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ROBUST FAULT DETECTION AND ISOLATION IN CONSTRAINED NONLINEAR SYSTEMS VIA A SECOND ORDER SLIDING MODE OBSERVER

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Abstract: A robust fault detection and isolation (FDI) approach for uncertain constrained nonlinear systems (CNS) is outlined. The FDI scheme is based on a second order sliding mode observer (SOSMO). First, a closed-loop system is formed by incorporating the constraint terms into the system's dynamic equation and then the observer is designed based on the resulting closed loop system. Stability of the SOSMO resulting from the utilization of equivalent control concept is then proved by assuming that the considered uncertain CNS has a single output, and then two outputs, respectively. An attractive feature of the FDI methodology is that the diagnostic observer can directly supply the estimate of the faults, hence, making the fault isolation a simple task. Finally, an example is given to show the effectiveness of the proposed SOSMO based FDI strategy.

1. INTRODUCTION

The growing need for fault detection and isolation (FDI) in complex systems such as automotive, manufacturing, autonomous vehicles and robots, has generated a great deal of research studies in this area (Cho, 1990; Ge, 1988; Isermann, 1995; Schneider, 1993). Observer-based FDI methodology is one of the most commonly used FDI strategies. Amongst various techniques for designing observers one is based on the theory of variable structure systems (Sreedhar, 1993; Xiong, 2000a; Xiong, 2000b). Hermans, et al. (Hermans, 1996) introduce a sliding mode observer on the basis of transforming the considered systems into canonical forms. The effects of faults on the sliding mode observer is also investigated. Edwards et al. (Edwards, 2000) consider the application of a particular sliding mode observer to fault detection and isolation problems. The novelty of their work lies in the reconstruction of the fault signals by the equivalent injection concept. Equivalent control concept is also used in (Wang, 1997) to prove the convergence of the proposed sliding mode observer that is used to estimate the states of nonlinear systems. Sreedhar, et al., 1993 present the robust detection of a subset of sensor, actuator and process faults using sliding mode observers. The performance of sliding mode observer-based FDI technique was shown to be robust to parameter uncertainties in the system model.

In this paper, a SOSMO is constructed for a class of uncertain constrained nonlinear systems (CNS). Many practical systems can be modelled as constrained nonlinear systems. The motion of a mobile robot moving on a surface, for example, describes a typical constrained nonlinear system. The constraint here is the specified surface. The observer is motivated from the second order sliding mode control (Chang, 1991; Elmali, 1992). Second order sliding mode concept is employed since

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this kind of second order sliding surface dynamics can sharply filter unwanted high frequency caused by disturbances and uncertainties, which makes our sliding mode observer robust to disturbances.

2. PRELIMINARIES

Consider a class of uncertain constrained nonlinear dynamic systems (CNS) described by

$$\begin{aligned}
\dot{x} &= f(x) + \sum_{j=1}^{m} g_j(x)u_j(t) + \omega(x)d(t) + E(x)f_a(t) \\
z_i &= k_i(x) = 0, \qquad i = 1, \dots, l; \\
y_i &= h_i(x), \qquad\quad i = 1, \dots, p;
\end{aligned} \tag{1}$$

where $x \in U \subset R^n$, $f(x), g_i(x), k_i(x)$ and $h_i(x)$ are analytic functions. Signals $d(t) \in R^n$ and $f_a(t) \in R^\rho$ represent disturbance and actuator faults, respectively. For convenience, we denote $G(x) = [g_1(x), \dots, g_m(x)]$, $k(x) = [k_1(x), \dots, k_l(x)]^T$, $h(x) = [h_1(x), \dots, h_p(x)]^T$, and $u = [u_1, \dots, u_m]^T$. Suppose that for all $x \in U, g_1(x), \dots, g_m(x)$ are linearly independent vector fields, $[dk_1(x), \dots, dk_l(x)]$ and $[dh_1(x), \dots, dh_p(x)]$ are each linearly independent sets of co-vector fields and $m = l + p$. Finally, $\omega(x) \in R^{n\times n}$ and $E(x) \in R^{n\times\rho}$ are distribution matrices of disturbances and faults, and it is assumed that $E(x)$ is of full rank.

The following notations are used throughout the paper:

- The Lie Derivative of a scalar function $\phi(x)$ along a vector $f(x) = [f_1(x), \dots, f_n(x)]^T$ is $L_f\phi(x) = \frac{\partial\phi}{\partial x}f(x) = \sum_{i=1}^n \frac{\partial\phi}{\partial x_i}f_i(x)$ where $x = [x_1, \dots, x_n]^T$.
- The derivative of $\phi(x)$ taken first along $f(x)$ and then along a vector $g(x)$ is $L_gL_f\phi(x) = \frac{\partial(L_f\phi)}{\partial x}g(x)$.
- If $\phi(x)$ is differentiated j times along $f(x)$, the notation $L_f^j\phi(x)$ is used with $L_f^0\phi(x) = \phi(x)$.

