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OBSERVER BASED DYNAMIC VISUAL FEEDBACK CONTROL FOR NONLINEAR ROBOTICS SYSTEMS

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Abstract: This paper investigates a robot motion control problem with visual information. Firstly the model of the relative rigid body motion (positions and rotations) and the method for the estimation of the relative rigid body motion are presented in order to derive the visual feedback system. Secondly we consider the velocity observer and derive the dynamic visual feedback system which contains the manipulator dynamics. Finally the main result with respect to stability for the proposed dynamic visual feedback control is discussed.

Keywords: Robot control, Visual servoing, Lyapunov stability, Nonlinear Observer, Nonlinear control, Robust control

1. INTRODUCTION

Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision system to control the position of the robot end-effector in an efficient manner. The combination of mechanical control with visual information, so-called *visual feedback control* or *visual servo*, should become extremely important, when we consider a mechanical system working under *dynamical* environments. Research efforts toward this direction have been nicely collected in the tutorial(Hutchinson *et al.*, 1996).

This paper deals with a robot motion control with visual information. This control problem is standard and important, and has gained much attention of researchers for many years (Wilson *et al.*, 1996; Hashimoto *et al.*, 1997; Kawabata and Fujita, 1998; Maruyama and Fujita, 1999; Kelly *et al.*, 2000). The control objective is to track the target object in a three-dimensional workspace by using image information. The typical example is shown in Fig. 1. Hence the dynamics of the visual feedback system is described by the nonlinear systems in a 3-D workspace. Under the assumption that the objects' depths are known, a simple image-based controller for the 3-D visual feedback system has been considered(Kelly *et al.*, 2000). In a recent paper(Maruyama *et al.*, 2001), the authors have proposed the 3-D visual feedback control which has guaranteed stability without the known objects' depths from the theoretical standpoint. However few rigorous results have been obtained in terms of the dynamic visual feedback system which contains a manipulator dynamics in a 3-D workspace. Moreover the previous works have assumed that the joint velocities of the manipulator can be measured directly.

In this paper, we discuss stability for the dynamic visual feedback system in a 3-D workspace. In order to derive the dynamic visual feedback system, we will consider a relative rigid body motion dynamics and a nonlinear observer. Moreover the velocity observer which estimates the joint

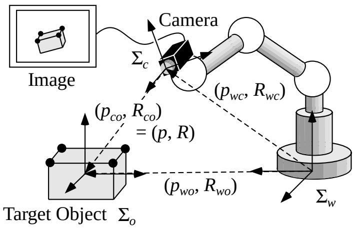

Fig. 1. Eye-in-hand visual feedback system.

velocities of the manipulator will be proposed. The key idea of this paper is to utilize an error function of the rotation matrix as a Lyapunov function. The work presented in this paper extends our previous researches(Maruyama and Fujita, 1999; Maruyama *et al.*, 2001).

This paper is organized as follows. In Section 2, we consider a model of the relative rigid body motion. Section 3 presents a method for the estimation of the relative rigid body motion. The main result concerned with stability for the dynamic visual feedback system considering the proposed velocity observer is derived in Section 4. Finally, we offer some conclusions in Section 5.

Let a rotation matrix $R_{ab} \in \mathcal{R}^{3\times 3}$ represent the change of the principal axes of a frame b relative to a frame a. Then, R_{ab} is known to become orthogonal with unit determinant. Such a matrix belongs to a Lie group of dimension three, called $SO(3) = \{R_{ab} \in \mathcal{R}^{3\times 3} | R_{ab}R_{ab}^T = R_{ab}^T R_{ab} = I, \det(R_{ab}) = +1\}$. The configuration space of the rigid body motion is the product space of $\mathcal{R}^3$ with $SO(3)$, which should be denoted as $SE(3)$ throughout this paper (see, e.g. (Murray *et al.*, 1994)).

