

AEROSPACE LAUNCH VEHICLE CONTROL: A GAIN SCHEDULING APPROACH

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Abstract: this paper presents a design methodology for an aerospace launch vehicle autopilot. The non stationary characteristics during the atmospheric flight impose to use a gain scheduled approach for a Linear Time Varying System (LTV). The important result is that the chosen interpolation is forced to be linear and the stability is guaranteed. *Copyright © 2002 IFAC*

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1. INTRODUCTION

This paper presents a gain scheduling application for an aerospace launch vehicle; it concerns the atmospheric flight control which is a non stationary step of the mission due to the physical parameters evolution (mass, velocity, gravity,...). Section 2 gives a short presentation of the plant and the control objectives; section 3 is dedicated to the gain scheduling problem and particularly to the interpolation method performed. Then section 4 shows the results obtained with a simulator which has been developed with the *CNES* (French Space Agency) and *EADS Launch Vehicles*.

2. LAUNCH VEHICLE CONTROL

The launch vehicle control during the atmospheric flight has been few investigated in the automatic control literature; except these last years with the *CNES* (French Space Agency) and *EADS Launch Vehicles* support (Mauffrey & Schoeller 1998, Clement *et al.* 2001, Voinot *et al.* 2001, Mauffrey *et*

al. 2001). The presented methodology for this kind of issue can be extended to other applications.

In order to simplify, the application is developed for the yaw axis whose dynamics include a rigid mode, bending modes (sloshing modes are not considered), actuators and sensors (figure 1). The automatic control of the launcher has the function of keeping the process stable around its center of gravity, following the guidance reference trajectory.

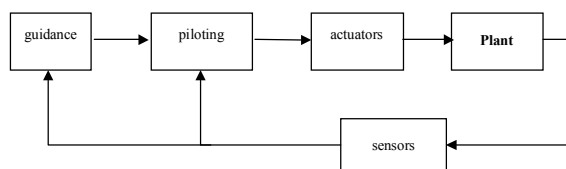


Figure 1. Control loops

The launcher control objectives, during the atmospheric flight phase, are the following:

- *Frequency specifications:*
 - closed-loop stability with sufficient margins: decreasing and increasing gain margins (ΔG_{BF} and ΔG_{HF}) have to stay higher than given specifications.
 - control the destabilizing bending modes: the aim is to attenuate these modes under a gain limit (X_{dB}) except for the first one which can be controlled in phase with a sufficient delay margin (at least one sample period T_s).
- *Time specifications:*
 - limit the angle of attack i in case of wind (a typical wind profile will be given on figure 7).
 - limit the angle of deflection β and its velocity $\dot{\beta}$
 - the consumption C must be limited to C_{max} where:
$$C = \sum_{k=T_{init}}^{T_{end}} |\beta(k+1) - \beta(k)|$$
- *Robustness*
 - all these objectives have to be robust against uncertainties (which affect rigid and bending modes)

A simplified launcher scheme is given in figure 2 where the angle of attack between the launcher axis and the relative speed V_R is noted i , the attitude ψ , the angle of deflection β (control input) and the wind velocity W (disturbance). The sensors allow to measure the attitude and its velocity whereas the actuators allow to control the angle of deflection for the thrusters.

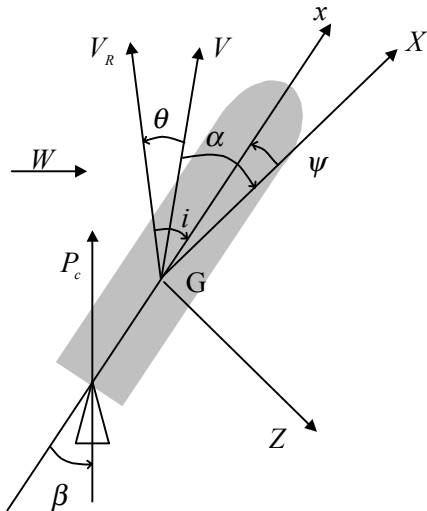


Fig. 2 – Simplified launcher representation

The challenge of such a control issue is to minimize the angle of attack i since it cannot be measured. The closed-loop structure is given in figure 3.

