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OPTIMAL ACCOMMODATION OF FAULTS IN SENSORS AND ACTUATORS

Giacomo Tortora [*,1] **Basil Kouvaritakis** [*]
David W. Clarke [*]

[*] *Department of Engineering Science, University of Oxford, Oxford OX1 3PJ, UK*

Abstract: A fault tolerant control scheme that accommodates sensor and actuator faults while maintaining optimal performance with respect to reference tracking is devised. Optimal fault-accommodation is achieved by solving a norm minimization problem which has been extensively studied in the literature. Here attention is focused on multiple-input single-output (MISO) systems. It is shown that, if the interest lies in the fault-tolerance of the output only, then the optimal tracking and fault-accommodation problems are totally decoupled. Motivated by the need to update in real-time the controller parameters as faults develop, an alternative scheme is presented which trades-off some optimality for computational simplicity. *Copyright ©2002 IFAC*

Keywords: Fault tolerance, intelligent instrumentation, sensor failures, actuators, multiobjective optimisation.

1. INTRODUCTION

Fault-tolerant control (FTC) is a relatively new field of research addressing the issue of designing controllers that are able to ensure the safe and efficient operation of the controlled system despite the occurrence of faults (Blanke *et al.*, 2000).

Typically, fault-tolerance is achieved by implementing an automatic fault-detection and diagnosis scheme followed by a fault-accommodation or controller-reconfiguration algorithm. Tortora *et al.* (2002) and Tortora *et al.* (2001) present a different approach to fault-tolerance and apply it to the case of sensor-fault accommodation in SIMO systems where the control aim is to minimize the effect on a primary output of faults occurring on all available sensors while maintaining optimal performance in respect of reference tracking. This approach is based on the assumption that intelligent instrumentation (such as SEVA sensors) is deployed. These devices return to the user not only a working/faulty flag, but also a measure of the reliability of the instrumentation which can therefore be constantly and easily monitored. In Tortora *et al.* (2001) it is assumed that the sensor reading is corrupted at all times by additive noise, the size of the bounds on the noise giving a measure of the performance of the sensor. It is then possible to model sensor faults as an increase in the size of the bounds of the noise superimposed on the measurement signal, leading to a fault-accommodation algorithm that minimizes the effect on the controlled output of deterioration in sensor performance. The sensitivity of the controlled output to sensor faults is measured by the induced l_∞ norm (i.e. the l_1 norm) of the transfer function from noise to output. Moreover, the class of controllers that give optimal tracking performance (measured by an l_2 norm) is defined so that restricting the choice of controllers to this class allows to accommodate faults without affecting the optimal tracking properties of the closed loop. In

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Tortora *et al.* (2001) it is proved that the two optimizations of tracking and fault-accommodation performance are completely decoupled. This is a very desirable property, meaning that both optima can be obtained simultaneously.

The current work uses the same $l_1 - l_2$ framework but extends it to consider the effect of both sensor and actuator faults on both the input and output variables. Similarly to sensor faults, actuator faults are modelled as an increase in an input disturbance. Again the disturbance is assumed to be additive and bounded. The systems under consideration are MISO, so that actuator redundancy is used to accommodate faults. The optimal tracking problem is set and solved in §2. The optimal fault-accommodation problem is solved in §3. It is furthermore established that, if we are only interested in the fault-tolerance of the output, the decoupling property of Tortora *et al.* (2001) can be extended to the case where fault-tolerance performance is measured with respect to both sensor and actuator faults. Moreover, the simultaneous optimization of reference tracking and output fault-tolerance still allows degrees of freedom in the controller. These can then be deployed to minimize input fault-tolerance. In §4 a computationally inexpensive and transparent (albeit suboptimal) alternative method of achieving fault-tolerance is presented. Numerical examples in §5 show that the suboptimality of this alternative method can be small.

Throughout this paper a discrete time system is assumed.The following convention will be used:

- no distinction between transfer functions and polynomials;
- lower case (l.c.) = scalar or scalar ransfer function;
- l.c. underlined = vector of scalars;
- upper case (u.c). = matrix;
- bold l.c = vector of transfer functions;
- bold u.c = for matrix of transfer functions;
- $\cdot_i = i^{th}$ element of a vector;
- $\cdot_{i,j} = i, j^{th}$ element of a matrix;
- $\cdot^{i \to j}$ = transfer function from i to j;
- $\tilde{(\cdot)}$ denotes t-domain;
- C_a denotes the the Toeplitz convolution matrix of the polynomial $a(z^{-1})$. This a lower triangular matrix where the i, j^{th} element is given by the coefficient of the $(i-j)^{th}$ power of z^{-1} in the polynomial $a(z^{-1})$;
- $||.||_1$ = induced l_∞ -norm for TFs or ratios of z-transforms and l_1-norms for t-domain.