2. RELATIVE RIGID BODY MOTION

Let us consider the eye-in-hand system(Hutchinson *et al.*, 1996) depicted in Fig. 1, where the coordinate frame Σ_w represents the world frame, Σ_c represents the camera (end-effector) frame, and Σ_o represents the object frame, respectively. Let $p_{co} \in \mathcal{R}^3$ and $R_{co} \in \mathcal{R}^{3\times 3}$ denote the position vector and the rotation matrix from the camera frame Σ_c to the object frame Σ_o. Then, the relative rigid body motion from Σ_c to Σ_o can be represented by $(p_{co}, R_{co}) \in SE(3)$. Similarly, we will define the rigid body motion (p_{wc}, R_{wc}) from Σ_w to Σ_c, and (p_{wo}, R_{wo}) from Σ_w to Σ_o, respectively, as in Fig. 1.

The objective of the visual feedback control is to bring the actual relative rigid body motion (p_{co}, R_{co}) to a given reference (p_d, R_d) (see, e.g. (Hutchinson *et al.*, 1996)). The reference (p_d, R_d) for the rigid motion (p_{co}, R_{co}) is assumed to be constant in this paper.

In this section, let us derive a model of the relative rigid body motion. The rigid body motion (p_{wo}, R_{wo}) of the target object, relative to the world frame Σ_w, is given by

$$p_{wo} = p_{wc} + R_{wc}p_{co} \tag{1}$$

$$R_{wo} = R_{wc}R_{co} \tag{2}$$

which is a direct consequence of a transformation of the coordinates(Murray *et al.*, 1994) in Fig. 1. Using the property of a rotation matrix, i.e. $R^{-1} = R^T$, the rigid body motion (1) and (2) is now rewritten as

$$p_{co} = R_{wc}^T(p_{wo} - p_{wc}) \tag{3}$$

$$R_{co} = R_{wc}^T R_{wo}. \tag{4}$$

The dynamic model of the relative rigid body motion involves the velocity of each rigid body. To this aid, let us consider the velocity of a rigid body(Murray *et al.*, 1994). Let $\hat{\omega}_{wc}$ and $\hat{\omega}_{wo}$ denote the instantaneous body angular velocities from Σ_w to Σ_c, and from Σ_w to Σ_o, respectively. Here the operator '$\wedge$' (wedge), from $\mathcal{R}^3$ to the set of 3×3 skew-symmetric matrices $so(3)$, is defined as

$$\hat{a} = (a)^\wedge := \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}.$$

The operator '$\vee$' (vee) denotes the inverse operator to '$\wedge$': i.e. $so(3) \to \mathcal{R}^3$. With these, it is possible to specify the velocities of each rigid body as follows (Murray *et al.*, 1994)(Chap.2, Eq.(2.55)).

$$\dot{p}_{wc} = R_{wc}v_{wc}, \quad \dot{R}_{wc} = R_{wc}\hat{\omega}_{wc} \tag{5}$$

$$\dot{p}_{wo} = R_{wo}v_{wo}, \quad \dot{R}_{wo} = R_{wo}\hat{\omega}_{wo}. \tag{6}$$

Differentiating (3) and (4) with respect to time, we can obtain

$$\dot{p}_{co} = -v_{wc} + \hat{p}_{co}\omega_{wc} + R_{co}v_{wo} \tag{7}$$

$$\dot{R}_{co} = -\hat{\omega}_{wc}R_{co} + R_{co}\hat{\omega}_{wo}. \tag{8}$$

Now, let us denote the body velocity of the camera relative to the world frame Σ_w as

$$V_{wc} := [v_{wc}^T \ \omega_{wc}^T]^T. \tag{9}$$

Further, the body velocity of the target object relative to Σ_w should be denoted as

$$V_{wo} := [v_{wo}^T \ \omega_{wo}^T]^T. \tag{10}$$

Then we can rearrange Eqs.(7) and (8) in a matrix form as follows
(Relative Rigid Body Motion)

$$\begin{bmatrix} \dot{p} \\ (\dot{R}R^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{p} \\ 0 & -I \end{bmatrix} V_{wc} + \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} V_{wo}. \tag{11}$$

Here (p, R) denotes (p_{co}, R_{co}) for short. Eq.(11) should be the model of the relative rigid body motion(Maruyama and Fujita, 1999).