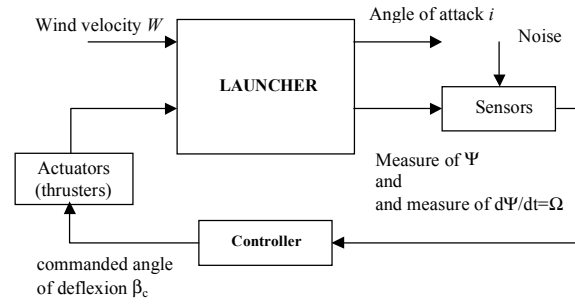


Figure 3. Control structure

The control design is performed along the guidance trajectory with several operating points: for each of them, a linear controller is performed according to the multiobjective algorithm presented in (Clement & Duc, 2000a) which uses Youla parameterization and LMI optimization. This algorithm reduces the conservatism of existing methods. Method and results for a given operating point are described in (Clement & Duc, 2000b).

The proposed interpolation method is particularly adapted to the launcher problem because the simplified model is already LTV i.e. Linear Time Varying (only small angles are considered and saturations have to be avoided) and the parameters evolutions are known as continuous time functions. Moreover, the parameters are slowly varying, then a linear interpolation is performed in order to reduce the number of on-line calculus.

3. GAIN SCHEDULING

The gain scheduling approach is a very classical nonlinear control technique which first appears in industrial fields (in particular aeronautic and military applications) and have been theoretically investigated since the '90 (Rugh, 1991). Roughly speaking, it can be described as a six-step procedure:

1. get a linear parameter-varying model;
2. choose the parameters to schedule;
3. choose a family of operating points
4. compute controllers for each point with linear design methods;
5. interpolate these controllers;
6. check the performance assessment.

Though differing technical avenues are available in each step, the scheme stays the same. Among papers that have investigated gain scheduling (Rugh & Shamma, 2000), very little focus in the interpolation problem, which is a fundamental step in the synthesis of a scheduled control law. In the literature, a number of *ad hoc* approaches have been presented (Nichols *et al.* 1993, Reichert 1992, Hyde & Glover 1993, Buschek, 1997) but the specifications are checked a posteriori.

These considerations are the main motivation of our

work about interpolation methods for discrete-time systems, more precisely sampled systems. We use, in this paper, an interpolation method that guarantees the stability of the nonlinear system along its trajectory with LMIs conditions.

3.1. Notations and definitions

The considered model is LTV finite dimensional, and described by a discrete-time state-space representation:

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)\beta(k) + B_2(k)W(k) \\ \begin{pmatrix} \psi(k) \\ \Omega(k) \end{pmatrix} = C(k)x(k) \end{cases} \quad (1)$$

All state-space parameters are time varying. The theoretical results (Shamma, 1988, Shamma & Athans, 1990, Fromion *et al.*, 1996) concerning gain scheduling are hard to be used in a practical way because of the generality of the field of gain scheduling. The choice of a closer class of systems seems to be a way to particularize some general results and to justify rigorously practical applications. According to the space launcher problem, the presented results consider an LTV system following a known trajectory. It means that all launcher parameters are known as time functions.

According to the general gain scheduling procedure, some operating points are chosen and corresponding controllers are designed. Then the interpolation problem can be considered; the proposed method is presented in the next paragraph jointly with a sufficient condition to preserve stability with the nonlinear plant. For ease of understanding we focus on the state feedback case and a generalization will be given with the application.

