atmosphere.

The control of rigid body has always been of a great interest all over the literature research work. Several methods have been suggested (one may see for instance (Aeyles, 1992; Banks, *et al.*, 1997; Bloch, et al., 1990; Debs and Athans, 1969; Krshman, *et al.*, 1992), (Siguerdidjane, 1991, 1992a, 1992b, 1994), Sira-Ramirez and Siguerdidjane, 1996). One may also be referred to the basic solutions described in books as for example (Geensite, 70). Since, more and more performance are required by an industrial context subject to a great competitiveness therefore one has to make more research work in order to meet the desired improvements .

We are particularly focused on rigid body dynamics described by Euler equations since they are involved in our main problems of interest area by means of spacecrafts, missiles and launchers.
In previous work, we have emphasized the connection between the analytical solutions of the Euler equations under no forces and under forces and some characteristic values and vectors. These values and vectors satisfy an algebraic non linear equation which may be derived using some algebraic manipulations.
In references (Siguerdidjane, 1991, 1992-b, 1994), it is presented the analytical solutions of the rigid body under three input control torques, two failure modes and one failure mode respectively. The solutions are expressed in terms of Jacobi elliptic functions and those characteristic values and vectors.

It is here shown that the corresponding feedback laws are a particular case of the ones derived by shaping the stored energy of the system in the Port Controlled Hamiltonian approach.

The use of a hamiltonian structure seems to be more significant from practical point of view. From theoretical point of view, it consists of conserving quantities and energy transfer and dissipation. Since also, one desires that the closed loop system possess a hamiltonian structure so then the feedback stabilizing control law is then derived. The theory based on the structure preserving stabilization is known as the so-called Interconnection and Damping Assignment, it has been developed by R. Ortega and his co-workers.

One proceeds in three steps. First, fix a desired hamiltonian function. So then, the energy generated by the controller is provided through an added hamiltonian. Second, in order to get the expression of the added hamiltonian, one has to solve a partial differential equation satisfied by the added function. For some systems, this is not always an easy task. Some applications have already been performed (Rodriguez *et al*., 2000a-b). Concerning the rigid body, one may also find some feedback control laws in (Ortega, 2000).

Third, the stabilizing feedback loop is derived by making equal the desired system dynamics and the original one. The stabilization is studied by taking the desired total hamiltonian as the Lyapunov function.
The paper is organized as follows. The first section describes the vehicle dynamics and the procedure design. Section 2 is devoted to the control design by using the port controlled Hamiltonian structure. Section 3 presents the feedback laws though the desired analytical solutions of the system. Section 4 shows the use of the procedure to derive the feedback control laws of a launcher in the phase outside the atmosphere. Simulation results, discussion and a brief conclusion are given in the last sections.

1.1 Vehicle dynamics

Consider a rigid body in an inertial reference frame. Let us start with the satellite. Its dynamics, using reaction jets only, are described by Euler equations

$$\begin{aligned} I_x \dot{p} + (I_z - I_y)\, qr &= T_x \\ I_y \dot{q} + (I_x - I_z)\, pr &= T_y \\ I_z \dot{r} + (I_y - I_x)\, pq &= T_z \end{aligned} \qquad (1)$$

where I_x, I_y and I_z denote the moments of inertia of the rigid body about the principal body axes, p, q and r denote the angular velocities of the satellite and T_x, T_y and T_z are the torques generated by the reaction jets. Assume that $I_x > I_y > I_z$.

1.2 Procedure design

A system is said to be having a Port Controlled Hamiltonian structure if it can be presented in local coordinates as

$$\dot{x} = [J(x) - R(x)]\frac{\partial H}{\partial x} + g(x)u \qquad (2)$$

$$y = g^T(x)\frac{\partial H}{\partial x}(x) \qquad (3)$$

where x is the state vector, $x \in R^n$, u is the control vector, $u \in R^m$, $y \in R^m$, $H(x)$ is the internal Hamiltonian, $J(x)$, $R(x)$ are the so-called interconnection and the damping matrices respectively, with the appropriate dimensions. $g(x)$ is the input matrix. $J(x)$ is a skew symmetric matrix, namely $J(x) = -J^T(x)$, $R(x)$ is a positive semi definite matrix, $R(x) \geq 0$.
Let x_e be the equilibrium point.