3. ESTIMATION OF RELATIVE RIGID BODY MOTION

The visual feedback control task should require the information of the relative rigid body motion (p, R). However, the available information that can be measured in the visual feedback systems is only image information. Hence, let us consider a nonlinear observer which will estimate the relative rigid body motion via image information.

We shall consider the following dynamic model which just comes from the actual relative rigid body motion (11).

$$\begin{bmatrix} \dot{\bar{p}} \\ (\dot{\bar{R}}\bar{R}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \end{bmatrix} V_{wc} + \begin{bmatrix} I & 0 \\ 0 & \bar{R} \end{bmatrix} u_e \quad (12)$$

where $(\bar{p}, \bar{R})$ is the estimated value of the relative rigid body motion, and the new input u_e for the estimation is to be determined in order to converge the estimated value to the actual relative rigid body motion.

Next let us derive a pinhole camera model as shown in Fig. 2. Let λ be a focal length. Let p_{oi}

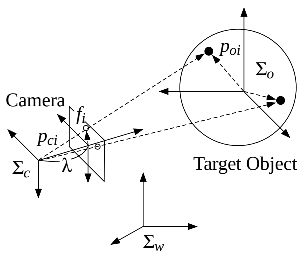

Fig. 2. Pinhole camera model.

and p_{ci} be coordinates of the target object's i-th feature point relative to Σ_o and Σ_c, respectively. Then, from a transformation of the coordinates, we have

$$p_{ci} = p + Rp_{oi}. \quad (13)$$

The perspective projection of the i-th feature point onto the image plane gives us the image plane coordinate as follows

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \quad (14)$$

where $p_{ci} := [x_{ci}\ y_{ci}\ z_{ci}]^T$. It is straightforward to extend this model to the n image points case by simply stacking the vectors of the image plane coordinate, i.e. $f := [f_1^T\ \cdots\ f_n^T]^T \in \mathcal{R}^{2n}$.

Now, we define the estimation error between the estimated value $(\bar{p}, \bar{R})$ and the actual relative rigid motion (p, R) as

$$(p_{ee}, R_{ee}) := (p - \bar{p}, \bar{R}^T R). \quad (15)$$

Note that, if $p = \bar{p}$ and $R = \bar{R}$, then it follows $p_{ee} = 0$ and $R_{ee} = I$. Let the matrix $\mathrm{sk}(R)$ denote $\frac{1}{2}(R - R^T)$ and let

$$e_R(R) := \mathrm{sk}(R)^\vee \quad (16)$$

represent the error vector of the rotation matrix R. Then the vector of the estimation error is given by

$$e_e := \begin{bmatrix} p_{ee}^T & e_R^T(R_{ee}) \end{bmatrix}^T \quad (17)$$

where $e_e = 0$ holds when $p_{ee} = 0$ and $R_{ee} = I$.

Next, we will derive the measurement equation from Eqs.(13) and (14). Suppose the estimation error is *small* enough that we can let $R_{ee} \simeq I + \mathrm{sk}(R_{ee})$, then Eq.(13) becomes

$$p_{ci} = \bar{p}_{ci} - \bar{R}\hat{p}_{oi} e_R(R_{ee}) + p_{ee} \quad (18)$$

where $\bar{p}_{ci} := \bar{p} + \bar{R}p_{oi}$. Using Taylor expansion, Eq.(14) can be written as

$$f_i = \bar{f}_i + \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} (p_{ci} - \bar{p}_{ci}) \quad (19)$$

where $\bar{p}_{ci} = [\bar{x}_{ci}\ \bar{y}_{ci}\ \bar{z}_{ci}]^T$ and $\bar{f}_i := \frac{\lambda}{\bar{z}_{ci}}[\bar{x}_{ci}\ \bar{y}_{ci}]^T$.