Consider the following definitions (Li, 1995)

Definition 1. The constrained characteristic index r_i^ω of disturbances $d(t)$ is defined to be the least positive integer such that $L_{\omega_j}L_f^{r_i^\omega-1}k_i(x) \neq 0$ for some $x \in U \subset R^n$, $i = 1, \cdots, l, j = 1, \cdots n$.

Definition 2. The constrained characteristic index r_i^a of fault $f_a(t)$ is defined to be the least positive integer such that $L_{E_j}L_f^{r_i^a-1}k_i(x) \neq 0$ for some $x \in U \subset R^n$, $i = 1, \cdots, l, j = 1, \cdots \rho$.

Definition 3. The constrained characteristic r_i^c of system input $u(t)$ is defined to be the least positive integer such that $L_{g_j}L_f^{r_i^c-1}k_i(x) \neq 0$, $i = 1, \cdots, l, j = 1, \cdots m$.

The following assumptions with respect to system (1) are used throughout

A1) $h(x) = [h_1(x), \cdots, h_p(x)]^T$, and $k(x) = [k_1(x), \cdots, k_l(x)]^T$ are C^∞.

A2) The system input $u = [u_1, \cdots, u_m]$ is bounded.

A3) The constrained characteristic indices satisfy $r_i^\omega > r_i^c, r_i^a > r_i^c$.

A4) Matrices $\dfrac{\partial h}{\partial\hat{x}}L_1(t)$ and $\dfrac{\partial h}{\partial\hat{x}}L_2(t)$ are nonsingular. Here $L_1(t)$ and $L_2(t)$ are sliding mode observer gains to be determined later.

A5) Signals $d(t)$ and $f_a(t)$ are bounded with b_d and b_f, respectively.

Assumption A3) implies that the disturbance and fault don't effect the the derivative of constraint term, and this assures that the $d(t)$ and $f_a(t)$ terms do not appear in derivatives of k_i of order r_i^c.
Using assumption A3) and differentiating constraint term $k_i(x)$ as in (Chen, 1992), we get

$$\begin{aligned}
\frac{dz_i}{dt} &= L_fk_i(x) = 0 \\
&\ \vdots \\
\frac{d^{r_i^c-1}z_i}{dt^{r_i^c-1}} &= L_f^{r_i^c-1}k_i(x) = 0 \\
\frac{d^{r_i^c}z_i}{dt^{r_i^c}} &= L_f^{r_i^c}k_i(x) + \sum_{j=1}^{m} L_{g_j}L_f^{r_i^c-1}k_i(x)u_j = 0
\end{aligned} \tag{2}$$

i.e.

$$\begin{aligned}
k_i(x) &= L_fk_i(x) = \ldots = L_f^{r_i^c-1}k_i(x) = 0 \\
b(x) &+ A(x)u = 0
\end{aligned} \tag{3}$$

where $b(x) = [L_f^{r_1^c}k_1(x), \dots, L_f^{r_l^c}k_l(x)]^T$, $A(x) = [L_{g_j}L_f^{r_i^c-1}k_i(x)]_{l\times m}$. The solution of (3) can be written as a feedback law

$$u = -A^+(x)b(x) + (I - A^+(x)A(x))\bar{u}, \ \ \bar{u} \in R^m \tag{4}$$

where $A^+(x) = A^T(x)(A(x)A^T(x))^{-1}$ is the pseudo-inverse of $A(x)$, I is an identity matrix and $\bar{u}$ is a reference input.
Substituting the feedback law (4) into the uncertain CNS (1) a closed loop system is formed

$$\begin{aligned}
\dot{x} &= (f(x) - G(x)A^+(x)b(x)) + G(x)(I - A^+(x)A(x))\bar{u} \\
&\quad + \omega(x)d(t) + E(x)f_a(t)
\end{aligned} \tag{5}$$

The proposed SOSMO will be designed based on the above system.

Remark 1. In assumption A3), condition $r_i^a > r_i^c$ guarantees that faults would not appear in the feedback law (4). This allows incorporating the constraint term into the CNS system equation. In fact, this condition can be relaxed when the constrained characteristic index $r_i^c = 1$. Under this circumstance, feedback law (4) becomes

$$u = -A^+(x)(b(x) + \frac{\partial k}{\partial x}E(x)f_a) + (I - A^+(x)A(x))\bar{u} \tag{6}$$

which means that condition $r_i^a > r_i^c$ isn't needed.
Consequently, the closed-loop system equation (5) has a new form

$$\begin{aligned}
\dot{x} &= (f(x) - G(x)A^+(x)b(x)) + G(x)(I - A^+(x)A(x))\bar{u} \\
&\quad + \omega(x)d(t) + (I - G(x)A^+\frac{\partial k}{\partial x})E(x)f_a(t).
\end{aligned} \tag{7}$$