3.2. Interpolation Conditions

For brevity, we consider the state feedback case with only two models corresponding to different instants k_1 and k_2 . Let consider the evolution and control LTV state equation:

$$x(k+1) = A(k)x(k) + B(k)\beta(k) \quad (3)$$

Let two state feedback gains K_1 and K_2 such that $A(k) + B(k)K_1$ et $A(k) + B(k)K_2$ are Schur-Cohn for each frozen value of k in a discrete temporal neighborhoods $[a \ c]$ and $[b \ d]$ of k_1 and k_2 respectively (figure 4). Assume that $b < c$, such that K_1 and K_2 are simultaneously stabilizing for each frozen k in $[b \ c]$. Moreover, stability intervals (with a frozen time) intersection is a closed interval. The a , b , c and d temporal (discrete) instants are defined on figure 1:

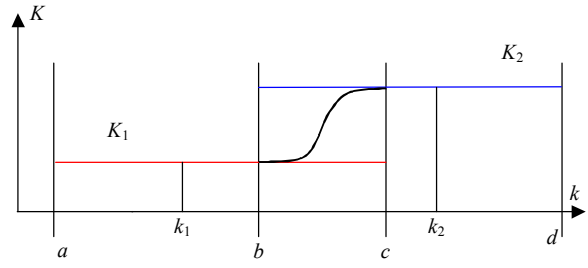


Figure 4. State feedback gain evolution

Theorem (Clement & Duc 2001)

If there exist two symmetric positive definite matrices X_1 et X_2 and a strictly positive $\gamma \in \mathbb{R}$ such that the following relations hold:

for every k such that $a \leq k \leq b$,

$$\begin{pmatrix} -X_1 & (A(k) + B(k)K_1)X_1 \\ X_1(A(k) + B(k)K_1)^T & -X_1 \end{pmatrix} < -\gamma I \quad (4)$$

for every k such that $b \leq k \leq d$,

$$\begin{pmatrix} -X_2 & (A(k) + B(k)K_2)X_2 \\ X_2(A(k) + B(k)K_2)^T & -X_2 \end{pmatrix} < -\gamma I \quad (5)$$

$$X_1 - X_2 < (c - b)\gamma I \quad (6)$$

then the scheduled state feedback gain $K(k)$:

$$K(k) = \begin{cases} K_1 & \text{if } a \leq k \leq b \\ \left(\frac{c-k}{c-b} K_1 X_1 + \frac{k-b}{c-b} K_2 X_2 \right) \left(\frac{c-k}{c-b} X_1 + \frac{k-b}{c-b} X_2 \right)^{-1} & \text{if } b \leq k \leq c \\ K_2 & \text{if } c \leq k \leq d \end{cases}$$

is such that the closed-loop LTV system is exponentially stable. ■

Remarks:

- The interpolation is performed only on the discrete time interval $[b \ c]$ when both controllers are stabilizing.
- The constraints are LMIs which allow to ensure stability with convex optimization.
- The condition (6) is original because no norm is considered. This is made possible by the knowledge of the trajectory.

Note that the interpolation turns into linear interpolation as a particular case. Indeed, if there is a solution such that $X_1 = X_2 = X$, the gain becomes a linear interpolation:

$$K(k) = \begin{cases} K_1 & \text{if } a \leq k \leq b \\ \frac{c-k}{c-b} K_1 + \frac{k-b}{c-b} K_2 & \text{if } b \leq k \leq c \\ K_2 & \text{if } c \leq k \leq d \end{cases} \quad (9)$$

This is possible when the two consecutive gains and frozen time plants are rather closed. In this case there exist a common Lyapunov function such that the inequalities (4) and (5) hold while (6) is obviously satisfied.

A dual problem is the interpolation of a full order observer which is not presented for brevity.

The last point is to connect this approach with the interpolation of non structured controller. Let n be the order of system (3) and n_c the controller order. In the case $n_c \geq n$, it is known that the controller can be interpreted as an observed state feedback structure connected with a Youla Parameter (Alazard & Apkarian, 1999). Then the state feedback interpolation case can be extended for controllers designed with various methods by interpolating the state feedback gains, the observer gains and the Youla parameter of the equivalent observer / state feedback structure.