An approximation of image information f around the estimated value $(\bar{p}, \bar{R})$ is given by

$$f - \bar{f} = J(\bar{p}, \bar{R}) e_e \quad (20)$$

where the matrix $J(\bar{p}, \bar{R})$ is defined as

$$J(\bar{p}, \bar{R}) := \begin{bmatrix} L(\bar{p}, \bar{R}; p_{o1}) \\ L(\bar{p}, \bar{R}; p_{o2}) \\ \vdots \\ L(\bar{p}, \bar{R}; p_{on}) \end{bmatrix} \quad (21)$$

$$L(\bar{p}, \bar{R}; p_{oi}) := \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \begin{bmatrix} I, & -\bar{R}\hat{p}_{oi} \end{bmatrix} . \quad (22)$$

Note that the matrix $J(\bar{p}, \bar{R})$ can be considered as the image Jacobian(Hutchinson *et al.*, 1996).

The following assumption will be made.

Assumption 1. For all $(\bar{p}, \bar{R}) \in SE(3)$, the matrix $J(\bar{p}, \bar{R})$ is full column rank.

Remark 2. Under Assumption 1, the relative rigid body motion can be uniquely defined by the image feature vector. Moreover it is known that $n > 4$ is desirable for the visual feedback systems.

The above discussion shows that we can derive the vector of the estimation error e_e from image information f and the estimated value of the relative rigid body motion $(\bar{p}, \bar{R})$,

$$e_e = J^\dagger(\bar{p}, \bar{R})(f - \bar{f}) \quad (23)$$

where $\dagger$ denotes the pseudo-inverse.

In the next section, Eqs.(12) and (23) will be exploited in order to estimate the relative rigid body motion.

4. DYNAMIC VISUAL FEEDBACK CONTROL

4.1 Dynamic Visual Feedback System

Let us derive a model of the visual feedback system. First, we consider the manipulator dynamics as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{24}$$

where q, $\dot{q}$ and $\ddot{q}$ are the joint angles, velocities and accelerations, τ is the vector of the input torques. The matrices $M(q)$ and $C(q,\dot{q})$ have some important properties which will be used in the sequel.

Property 3. By defining $C(q,\dot{q})$ using the Christoffel symbols, $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric. Furthermore, for this choice $C(q,\dot{q})$ satisfies

$$C(q,x)y = C(q,y)x \quad \forall x,y \tag{25}$$

$$C(q,z+\alpha x)y = C(q,z)y + \alpha C(q,x)y \quad \forall x,y \tag{26}$$

where α is scalar.

Property 4. The matrices $M(q)$ and $C(q,\dot{q})$ are bounded with respect to q,

$$0 < M_m \leq \|M(q)\| \leq M_M \qquad \forall q \tag{27}$$

$$\|C(q,x)\| \leq C_M\|x\| \qquad \forall q,x \tag{28}$$

where we define the norm of a vector x as

$$\|x\| = \sqrt{x^T x}. \tag{29}$$

The norm of a matrix A as

$$\|A\| = \sqrt{\lambda_{max}(A^T A)} \tag{30}$$

where λ_{max} denotes the maximum eigenvalue.

Here we consider the body velocity of the camera V_{wc}. Since the camera is mounted on the end effector of the manipulator, the body velocity of the camera V_{wc} is given by

$$V_{wc} = J_b(q)\dot{q} \tag{31}$$

where $J_b(q)$ is the manipulator body Jacobian (Murray *et al.*, 1994). Before formulating the visual feedback control problem, we make the following assumption.

Assumption 5. The manipulator has 6 degrees of freedom, and the manipulator Jacobian $J_b(q)$ is the nonsingular matrix.

Under Assumption 5, Eq.(31) can be transformed into

$$\dot{q} = J_b^{-1}(q)V_{wc}. \tag{32}$$

Henceforth, we define the reference of the joint velocities as follows

$$\dot{q}_d := J_b^{-1}(q)u_c. \tag{33}$$

where u_c represents an ideal body velocity of the camera, which will be proposed later.