3. MAIN RESULTS

To achieve robust fault detection for the uncertain constrained nonlinear systems despite the existence of uncertainties and disturbances, consider a SOSMO (Chang, 1991; Elmali, 92):

$$
\begin{aligned}
\dot{\hat{x}} &= (f(\hat{x}) - G(\hat{x})A^{+}(\hat{x})b(\hat{x})) + G(\hat{x})(I - A^{+}(\hat{x})A(\hat{x}))\bar{u} \\
&\quad - L_1(t)(\frac{\partial h}{\partial \hat{x}}L_1(t))^{-1}v + L_2(t)(\frac{\partial h}{\partial \hat{x}}L_2(t))^{-1}D\hat{f}_a \\
\hat{y} &= h(\hat{x}) \\
v &= (D - \Gamma^T)\hat{f}_a - ce + z_0\dot{S} - wS - \kappa sgn(\dot{S}) \\
\dot{\hat{f}}_a &= \Gamma\dot{S}
\end{aligned}
\tag{8}
$$

where D and Γ are $p \times \rho$ and $\rho \times p$ matrices, and $L_1(t)$ and $L_2(t)$ are $n \times p$ observer gain matrices, respectively. w, c and z_0 are some constants, $\dot{S}(t) \in R^p$ is the sliding surface vector, sgn is a sign function and κ is a switching gain to be determined. The estimate of the fault is denoted by $\hat{f}_a$, v is the discontinuous term, and finally, $e(t) = y - \hat{y}$, is the output error. The output error dynamics can be obtained by differentiating (5) and (8)

$$
\begin{aligned}
\dot{e} &= (\frac{\partial h}{\partial x}f - \frac{\partial h}{\partial x}GA^{+}b) + \frac{\partial h}{\partial x}G(I - A^{+}A)\bar{u} + \frac{\partial h}{\partial x}\omega(x)d(t) \\
&\quad + \frac{\partial h}{\partial x}E(x)f_a(t) - (\frac{\partial h}{\partial \hat{x}}f(\hat{x}) - \frac{\partial h}{\partial \hat{x}}G(\hat{x})A^{+}(\hat{x})b(\hat{x})) \\
&\quad - \frac{\partial h}{\partial \hat{x}}G(\hat{x})(I - A^{+}(\hat{x})A(\hat{x}))\bar{u} + v - D\hat{f}_a \\
&= \Delta A + \frac{\partial h}{\partial x}\omega(x)d(t) + \frac{\partial h}{\partial x}E(x)f_a(t) + v - D\hat{f}_a
\end{aligned}
\tag{9}
$$

where

$$
\frac{\partial h}{\partial x} \triangleq \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \cdots & \frac{\partial h_1}{\partial x_n} \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \cdots & \frac{\partial h_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \\ \frac{\partial h_p}{\partial x_1} & \frac{\partial h_p}{\partial x_2} & \cdots & \frac{\partial h_p}{\partial x_n} \end{bmatrix}, \tag{10}
$$

$$
\begin{aligned}
\Delta A &= (\frac{\partial h}{\partial x}f - \frac{\partial h}{\partial x}GA^{+}b) + \frac{\partial h}{\partial x}G(I - A^{+}A)\bar{u} \\
&\quad - (\frac{\partial h}{\partial \hat{x}}f(\hat{x}) - \frac{\partial h}{\partial \hat{x}}G(\hat{x})A^{+}(\hat{x})b(\hat{x})) \\
&\quad - \frac{\partial h}{\partial \hat{x}}G(\hat{x})(I - A^{+}(\hat{x})A(\hat{x}))\bar{u}
\end{aligned}
\tag{11}
$$

The relationship between sliding surface dynamics and output error can be taken as:

$$\ddot{S} + z_0\dot{S} = \dot{e} + ce \tag{12}$$

where $S = col[s_1, s_2, \cdots, s_p]$, the sliding surface. Substituting (9) into (12),

$$
\begin{aligned}
&\ddot{S} + z_0\dot{S} = \dot{e} + ce \qquad (13) \\
&= \Delta A + \frac{\partial h}{\partial x}\omega(x)d(t) + \frac{\partial h}{\partial x}E(x)f_a(t) + v - D\hat{f}_a + ce.
\end{aligned}
$$

Now a Lyapunov function candidate can be chosen to create the attractivity condition as follows:

$$V = \frac{1}{2}(\dot{S}^T\dot{S} + S^T\Omega S) + \frac{1}{2}\hat{f}_a^{\,T}\hat{f}_a,\ \Omega = Diag(w). \tag{14}$$