4. AEROSPACE LAUNCHER CONTROL

4.1. Gain scheduling procedure

This section describes the six steps of the gain scheduling procedure given in section 3.

- Steps 1 and 2 are given by the choice of the model. Indeed the LTV structure of the model is known as a table giving all parameters of the launcher at each instant k .
- Step 3: the set of operating points is chosen in order to take into account the critical points; 10 points are chosen with constant intervals:

$$k_i = T_{init} + i \frac{T_{end} - T_{init}}{10}$$

Some intervals can be split if the procedure failed. This case occurs when the parametric variations are high.

- Step 4: 10 controllers are performed according to (Clement & Duc, 2000b). This synthesis consists in a multiobjective design based on LMI optimization. A first H_∞ synthesis is performed; the second step consists in the controller transformation into an observer based controller interconnected with a Youla parameter. This transformation requires some critical choices (which dynamics are from the observer and which ones are from the state feedback) and they are done to insure the continuity of the closed loop dynamics partition as shown on figure 5. Then the multiobjective synthesis is performed with the Youla parameter optimisation. Note that the synthesis structure is the same for all operating points

which insure that the evolutions of the controllers parameters stay smooth as indicated in the eigenvalues evolution.

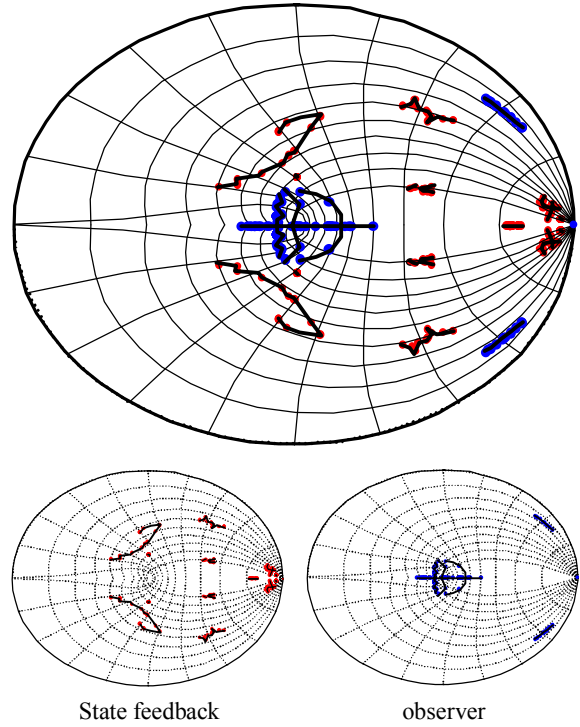


Figure 5. Closed loop eigenvalues map and affection

- Step 5: the main step of the procedure is to insure stability with linear interpolation (i.e. with a common X). For each interval and according to theorem 1, the instants are chosen as $a = b = k_i$ and $c = d = k_{i+1}$ which means that the interpolation is performed all along the trajectory. Then the feasibility LMI problem becomes simpler:

$$\begin{pmatrix} -X & (A(k_i) + B(k_i)K_i)X \\ X(A(k_i) + B(k_i)K_i)^T & -X_i \end{pmatrix} < -\gamma I$$

$$\begin{pmatrix} -X & (A(k_{i+1}) + B(k_{i+1})K_{i+1})X \\ X(A(k_{i+1}) + B(k_{i+1})K_{i+1})^T & -X \end{pmatrix} < -\gamma I$$

This LMI problem is then checked for every k between k_i and k_{i+1} to insure stability for every frozen system. This is possible because it is known that the parameters are slowly varying. Note that the same interpolation is performed with the observer gains.

- Steps 6 is useless for stability because it is guaranteed *a priori*. But the performances have to be checked; this is done in the next section with graphical interpretations.