Let us define the error vector with respect to the joint velocities of the manipulator dynamics as

$$\xi := \dot{q} - \dot{q}_d. \tag{34}$$

Now, we consider the velocity observer approach (Berghuis and Nijmeijer, 1993) in order to estimate the joint velocities. Here we propose the control law as the input torques and the velocity observer for the dynamic visual feedback system as follows.

$$\text{Controller} \quad \begin{cases} \tau = M(q)\ddot{q}_d + C(q,\dot{q}_0)\dot{q}_d \\ \qquad\qquad\qquad +g(q) + u_\xi \\ \dot{q}_0 = \dot{\bar{q}} - \Lambda\tilde{q} \end{cases} \tag{35}$$

$$\begin{matrix}\text{Velocity}\\ \text{Observer}\end{matrix} \quad \begin{cases} \dot{\bar{q}} = z + L_d\tilde{q} \\ \dot{z} = \ddot{q}_d + L_p\tilde{q} \end{cases} \tag{36}$$

where $[\bar{q}^T \ z^T]^T$ is the observer state, $\bar{q}$ denotes the estimated velocities, $\Lambda = \Lambda^T > 0$, $L_d = L_d^T > 0$, $L_p = L_p^T > 0$. The new input u_ξ is to be determined in order to achieve the control objectives. Moreover, $\tilde{q}$ is defined as

$$\tilde{q} := q - \bar{q} \tag{37}$$

which represents the error between the actual joint angles and the estimated joint angles. The following assumptions will be made.

Assumption 6. Λ, L_d and L_p are constant and diagonal. Moreover, L_d and L_p can be written as

$$L_d = l_d I + \Lambda, \quad L_p = l_d\Lambda$$

where $l_d > 0$ is scalar.

Assumption 7. The desired velocities $\dot{q}_d$ are bounded by V_M, i.e.,

$$V_M = \sup_t \|\dot{q}_d(t)\|. \tag{38}$$

Here we consider the error dynamics of the manipulator. Substituting Eq.(35) into Eq.(24) yields

$$M(q)(\ddot{q} - \ddot{q}_d) = -C(q,\dot{q})(\dot{q} - \dot{q}_d) \\ +C(q,\dot{q}_0 - \dot{q})\dot{q}_d + u_\xi \tag{39}$$

where Property 3 has been used. From Eqs.(34) and (39), we have

$$M(q)\dot{\xi} = -C(q,\dot{q})\xi - C(q,s)\dot{q}_d + u_\xi \tag{40}$$

where s is defined as

$$s := \dot{q} - \dot{q}_0 = \dot{\tilde{q}} + \Lambda\tilde{q}. \tag{41}$$

Next we derive the observer error dynamics. Under Assumption 6, we have

$$\ddot{q}_0 - \ddot{q}_d = l_d s. \tag{42}$$

From Eqs.(40)–(42), the observer error dynamics can be written as

$$M(q)\dot{s} = -M(q)l_d s + C(q, s-\dot{q})\xi \\ -C(q,\dot{q})s + u_\xi. \quad (43)$$

Next, we define the control error as follows.

$$(p_{ec}, R_{ec}) := (\bar{p} - p_d, \bar{R}R_d^T) \quad (44)$$

represents the error between the estimated value $(\bar{p}, \bar{R})$ and the reference of the relative rigid body motion (p_d, R_d). It should be remarked that, if $p_d = \bar{p}$ and $R_d = \bar{R}$, then $p_{ec} = 0$ and $R_{ec} = I$ hold. Using the notation $e_R(R)$, the vector of the control error is defined as

$$e_c := \left[p_{ec}^T \ e_R^T(R_{ec}) \right]^T \quad (45)$$

where $e_c = 0$ gives $p_{ec} = 0$ and $R_{ec} = I$.

From Eqs.(12) and (44), the dynamics of the control error can be given by

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \end{bmatrix} V_{wc} + R_1 u_e \quad (46)$$

where $R_1 = \mathrm{diag}\{I, \bar{R}\}$.