The time derivative of Lyapunov function is

$$\dot{V} = \dot{S}^T(\ddot{S} + wS) + \dot{\hat{f}}_a^{\,T}\hat{f}_a. \tag{15}$$

To ensure attractivity condition for the sliding mode, the following inequality should hold:

$$\dot{S}^T(\ddot{S} + wS) + \dot{\hat{f}}_a^{\,T}\hat{f}_a \leq 0. \tag{16}$$

Substituting equation (13) into above equation, we have:

$$
\begin{aligned}
&\dot{S}^T[\Delta A + \frac{\partial h}{\partial x}\omega(x)d(t) + \frac{\partial h}{\partial x}E(x)f_a(t) + v - D\hat{f}_a \\
&+ ce - z_0\dot{S} + wS + \Gamma^T\hat{f}_a] \leq 0.
\end{aligned}
\tag{17}
$$

Substituting discontinuous term v from equation (8) into equation(17) yields:

$$\dot{S}^T[\Delta A + \frac{\partial h}{\partial x}\omega(x)d(t) + \frac{\partial h}{\partial x}E(x)f_a(t) - \kappa sgn(\dot{S})] \leq 0. \tag{18}$$

Noticing that $\dot{S}^T sgn(\dot{S}) \geq \|\dot{S}\|$ and consequently $-\kappa\dot{S}^T sgn(\dot{S}) \leq -\kappa\|\dot{S}\|$, therefore using this inequality, equation (18) can be further derived by using vector norms

$$
\begin{aligned}
\dot{V} &\leq \|\dot{S}\|(\|\Delta A\| + \|\frac{\partial h}{\partial x}\omega(x)\|\|d(t)\| \\
&\quad + \|\frac{\partial h}{\partial x}E(x)\|\|f_a(t)\| - \kappa) \leq 0
\end{aligned}
\tag{19}
$$

therefore, if

$$\kappa \geq \|\Delta A\| + \|\frac{\partial h}{\partial x}\omega(x)\|b_d + \|\frac{\partial h}{\partial x}E(x)\|b_f, \tag{20}$$

then, $\dot{V} \leq 0$, which means that output error is kept sliding on the sliding surface if switching gain κ is selected according to equation (20).

As we can see from equation (13) that $v(t)$ is discontinuous across $\dot{S} = 0$, the sliding surface $\dot{S}$ accordingly is discontinuous, which leads chattering. If we use $\dot{S}$ as a residual, the chattering is not desirable. To smooth out the discontinuity, a boundary layer ϕ neighboring the switching surface $\dot{S}$ is introduced (Chang, 1991). A Saturation function (sat) is used to replace signum function, the saturation function is defined as

$$sat(\frac{\dot{S}}{\phi}) = \left[sat(\frac{\dot{s}_1}{\phi}), sat(\frac{\dot{s}_2}{\phi})\cdots, sat(\frac{\dot{s}_p}{\phi})\right]^T \tag{21}$$

and

$$sat(\frac{\dot{s}_i}{\phi}) = \begin{cases} sign(\frac{\dot{s}_i}{\phi}), & \text{when } |\dot{s}_i| \geq \phi \\ \frac{\dot{s}_i}{\phi}, & \text{when } |\dot{s}_i| < \phi \end{cases} \tag{22}$$

The S dynamics outside the boundary layer can be obtained by substituting v into equation (13) as

$$
\begin{aligned}
\ddot{S} + wS + \kappa sign(\dot{S}) &= \Delta A + \frac{\partial h}{\partial x}\omega(x)d(t) \\
&\quad + \frac{\partial h}{\partial x}E(x)f_a(t) - \Gamma^T\hat{f}_a
\end{aligned}
\tag{23}
$$

Within the boundary layer, the S dynamics have the form of

$$\ddot{S} + wS + \kappa\frac{\dot{S}}{\phi} = \Delta A + \frac{\partial h}{\partial x}\omega(x)d(t) + \frac{\partial h}{\partial x}E(x)f_a(t) - \Gamma^T\hat{f}_a. \tag{24}$$

Equation (24) represents a low-pass filter (Chang,1991; Chirlian,1994). So, as long as the fault is not a high-frequency signal, it will have impact on sliding surface $\dot{S}$. Therefore, sliding surface $\dot{S}$ can be selected as a residual.

Remark 2. The proposed observer maintains sliding motion even in the presence of faults, which is quite different from the approaches of (Ge,1988;Hermans, 1996; Sreedhar, 1993), in which fault detection is based on the requirement that sliding motion should cease to exist once a fault occurs. When disturbances or uncertainties are present in the system, it is in general difficult to appropriately design the gain of these observers to accomplish their intended task.

Remark 3. At present, there does not exist a systematic methodology to design the parameters w, c and z_0 (Elmali, 92) and additional research in this direction is desirable.

Remark 4. Based on equation (20), one can observe that the selection of switching gain κ is only related to some norm bounds. Therefore, one can claim that if these norm bounds are known, then κ can be chosen easily.