This procedure success because of the parameters evolutions are smooth. Next paragraph shows the details of the final results.

4.2. Results

The efficiency of the method is considered with the frequency open loop responses of the frozen systems in a Black-Nichols chart (figure 6); all specifications (gain margins, first mode phase, roll-off) are almost satisfied. Time responses of the closed loop plant are given with a typical worst case wind profile (figure 7). The angle of attack i specification is almost satisfied (figure 8), the deflection angle β is far from the saturations (figure 10), and it is the same for the velocity of the deflection angle $d\beta/dt$ (figure 11). The consumption constraint is clearly respected as shown on figure 12. The attitude signal is given to show its stability (figure 9).

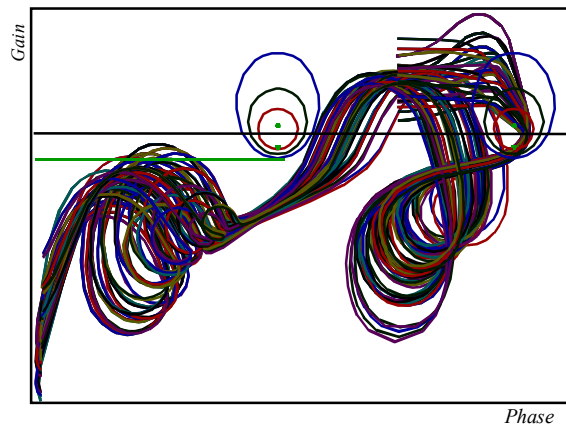


Figure 6. Nichols chart for every linearized point