Further, we consider the dynamics of the estimation error. Using Eqs.(11), (12) and (15), the dynamics of the estimation error can be obtained as follows

$$\begin{bmatrix} \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} 0 & \hat{p}_{ee} \\ 0 & 0 \end{bmatrix} V_{wc} - u_e + R_2 V_{wo} \quad (47)$$

where $R_2 = \mathrm{diag}\{R, R_{ee}\}$.

Using Eqs.(40), (43), (46) and (47), the visual feedback system can be derived as follows

$$M(q)\dot{\xi} = -C(q,\dot{q})\xi - C(q,s)\dot{q}_d + u_\xi \quad (48)$$

$$M(q)\dot{s} = -M(q)l_d s + C(q, s-\dot{q})\xi \\ -C(q,\dot{q})s + u_\xi \quad (49)$$

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \\ \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \\ 0 & \hat{p}_{ee} \\ 0 & 0 \end{bmatrix} J_b(q)\xi + \begin{bmatrix} 0 \\ R_2 \end{bmatrix} V_{wo} \\ + \begin{bmatrix} -I & \hat{\bar{p}} & I & 0 \\ 0 & -I & 0 & \bar{R} \\ 0 & \hat{p}_{ee} & -I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} u_c \\ u_e \end{bmatrix} . (50)$$

Eqs.(48) and (49) represent the manipulator dynamics with the velocity observer. Eq.(50) denotes the relative rigid body motion with the nonlinear observer.

4.2 Stability of Dynamic Visual Feedback Control

Let us define the error vector of the dynamic visual feedback system as

$$x := \left[\xi^T \ s^T \ e_c^T \ e_e^T\right]^T.$$

Then the dynamic visual feedback control problem can be formulated as follows.

Problem 8. Find a input vector $u = [u_\xi^T \ u_c^T \ u_e^T]^T$ such that the closed-loop system satisfies the control objectives as follows: (Internal stability) If the target object is static, i.e. $V_{wo} = 0$, then the equilibrium point $x = 0$ for the closed-loop system is asymptotically stable.

We propose the following dynamic visual feedback control law

$$u_\xi = -K_\xi(\xi - s) - J_b^T(q)B(p_d)e_c \quad (51)$$

$$\begin{bmatrix} u_c \\ u_e \end{bmatrix} = -\begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} -B(p_d) & 0 \\ R_1^T & -I \end{bmatrix} e \quad (52)$$

where K_ξ, K_c and K_e are 6×6 positive definite matrices. B is defined as

$$B(a) = \begin{bmatrix} I & 0 \\ \hat{a} & I \end{bmatrix}$$

for any $a \in \mathcal{R}^3$. The error vector e is defined as

$$e := \left[e_c^T \ e_e^T \right]^T.$$

Remark 9. The control input u contains the error vectors ξ, s, e_c and e_e. ξ and s cannot be realized, whereas the difference $\xi - s$ can be obtained from known signals, i.e., $\xi - s = \dot{\tilde{q}} - \lambda\tilde{q} - \dot{q}_d$. e_c is derived from the nonlinear observer. And e_e can also be obtained from Eq.(23). Hence we can exploit the dynamic visual feedback control law u.

Now, let us define

$$K_{ce} := \begin{bmatrix} B^T(p_d) & -R_1 \\ 0 & I \end{bmatrix} \begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} B(p_d) & 0 \\ -R_1^T & I \end{bmatrix}$$

$$K_J(q) := (J_b^T(q)B(p_d))^T (J_b^T(q)B(p_d)).$$

Moreover, for any matrix $A(x) = A^T(x) > 0$ and for all x, A_m and A_M represent the minimum and maximum eigenvalue of $A(x)$, respectively.

Using Property 4 and Assumptions 6, 7, the result with respect to asymptotic stability for the closed-loop system can be established as follows.