The procedure consists of finding a modified Hamiltonian function $H_a(x)$, a modified interconnection matrix $J_a(x)$ and a modified damping matrix $R_a(x)$ and finally a feedback control $u(x)$ such that the closed loop system preserves the Port Controlled Hamiltonian structure

$$\dot{x} = [J_d(x) - R_d(x)]\frac{\partial H_d}{\partial x} \quad (4)$$

where $H_d(x)$, $J_d(x)$ and $R_d(x)$ are the desired hamiltonian function, interconnection and damping matrices respectively, such that
$H_d(x) = H(x) + H_a(x)$,
$J_d(x) = J(x) + J_a(x)$
and $R_d(x) = R(x) + R_a(x)$.

Along the trajectories, one has

$$\frac{dH_d(x)}{dt} = -\frac{\partial H_d(x)}{\partial x}^T R_d(x)\frac{\partial H_d}{\partial x}(x) \le 0 .$$

Moreover, one has to bear in mind that the aim is to reach the desired structure (4), the feedback control law is therefore derived by solving the following equation

$$\begin{aligned}\dot{x} &= [J_d(x) - R_d(x)]\frac{\partial H_d}{\partial x} \\ &= [J(x) - R(x)]\frac{\partial H}{\partial x} + g^T(x)u\end{aligned} \quad (5)$$

or,

$$\begin{aligned}\dot{x} &= [J_d(x) - R_d(x)]\frac{\partial H_a}{\partial x} \\ &= -[J_a(x) - R_a(x)]\frac{\partial H}{\partial x} + g^T(x)u\end{aligned} \quad (6)$$

Furthermore, it is necessary to guarantee that $\left.\frac{\partial H_d}{\partial x}\right|_{x=x_e} = 0$, which amounts to say that x_e is an equilibrium of the closed loop system. Moreover, the closed loop system is Lyapunov stable and the trajectories converge asymptotically to x_e.
In addition, if x_e is a minimum of $H_d(x)$, then the Hessian of $H_d(x)$ is positive semi definite, $\frac{\partial^2 H_d(x)}{\partial^2 x} \ge 0$.

2. NONLINEAR FEEDBACK CONTROL

Let us now back to the rigid body and first use, for convenience, the components of the angular momentum as the state variables.

So define $x = (x_1, x_2, x_3)^T$ such that $x_1 = I_x p$, $x_2 = I_y q$ and $x_3 = I_z r$ and the control vector $u = (u_1, u_2, u_3)^T$ such that $u_1 = T_x$, $u_2 = T_y$ and $u_3 = T_z$.

System (1) may be rewritten down as

$$\begin{aligned}\dot{x}_1 &= x_2 x_3\left(\frac{1}{I_z} - \frac{1}{I_y}\right) + u_1 \\ \dot{x}_2 &= x_3 x_1\left(\frac{1}{I_x} - \frac{1}{I_z}\right) + u_2 \\ \dot{x}_3 &= x_1 x_2\left(\frac{1}{I_y} - \frac{1}{I_x}\right) + u_3\end{aligned} \quad (7)$$

The Hamiltonian is given by the kinetic energy

$$H = \frac{1}{2}\left(\frac{x_1^2}{I_x} + \frac{x_2^2}{I_y} + \frac{x_3^2}{I_z}\right) \quad (8)$$

Using equation (8), system (7) becomes

$$\begin{pmatrix}\dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3\end{pmatrix} = \begin{pmatrix}0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0\end{pmatrix}\begin{pmatrix}\frac{\partial H}{\partial x_1} \\ \frac{\partial H}{\partial x_2} \\ \frac{\partial H}{\partial x_3}\end{pmatrix} + \begin{pmatrix}u_1 \\ u_2 \\ u_3\end{pmatrix} \quad (9)$$

or in short

$$\dot{x} = J(x)\frac{\partial H}{\partial x} + g^T u \quad (10)$$

where

$J(x) = \begin{pmatrix}0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0\end{pmatrix}$ and g^T is the identity matrix, we have here $R(x) = 0$.