2. THE TRACKING PROBLEM

Consider a MISO system with output y and l inputs denoted by the vector $\underline{u}$. The system is subjected to measurement noise (x) and input disturbances ($\underline{d}$) representing respectively sensor and actuator faults. The problem is to find a controller that minimizes the cost J_{track} defined as:

$$J_{\text{track}} = ||\tilde{r} - \tilde{y}||_2^2 + \sum_{i=1}^{l} \lambda_i ||\tilde{u}_i||_2^2, \qquad (1)$$
$$\text{assuming } x = 0,\ \underline{d} = \underline{0},$$

where $\tilde{u}_i$ denotes the i^{th} input and $||.||_2$ denotes the l_2 norm. Let $\mathbf{b}^T$, a be the plant zero and pole polynomial respectively. Standard l_2 theory can be used to find the controller parameters $\mathbf{h}$, m, $\mathbf{n}$ that, deployed in the loop configuration of figure 1, minimize J_{track}. The closed-loop transfer

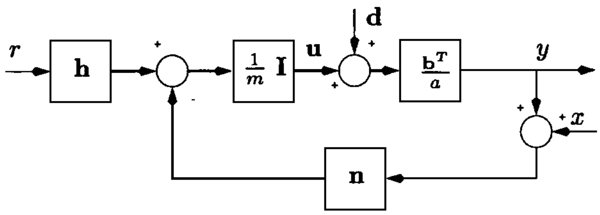

Fig. 1. Loop block diagram.

function from reference to output is given by:

$$g^{r \to y} = \frac{\mathbf{b}^T \mathbf{h}}{am + \mathbf{b}^T \mathbf{n}}. \qquad (2)$$

Let p denote the optimal closed loop pole polynomial:

$$p = am + \mathbf{b}^T \mathbf{n},\ p(\infty) = a(\infty). \qquad (3)$$

Considering eqn.2 it is apparent that the optimal J_{track} can be achieved by any m, $\mathbf{n}$ satisfying eqn.3. The general class of controllers satisfying eqn.3 can be obtained by simple transposition of the result in Tortora *et al.* (2002) which for brevity is given here without proof.

Lemma 2.1. (Tortora *et al.*, 2002). The whole class of stabilising m and $\mathbf{n}$ that minimizes the tracking cost of (2) is given as:

$$\begin{bmatrix} m \\ \mathbf{n} \end{bmatrix} = \begin{bmatrix} m_o \\ \mathbf{n}_o \end{bmatrix} + \begin{bmatrix} \mathbf{e}^T \\ \mathbf{E} \end{bmatrix} \mathbf{q},\ \mathbf{q} \in l_1 \qquad (4)$$

where:

$$am_o + \mathbf{b}^T \mathbf{n}_o = p,\ \mathbf{e} = -\mathbf{b},\ \mathbf{E} = -(aI + \mathbf{K}_b \mathbf{E}_1) \qquad (5)$$

and $\mathbf{K}_b$ is a matrix representation of the kernel of $\mathbf{b}^T$, while $\mathbf{E}_1$ is such that the zeros of $\mathbf{det}\,(aI + \mathbf{K}_b \mathbf{E}_1)$ are all within the unit circle.

3. OPTIMAL REFERENCE TRACKING WITH FAULT-ACCOMMODATION

3.1 *Optimization of mixed output and input fault tolerance*

As explained in the introduction, sensor and actuator faults are modelled as an increase in the bounds of (respectively) the measurement noise and the input disturbances. The reliability of the sensor and of the actuators can then be expressed by the scalar s^x and the vector $\underline{s}^d$ defined as:

$$s^x = \|\tilde{x}\|_\infty, \qquad \underline{s}^d = \left[\|\tilde{d}_1\|_\infty \cdots \|\tilde{d}_l\|_\infty\right]. \quad (6)$$

Fault-tolerance of the inputs and output is measured by the ∞-norms of $\tilde{\underline{u}}$, $\tilde{y}$ for the worst case over the whole class of instrumentation faults (i.e. the whole class of $\tilde{x}$, $\tilde{\underline{d}}$ satisfying eqn. 6). The indices of fault-tolerance J_{y} and J_{u} of the output and inputs are then defined as:

$$J_{\mathrm{y}} = \|\tilde{y}^*\|_\infty, \; J_{\mathrm{u}} = \|\tilde{\underline{u}}^*\|_\infty \text{ (assuming r = 0)}, \quad (7)$$

where superscript $.^*$ is used to denote the worst case over the whole class of faults. Assuming that the measures of sensor and actuator performance s^x and $\underline{s}^d$ are known, J_{y} and J_{u} can be easily computed using the l_1 norms of the transfer functions from x and $\underline{d}$ to y and $\underline{u}$:

$$J_{\mathrm{y}} = \sum_i \left(\|\mathbf{g}_i^{d\to y}\|_1 \, \underline{s}_i^d\right) + \|g^{x\to y}\|_1 \, s^x, \quad (8)$$

$$J_{\mathrm{u}} = \max_j \left[\sum_i \left(\|\mathbf{G}_{j,i}^{\underline{d}\to\underline{u}}\|_1 \, \underline{s}_i^d\right) + \|\mathbf{g}_j^{x\to\underline{u}}\|_1 \, s^x\right] \quad (9)$$