Theorem 10. Suppose that the following conditions hold.

$$K_{\xi,m} > C_M V_M \quad (53)$$

$$l_d > M_m^{-1}(K_{\xi,M} + \frac{1}{2} + C_M V_M) \quad (54)$$

$$K_{ce,m} > \frac{1}{2} K_{J,M} \quad (55)$$

If $V_{wo} = 0$, then the equilibrium point $x = 0$ for the closed-loop system (48)-(52) is asymptotically stable. Moreover, a region of attraction is given by

$$D = \left\{ x \in \mathcal{R}^{24} \ \Big| \ \frac{K_{\xi,m}}{C_M} - V_M > \|s\|, \\ \frac{2l_d M_m - 2K_{\xi,M} - 1}{2C_M} - V_M > \|\xi\| \right\} (56)$$

PROOF. Consider the following positive definite function

$$V = \frac{1}{2}\xi^T M(q)\xi + \frac{1}{2}s^T M(q)s + \frac{1}{2}\|p_{ec}\|^2 + \phi(R_{ec}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(R_{ee}) \quad (57)$$

where ϕ is the error function of the rotation matrix. We refer to Appendix A for the error function on $SO(3)$. Differentiating (57) with respect to time along the trajectories of the system (48)-(52) yields

$$\begin{aligned}\dot{V} = &-\xi^T K_\xi \xi - e^T K_{ce} e \\ &-s^T\big(l_d M(q) - K_\xi - C(q,\xi)\big)s \\ &-s^T C(q,\xi)\xi - s^T C(q,\dot{q}_d)\xi \\ &-\xi^T C(q,\dot{q}_d)s - s^T J_b^T(q)B(p_d)e_c \quad (58)\end{aligned}$$

where Property 3 has been used. Next, using Property 4 and Assumption 7, Eq.(58) can be upper bounded by

$$\begin{aligned}\dot{V} \le &-\big(K_{\xi,m} - C_M(V_M + \|s\|)\big)\|\xi\|^2 \\ &-\big(l_d M_m - K_{\xi,M} - \frac{1}{2} - C_M(V_M + \|\xi\|)\big)\|s\|^2 \\ &-\big(K_{ce,m} - \frac{1}{2}K_{J,M}\big)\|e\|^2. \quad (59)\end{aligned}$$

Hence $\dot{V}$ is negative on $D - \{0\}$. This completes the proof. □

In the proof of Theorem 10, the positive definite function V plays the role of a Lyapunov function. In this section, we have obtained the rigorous result with respect to stability of the dynamic visual feedback system in a 3-D workspace. L_2-gain performance analysis for the visual feedback system has been discussed in the previous works(Maruyama and Fujita, 1999; Maruyama *et al.*, 2001). Further one of the experimental results of the visual feedback control has been presented by the authors(Kawabata and Fujita, 1998). In future work, dynamic visual feedback control will be discussed based on our recent result of the nonlinear receding horizon control approach(Kawai and Fujita, 2001).

5. CONCLUSIONS

This paper has discussed the dynamic visual feedback control which contains the manipulator dynamics from the theoretical standpoint. By using the representation of $SE(3)$, we have derived the relative motion dynamics. The nonlinear observer has been employed in order to estimate the relative rigid body motion. Furthermore, we have proposed the velocity observer with the aim of obtaining the joint velocities. Stability for the dynamic visual feedback system considering the proposed velocity observer has been discussed.

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Appendix A. ERROR FUNCTION ON $SO(3)$

Let us introduce the notation of the error function

$$\phi(R) := \frac{1}{2}\mathrm{tr}(I - R). \quad (A.1)$$

The error function ϕ has the following properties (Bullo and Murray, 1999).

Property 11. Let $R \in SO(3)$. The following properties hold.

(1) $\phi(R) = \phi(R^T) \ge 0$ and $\phi(R) = 0$ iff $R = I$.
(2) $\dot{\phi}(R) = e_R^T(R)(R^T\dot{R})^\vee = e_R^T(R)(\dot{R}R^T)^\vee$.