The objective is to possibly find simple expressions of the feedback control laws in order to facilitate their implementation as generally required in an industrial context. Consider the case where three control torques are acting on the body.

This case has been studied by several authors (see for instance (Debs and Athans, 1969, Siguerdidjane, 1991), using the optimal control, by minimizing the kinetic energy. We here show that the solutions may be derived quite simply.

We have $u_1 \ne 0$, $u_2 \ne 0$, $u_3 \ne 0$, let us make no modification in the interconnection matrix and fix the damping matrix as follows

$$R_a(x) = \begin{pmatrix}r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3\end{pmatrix}$$

r_1, r_2 and r_3 are some given arbitrary constants.

According to the preceding section, by using partial differential equation (6), the added Hamiltonian is found to be

$H_a(x) = \frac{k}{2}(x_1^2 + x_2^2 + x_3^2)$.

k is a constant which may satisfy a given condition in order to keep the system stable. Therefore using the added hamiltonian function leads to the feedback control laws

$$\begin{aligned} u_1(x) &= -r_1(k+k_1)x_1 \\ u_2(x) &= -r_2(k+k_2)x_2 \\ u_3(x) &= -r_3(k+k_3)x_3 \end{aligned} \qquad (11)$$

where

$k_1 = \frac{I_y - I_z}{I_y I_z}, k_2 = \frac{I_z - I_x}{I_x I_z}, \; k_3 = \frac{I_x - I_y}{I_y I_x}$

$r_1(k+k_1) > 0$, $r_2(k+k_2) > 0$

and $r_3(k+k_3) > 0$.

3. ANALYTICAL SOLUTIONS OF THE ANGULAR MOMENTUM

Up to now, it is quite difficult to find the analytic solutions using (11). Even though, in the particular case where

$r_1(k+k_1) = r_2(k+k_2) = r_3 \;(k+k_3) = \beta$,

the analytical solutions of the angular momentum components are found to be :

$$\begin{aligned} x_1(t) &= imv_1 e^{-\beta t}\omega_0 cn(\frac{\omega_0 \lambda e^{-\beta t}}{\beta}) \\ x_2(t) &= mv_2 \omega_0 e^{-\beta t} sn(\frac{\omega_0 \lambda e^{-\beta t}}{\beta}) \\ x_3(t) &= iv_3 \omega_0 e^{-\beta t} dn(\frac{\omega_0 \lambda e^{-\beta t}}{\beta}) \end{aligned} \qquad (12)$$

where $cn()$, $sn()$ and $dn()$ are the Jacobi functions of pole n. m is the so-called modulus of the Jacobi functions. ω_0 is an arbitrary constant, and i is the imaginary number.

The v_i's are the components of the eigenvector v of the closed loop system which has been seen to satisfy the nonlinear characteristic equation (Siguerdidjane, 91, 92, 94)

$$\begin{aligned} \lambda v_1 &= \frac{I_2 - I_3}{I_1} v_2 v_3 \\ \lambda v_2 &= \frac{I_3 - I_1}{I_2} v_3 v_1 \\ \lambda v_3 &= \frac{I_1 - I_2}{I_3} v_1 v_2 \end{aligned} \qquad (13)$$

λ denotes the eigenvalue associated to the eigenvector v.

One may mention that, in some hard cases, it may happen that it is not quite straightforward to compute the involved added Hamiltonian function, so finding the analytic solutions should be an alternative way to solve the problem and vice versa.