3.1 *Stability Analysis of the Second-Order Sliding Mode Observer with A Single Output*

In this subsection, we assume that $y = Cx(t)$, where C is a constant row vector. Without loss of generality, we will assume $y = x_1(t)$.

We start this stability analysis with expanding the sliding observer (8) according to each observer state variable

$$\begin{aligned}
\dot{\hat{x}}_1 &= f_1(\hat{x}) - k_{11}(t)v + k_{12}D\hat{f}_a \\
\dot{\hat{x}}_2 &= f_2(\hat{x}) - k_{21}(t)v + k_{22}D\hat{f}_a \\
&\vdots \\
\dot{\hat{x}}_n &= f_n(\hat{x}) - k_{n1}(t)v + k_{n2}D\hat{f}_a
\end{aligned} \tag{25}$$

where $[k_{11}(t), k_{21}(t), \cdots, k_{n1}(t)]^T = L_1(t)[\frac{\partial h}{\partial \hat{x}}L_1(t)]^{-1}$,
$[k_{12}(t), k_{22}(t), \cdots, k_{n2}(t)]^T = L_2(t)[\frac{\partial h}{\partial \hat{x}}L_2(t)]^{-1}$,
v is the discontinuous term in the sliding mode observer and

$$\begin{aligned}
[f_1(\hat{x}), f_2(\hat{x}), \cdots, f_n(\hat{x})]^T &= (f(\hat{x}) - G(\hat{x})A^+(\hat{x})b(\hat{x})) \\
&\quad + G(\hat{x})(I - A^+(\hat{x})A(\hat{x}))\bar{u}
\end{aligned} \tag{26}$$

Let $\tilde{x}_i = x_i - \hat{x}_i, i = 1, \cdots, n$, subtracting (25) from (5),

$$\begin{aligned}
\dot{\tilde{x}}_1 &= \Delta f_1 + \omega_1 d(t) + E_1 f_a + k_{11}(t)v - k_{12}D\hat{f}_a \\
\dot{\tilde{x}}_2 &= \Delta f_2 + \omega_2 d(t) + E_2 f_a + k_{21}(t)v - k_{22}D\hat{f}_a \\
&\vdots \\
\dot{\tilde{x}}_n &= \Delta f_n + \omega_n d(t) + E_n f_a + k_{n1}(t)v - k_{n2}D\hat{f}_a
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
&[\Delta f_1, \Delta f_2, \cdots \Delta f_n]^T \\
&= (f(x) - G(x)A^+(x)b(x)) - (f(\hat{x}) - G(\hat{x})A^+(\hat{x})b(\hat{x})) \\
&+ G(x)(I - A^+(x)A(x))\bar{u} - G(\hat{x})(I - A^+(\hat{x})A(\hat{x}))\bar{u},
\end{aligned} \tag{28}$$

and ω_i and E_i are ith rows of ω and E, respectively.

Lemma 1. The coefficients $k_{11}(t)$ and k_{12} in error dynamics equation (27) are equal to 1.

The proof is omitted because of the space limitation. Recall that inequality (20) guarantees that the output estimation error will reach the sliding surface and once there keeps sliding on it, i.e. output estimation error $\tilde{x}_1$ is zero on this surface. Applying the concept of equivalent dynamics in accordance with (Utkin, 1992; Wang, 1997), we have the reduced sliding mode observer error dynamics in the form of

$$\begin{aligned}
\dot{\tilde{x}}_2 &= \Delta f_2 + \omega_2 d(t) + E_2 f_a - \frac{l_2^1}{l_1^1}(\Delta f_1 + \omega_1 d(t) + E_1 f_a) \\
&\quad + (\frac{l_2^1}{l_1^1} - \frac{l_2^2}{l_1^2})D\hat{f}_a \\
\dot{\tilde{x}}_3 &= \Delta f_3 + \omega_3 d(t) + E_3 f_a - \frac{l_3^1}{l_1^1}(\Delta f_1 + \omega_1 d(t) + E_1 f_a) \\
&\quad + (\frac{l_3^1}{l_1^1} - \frac{l_3^2}{l_1^2})D\hat{f}_a \\
&\vdots \\
\dot{\tilde{x}}_n &= \Delta f_n + \omega_n d(t) + E_n f_a - \frac{l_n^1}{l_1^1}(\Delta f_1 + \omega_1 d(t) + E_1 f_a) \\
&\quad + (\frac{l_n^1}{l_1^1} - \frac{l_n^2}{l_1^2})D\hat{f}_a.
\end{aligned} \tag{29}$$