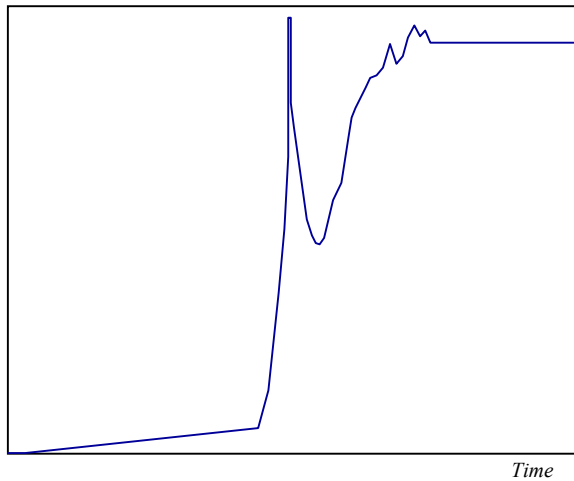


Figure 7. wind velocity profile

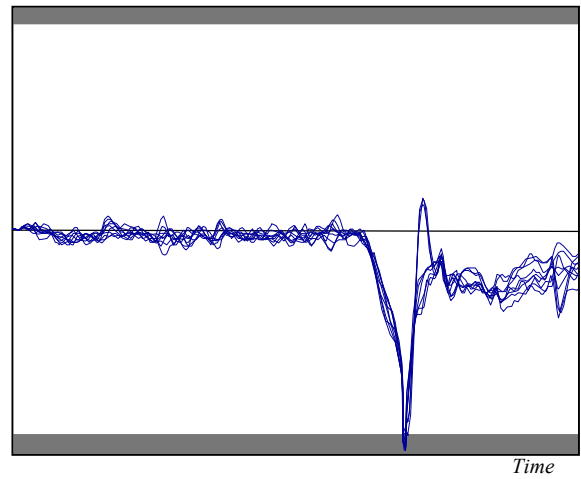


Figure 8. angle of attack i

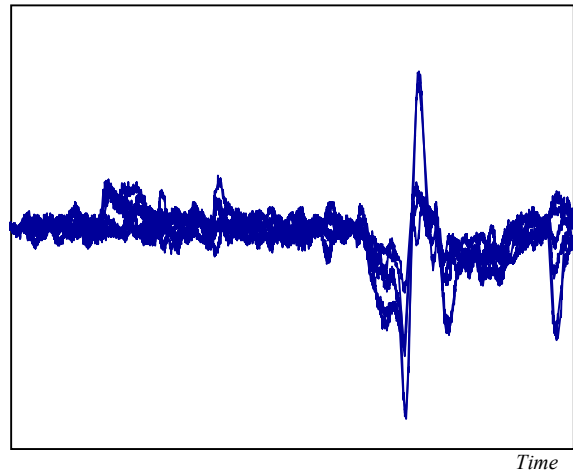


Figure 9. Attitude ψ

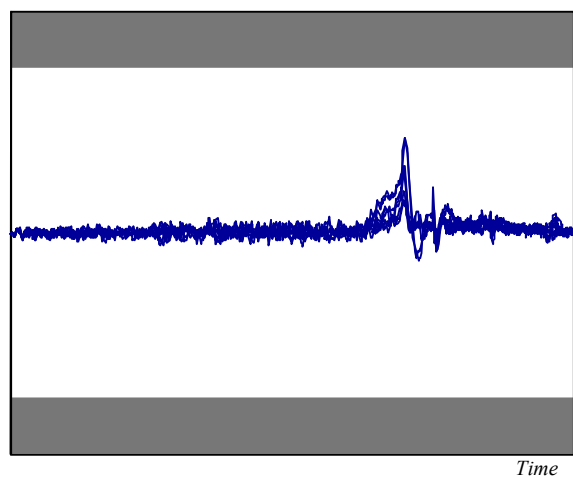


Figure 10. angle of deflection β (control)

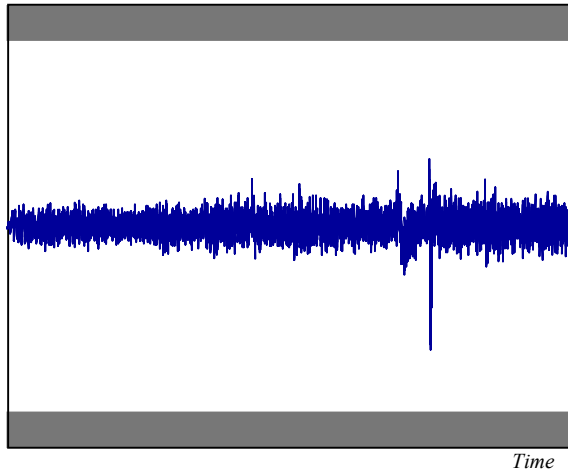


Figure 11. angle of deflection velocity $\dot{\beta}$

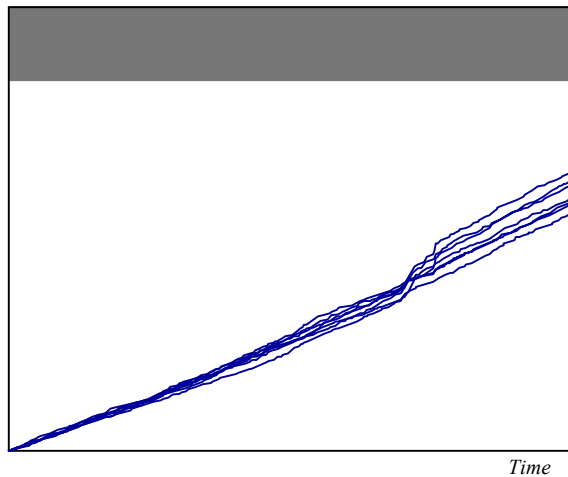


Figure 12. Consumption

5. CONCLUSION

An original methodology has been presented to control the atmospheric flight of an aerospace launch vehicle. It leads to a linear interpolation of the controller gains with a guaranteed stability along the trajectory. These results come from investigations about multiobjective design and gain scheduling which are largely discussed in the thesis (Clement 2001).

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