4. LAUNCHER DYNAMICS IN THE PHASE OUTSIDE THE ATMOSPHERE

Let us now consider the mathematical model of a launcher in the phase outside the atmosphere. It is described by the following differential equations

$$\begin{aligned} \dot{p} &= \frac{M_\phi}{I_x}\delta_r - \frac{(I_z - I_y)}{I_x} qr \\ \dot{q} &= \frac{M_\theta}{I_y}\delta_p + \frac{(I_x - I_z)}{I_y} pr \\ \dot{r} &= \frac{M_\psi}{I_z}\delta_y + \frac{(I_y - I_x)}{I_z} pq \end{aligned} \qquad (14a)$$

$$\begin{aligned} \dot{\theta} &= q\frac{\cos\phi}{\cos\psi} - r\frac{\sin\phi}{\cos\psi} \\ \dot{\psi} &= q\sin\phi + r\cos\phi \\ \dot{\phi} &= p - q\cos\phi \; tg\psi + r\sin\phi \; tg\psi \end{aligned} \qquad (14b)$$

The vehicle is flying without propulsion, there is no aerodynamic coupling effect acting on the vehicle.

p, q and r are the angular velocities about x, y and z body axis respectively. θ, ψ and ϕ are the pitch, yaw and roll angles respectively. M_θ, M_φ and M_ϕ are the control moments in pitch, yaw and roll respectively. δ_p, δ_y and δ_r are actuation signals for control moments in pitch, yaw and roll respectively.

Let us here suppose that one desires the closed loop system to have a linear behavior, the solution should obviously be

$$x = \omega_0 v e^{\lambda t} \qquad (15)$$

where here $x = (x_1, x_2, x_3)^T = (I_x p, I_y q, I_z r)^T$, ω_0 is an arbitrary constant, v denotes the eigenvector of the closed loop system and λ represents its associated eigenvalue. Set now,

$u = (u_1, u_2, u_3)^T = (M_\phi \delta_r, M_\theta \delta_p, M_\psi \delta_y)^T$

From the theory of eigenstructure described in (Siguerdidjane, 94), it comes that

$$\lambda v_1 = \frac{I_2 - I_3}{I_1} v_2 v_3 + u_1(v)$$
$$\lambda v_2 = \frac{I_3 - I_1}{I_2} v_3 v_1 + u_2(v) \qquad (16)$$
$$\lambda v_3 = \frac{I_1 - I_2}{I_3} v_1 v_2 + u_3(v)$$

where $u(v)$ is the unknown function $u(x)$ evaluated in v.

By differentiating equation (15), using equations (16) and by entering the solutions into equations (14a) it comes out the linearizing feedback controls

$$u_1(x) = k_1 x_2 x_3 + \beta_1 x_1$$
$$u_2(x) = k_2 x_1 x_3 + \beta_2 x_2 \qquad (17)$$
$$u_3(x) = k_3 x_1 x_2 + \beta_3 x_3$$

β_1, β_2 and β_3 are constants. k_1, k_2 and k_3 are defined as in preceding section.

5. SIMULATION RESULTS-DISCUSSION

5.a- Spacecraft vehicle

In order to perform simulations, the parameters of Spot 4 have been considered. $I_1 = 2500 kgm^2$, $I_2 = 6500 kgm^2$ and $I_3 = 8000 kgm^2$.
The parameter *k* must be chosen such that $k > max\,(abs(k_1), abs(k_3))$ in order to ensure the asymptotic stability (see Figs 1a-b). The damping coefficients are being $r_1 = r_2 = 5, r_2 = 2$.
The settling time depends on the choice of r_i.
The variation of r_2 has more effect on the system than r_1 and r_3. The oscillations of the angular velocities may be observed for $r_2 > 30$, $r_1 > 150$ and $r_3 > 100$ (see Figs 2a- 2b).
The system is shown to be not sensitive to large perturbations on the moments of inertia values (see Fig 3).