By expanding $\Delta f_1, \Delta f_2, \cdots, \Delta f_n$ into power series, we get the following differential form of the above equation:

$$\begin{aligned}
\dot{\tilde{x}}_2 &= [\frac{\partial f_2}{\partial x_2} - \frac{l_2^1}{l_1^1}\frac{\partial f_1}{\partial x_2}]\tilde{x}_2 + \cdots + [\frac{\partial f_2}{\partial x_n} - \frac{l_2^1}{l_1^1}\frac{\partial f_1}{\partial x_n}]\tilde{x}_n + \phi_2 \\
&\quad + (\frac{l_2^1}{l_1^1} - \frac{l_2^2}{l_1^2})D\hat{f}_a + ((\omega_2 - \frac{l_2^1}{l_1^1}\omega_1)d(t) + (E_2 - \frac{l_2^1}{l_1^1}E_1)f_a) \\
\dot{\tilde{x}}_3 &= [\frac{\partial f_3}{\partial x_2} - \frac{l_3^1}{l_1^1}\frac{\partial f_1}{\partial x_2}]\tilde{x}_2 + \cdots + [\frac{\partial f_3}{\partial x_n} - \frac{l_3^1}{l_1^1}\frac{\partial f_1}{\partial x_n}]\tilde{x}_n + \phi_3 \\
&\quad + (\frac{l_3^1}{l_1^1} - \frac{l_3^2}{l_1^2})D\hat{f}_a + ((\omega_3 - \frac{l_3^1}{l_1^1}\omega_1)d(t) + (E_3 - \frac{l_3^1}{l_1^1}E_1)f_a) \\
&\vdots \\
\dot{\tilde{x}}_n &= [\frac{\partial f_n}{\partial x_2} - \frac{l_n^1}{l_1^1}\frac{\partial f_1}{\partial x_2}]\tilde{x}_2 + \cdots + [\frac{\partial f_n}{\partial x_n} - \frac{l_n^1}{l_1^1}\frac{\partial f_1}{\partial x_n}]\tilde{x}_n + \phi_n \\
&\quad + (\frac{l_n^1}{l_1^1} - \frac{l_n^2}{l_1^2})D\hat{f}_a + ((\omega_n - \frac{l_n^1}{l_1^1}\omega_1)d(t) + (E_n - \frac{l_n^1}{l_1^1}E_1)f_a)
\end{aligned} \tag{30}$$

where $\phi_i, i = 2, \cdots, n$, are the terms of second and higher orders in $(x_i - \hat{x}_i)$. Let $\tilde{x} = [\tilde{x}_2, \tilde{x}_3, \cdots, \tilde{x}_n]^T$, we have:

$$\dot{\tilde{x}} = A(t)\tilde{x} + \Phi + \delta F + \Lambda D\hat{f}_a \tag{31}$$

where

$$\delta F = \begin{bmatrix} (\omega_2 - \frac{l_2^1}{l_1^1}\omega_1)d(t) + (E_2 - \frac{l_2^1}{l_1^1}E_1)f_a \\ (\omega_3 - \frac{l_3^1}{l_1^1}\omega_1)d(t) + (E_3 - \frac{l_3^1}{l_1^1}E_1)f_a \\ \vdots \\ (\omega_n - \frac{l_n^1}{l_1^1}\omega_1)d(t) + (E_n - \frac{l_n^1}{l_1^1}E_1)f_a \end{bmatrix}, \ \Lambda = \begin{bmatrix} \frac{l_2^1}{l_1^1} - \frac{l_2^2}{l_1^2} \\ \frac{l_3^1}{l_1^1} - \frac{l_3^2}{l_1^2} \\ \vdots \\ \frac{l_n^1}{l_1^1} - \frac{l_n^2}{l_1^2} \end{bmatrix}$$

$$A(t) = \begin{bmatrix} \frac{\partial f_2}{\partial x_2} - \frac{l_2^1}{l_1^1}\frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} - \frac{l_2^1}{l_1^1}\frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_3}{\partial x_2} - \frac{l_3^1}{l_1^1}\frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_3}{\partial x_n} - \frac{l_3^1}{l_1^1}\frac{\partial f_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_2} - \frac{l_n^1}{l_1^1}\frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} - \frac{l_n^1}{l_1^1}\frac{\partial f_1}{\partial x_n} \end{bmatrix}. \tag{32}$$

Let us choose gains $l_i^1(t), i = 1, \cdots, n$ such that matrix $A(t)$ is Hurwitz and there exists a positive define symmetric matrix $P(t)$ such that

$$P(t)A(t) + A^T(t)P(t) + \dot{P}(t) = -Q \tag{33}$$

where Q is a positive define matrix.
Consider now a Lyapunov function candidate of the form

$$V = \tilde{x}^T P \tilde{x}. \tag{34}$$

Differentiating V, we get:

$$\begin{aligned}\dot{V} &= \dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}} + \tilde{x}^T \dot{P}\tilde{x} = \tilde{x}^T(A^T P + PA + \dot{P})\tilde{x} \\ &\quad + \Phi^T P\tilde{x} + \delta F^T P\tilde{x} + \tilde{x}^T P\Phi + \tilde{x}^T P\delta F + \tilde{x}^T P\Lambda D\hat{f}_a \\ &\quad + \hat{f}_a{}^T D^T \Lambda^T P\tilde{x}.\end{aligned} \tag{35}$$