6. CONCLUSION

In linear systems, the relationship between the damping coefficients and the overshoot is known. So, from the simulations below, it can obviously be observed that the system behavior is more sensitive to some coefficients than the others of the damping matrix. Our next goal would be the investigations in order to solve the whole system, by means of the attitude behavior and to establish a relationship between the overshoot and the damping of the nonlinear system.

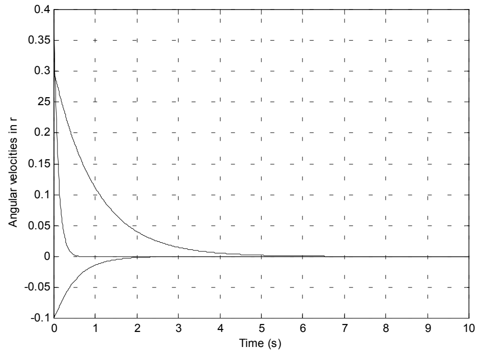

Fig. 1a. Angular velocities vs time (s)

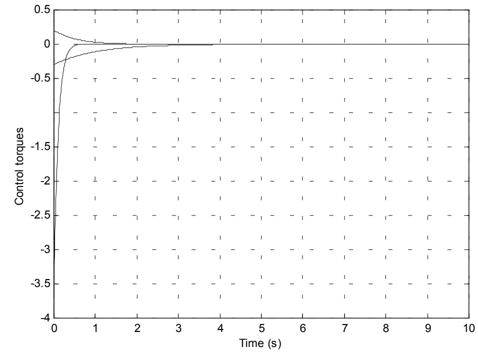

Fig. 1b. Control torques vs time (s)

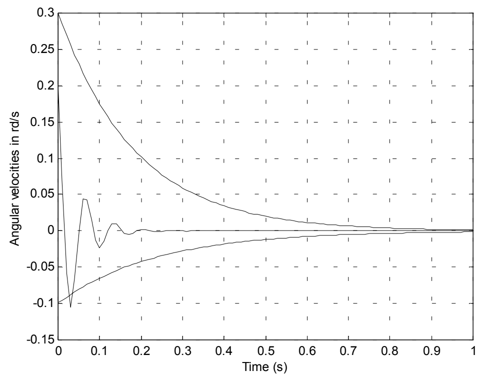

Fig. 2a. Angular velocities vs time (s)
$r_1 = r_3 = 10, r_2 = 30$

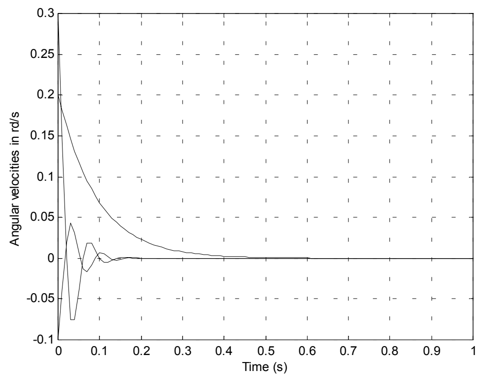

Fig. 2b. Angular velocities vs time (s)
$r_1 = 150, r_2 = 5, r_3 = 100$

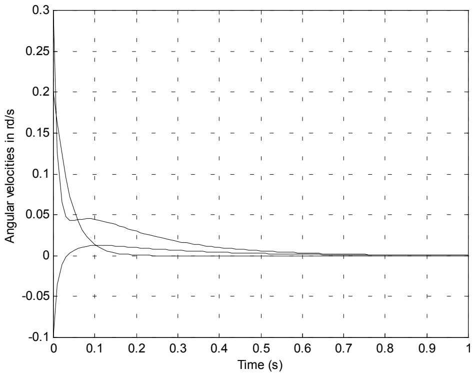

Fig. 3. Angular velocities vs time (s) under large uncertainties of the moments of inertia

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