Considering (33), the above can be further extended as

$$\begin{aligned}\dot{V} &\le -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2\gamma_\phi\|P\|\|\tilde{x}\|^2 + 2\gamma_{\delta F}\|P\|\|\tilde{x}\| \\ &\quad + 2\|\tilde{x}\|\|P\Lambda D\|\|\hat{f}_a\| \\ &\le ((-\lambda_{min}(Q) + 2\gamma_\phi\lambda_{max}(P))\|\tilde{x}\| + 2\gamma_{\delta F}\lambda_{max}(P) \\ &\quad + 2\lambda_{max}(P)\|\Lambda D\|\|\hat{f}_a\|)\|\tilde{x}\|.\end{aligned} \tag{36}$$

where λ_{min} and λ_{max} are minimum and maximum eigenvalues, respectively, $\gamma_{\delta F}$ is the norm bound of δF. In the derivation process above, inequality $\|\Phi\| \le \gamma_\phi\|\tilde{x}\|$ is used. Therefore, if inequality

$$\|\tilde{x}\| \ge \frac{2\gamma_{\delta F}\lambda_{max}(P) + 2\lambda_{max}(P)\|\Lambda D\|\|\hat{f}_a\|}{(\lambda_{min}(Q) - 2\gamma_\phi\lambda_{max}(P))}$$

holds, then $\dot{V} \le 0$, which means that the reduced SOSMO is stable. In a similar fashion to (Wang, 1997), the observer's gain $L_1(t)$ can be directly calculated.
The following theorem summarizes the results presented above.

Theorem 1. Consider constrained uncertain nonlinear system (5) with a single output and its SOSMO defined in equation (8), if inequality (20) and equation (33) hold, then the proposed SOSMO is stable.

Remark 5. As a result, the coefficient of v in the first error equation of equation (25) is 1 , which is due to the special structure of the proposed observer $L_1(t)[\frac{\partial h}{\partial \hat{x}}L_1(t)]^{-1}$. From this special structure, we can also conclude that if $y = x_i(t), i = 1, \cdots, n$, then the coefficient of v in the i-th error equation would be 1.

3.2 *Stability Analysis of the Second-Order Sliding Mode Observer with Multiple Outputs.*

A similar stability analysis for the multiple output case is omitted due to the space limitation.

3.3 *FDI Strategy*

In this paper, faults are directly estimated in the sliding mode observer. As soon as any of the components of the estimate of faults is greater than zero, then the alarm for corresponding fault component will be activated. The following algorithm summarizes the FDI process by sliding mode observer.

Step 1: Impose constraint into system equation by differentiating constraint term $k(x)$ under assumption A3) to form a closed-loop system.

Step 2: Construct a sliding mode observer for this closed-loop system.

- **step 2.1** Select switching gain κ from equation (20) under assumptions A4) and A5).
- **step 2.2** Choose gain matrices $L_1(t)$ and $L_2(t)$, $L_1(t)$ must make $A(t)$ or $B(t)$ stable, then solve equation (33) to get P or Π. If equation (3.1) holds, then go to step 3, otherwise, reselect gain matrices $L_1(t), L_2(t)$ and Q or R to solve P or Π

Step 3: Monitor the estimated fault vector to detect and isolate faults.

4. AN ILLUSTRATIVE EXAMPLE

In this section, we shall illustrate the proposed FDI strategy on a simple nonlinear system. As compared to the SOSMO, a standard SMO will also be presented for comparison purposes. Consider the system

$$\begin{aligned}\dot{x} &= \begin{bmatrix} -x_1x_2^2 \\ x_3 \\ x_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u + \begin{bmatrix} x_3 d_1(t) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_a^1 \\ f_a^2 \end{bmatrix} \\ y &= x_1 + x_3 \\ k &= x_2 + x_3 - const. = 0\end{aligned} \tag{37}$$

where *const.* is a constant, and the disturbance $d_1(t)$ is a random function. Obviously, the constrained characteristic index r^c=1, after calculation of $b(x)$ and $A(x)$, we form a feedback law as in the equation (6)

$$u = -\begin{pmatrix} \frac{1}{2}(x_1 + x_3) \\ \frac{1}{2}(x_1 + x_3) \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} f_a^2 \tag{38}$$

Substituting equation (38) into system equation (37), we obtain the closed-loop system with constraint incorporated into it

$$\dot{x} = \begin{bmatrix} -x_1x_2^2 \\ \frac{1}{2}x_3 - \frac{1}{2}x_1 + \frac{1}{2}\bar{u}_1 - \frac{1}{2}\bar{u}_2 \\ -\frac{1}{2}x_3 + \frac{1}{2}x_1 - \frac{1}{2}\bar{u}_1 + \frac{1}{2}\bar{u}_2 \end{bmatrix} + \begin{bmatrix} x_3 d_1(t) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -f_a^1 \\ \frac{1}{2}f_a^2 \\ -\frac{1}{2}f_a^2 \end{bmatrix} \tag{39}$$

Based on equation (39), the SOSMO is constructed as follows:

$$\begin{aligned}\dot{\hat{x}} &= \begin{bmatrix} -\hat{x}_1\hat{x}_2^2 \\ \frac{1}{2}\hat{x}_3 - \frac{1}{2}\hat{x}_1 + \frac{1}{2}\bar{u}_1 - \frac{1}{2}\bar{u}_2 \\ -\frac{1}{2}\hat{x}_3 + \frac{1}{2}\hat{x}_1 - \frac{1}{2}\bar{u}_1 + \frac{1}{2}\bar{u}_2 \end{bmatrix} - L_1(t)(\frac{\partial h}{\partial \hat{x}}L_1(t))^{-1}v \\ &\quad + L_2(t)(\frac{\partial h}{\partial \hat{x}}L_2(t))^{-1}D\hat{f}_a \\ \hat{y} &= \hat{x}_1 + \hat{x}_3 \\ v &= (D - \Gamma^T)\hat{f}_a - ce + z_0\dot{S} - wS - \kappa sat(\dot{S})\end{aligned} \tag{40}$$

The sliding mode observer will detect and estimate the actuator fault when f_a^1 occurs. Output estimation error or sliding surface dynamics $\dot{S}(t)$ can be taken as a residual. When a fault occurs, $\dot{S}(t)$ will deviate from zero and recover to zero because the switching gain κ has been properly selected. The simulation results are shown in Figure 1. Note that either the residual or the estimated fault signal could be used for fault detection

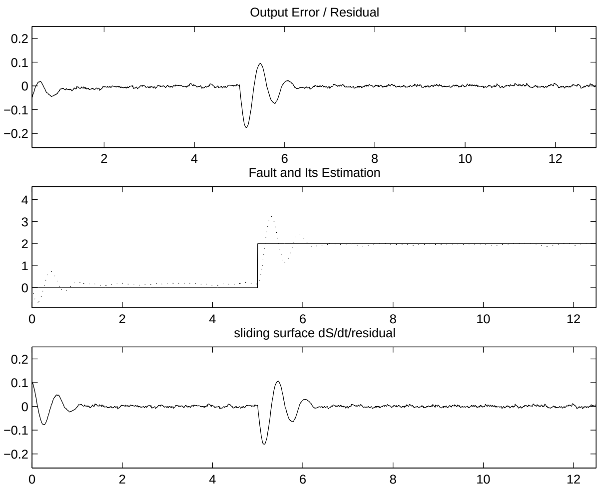

Fig. 1. Fault detection and estimate by SOSMO.

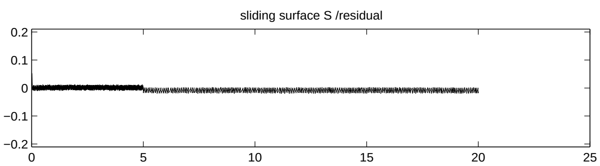

Fig. 2. Residual by First order sliding mode observer .

purposes.Finally, a first order sliding mode observer is constructed according to (Ali,1999; Wang,1997)

$$\dot{\hat{x}} = \begin{bmatrix} -\hat{x}_1\hat{x}_2^2 \\ \frac{1}{2}\hat{x}_3 - \frac{1}{2}\hat{x}_1 + \frac{1}{2}\bar{u}_1 - \frac{1}{2}\bar{u}_2 \\ -\frac{1}{2}\hat{x}_3 + \frac{1}{2}\hat{x}_1 - \frac{1}{2}\bar{u}_1 + \frac{1}{2}\bar{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 sign(y-\hat{y}) \\ k_2 sign(y-\hat{y}) \\ k_3 sign(y-\hat{y}) \end{bmatrix} \quad (41)$$

Note here that when the sliding surface is selected as a residual, small faults can't be efficiently detected due to the chattering as shown in Figure 2.

5. CONCLUSIONS

This paper explored use of a SOSMO scheme for fault detection and isolation in uncertain constrained nonlinear systems. It is demonstrated that while being robust to uncertainties and disturbances the SOSMO can be used to detect and estimate actuator faults with certain benefits over the standard SMOs. The ability to estimate the faults directly is very desirable for fault detection, especially isolation, and perhaps accommodation purposes.

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