For the loop configuration of Figure 1 these transfer functions are given by:

$$\mathbf{g}^{\underline{d}\to y} = \frac{m\mathbf{b}^T}{p}, \quad \mathbf{G}^{\underline{d}\to\underline{u}} = -\frac{\mathbf{n}\mathbf{b}^T}{p}, \quad (10a)$$

$$g^{x\to y} = -\frac{\mathbf{b}^T\mathbf{n}}{p}, \quad \mathbf{g}^{x\to\underline{u}} = -\frac{a\mathbf{n}}{p}. \quad (10b)$$

Fault-tolerance can in general be measured by a "mixed fault-tolerance" index J_{mft} defined as:

$$J_{\mathrm{mft}} = J_{\mathrm{y}} + \sigma J_{\mathrm{u}} \quad (11)$$

where σ is a constant. Mixed fault-tolerance can then be maximised subject to optimal tracking performance by minimizing J_{mft} over the class of eqn.4 using eqns.8,9,10. In general this will define a "bad-rank" l_1 minimization problem (McDonald and Pearson, 1991) so that the optimal $\mathbf{q}$ minimizing J_{mft} will be a vector of transfer functions. This creates numerical problems since the minimization of J_{mft} is given by an LP with an infinite number of variables and equality constraints. However, a variety of methods to get around this problem and obtain arbitrarily good approximations to the optimal have been devised. The most obvious approach is to optimize over a vector of polynomials $\mathbf{q}$ and to truncate the transfer functions giving the equality constraints. As n_q (the number of terms in each of the polynomial elements) tends to infinity then the solution to the LP will tend to the optimal J_{mft}. This is the Q-design method (Boyd and Barratt, 1991) an the resulting LP will not be discussed here. Recent research has concentrated on other methods of solving l_1 optimization problems which provide converging upper and lower bounds to the optimal solution (Khammash, 1997; Elia and Dahleh, 1998).

3.2 *Optimization of output fault-tolerance only*

An important special case arises when $J_{\mathrm{mft}} = J_{\mathrm{y}}$. Optimal reference tracking with fault-accommodatic then displays some important and desirable properties. The transfer functions affecting J_{y} are $\mathbf{g}^{d\to y}$ and $g^{x\to y}$. Using eqn.3 we can rewrite:

$$g^{x\to y} = -\left(1 - \frac{am}{p}\right), \quad (12)$$

so that the optimal fault accommodation problem is given by:

$$\min_{m,\mathbf{n}} J_{\mathrm{y}} \quad (13a)$$

$$\text{subject to: } am + \mathbf{b}^T\mathbf{n} = p, \quad m, \, \mathbf{n} \in l_1, \quad (13b)$$

where:

$$J_{\mathrm{y}} = \sum_i \left(\|\frac{\mathbf{b}_i m}{p}\|_1 \underline{s}_i^d\right) + \|1 - \frac{am}{p}\|_1 s^x. \quad (14)$$

Note that J_{y} of eqn.14 is a function of m only (a scalar transfer function) but we also have $\mathbf{n}$ appearing in condition 13b. However, $\mathbf{n}$ can be eliminated by noting that, given an $m \in l_1$, condition 13b is equivalent to requiring that the Bezout identity $\mathbf{b}^T\mathbf{n} = p - am$ has as a solution some $\mathbf{n} \in l_1$. This is equivalent to requiring that $p - am$ has as zeros the non-minimum phase zeros of $\mathbf{b}^T$ (where the zeros of the vector of polynomials $\mathbf{b}^T$ are defined as the roots of the GCD of all its elements). Note that $\mathbf{b}^T$ will have at least one non-minimum phase zero arising from a unit time delay. Assume that there exist n_β such zeros and denote them by $\beta_1, \beta_2, \ldots, \beta_{n_\beta}$. Then the fault accommodation problem can be rewritten as:

$$\min_m \left[\sum_i \left(\|\frac{\mathbf{b}_i m}{p}\|_1 \underline{s}_i^d\right) + \|1 - \frac{am}{p}\|_1 s^x\right] \quad (15a)$$

$$\text{subject to: } m \in l_1, \; m(\beta_i) = \frac{p(\beta_i)}{a(\beta_i)} \, \forall \, \beta_i. \quad (15b)$$

Let

$$m = pm' \quad (16)$$

where m' will in general be a transfer function. Then problem 15 becomes:

$$\min_{m'} \left[\sum_i \left(\|\mathbf{b}_i m'\|_1 \underline{s}_i^d \right) + \|1 - am'\|_1 s^x \right] \quad (17a)$$

$$\text{subject to: } m' \in l_1, \; m'(\beta_i) = \frac{1}{a(\beta_i)} \; \forall \, \beta_i. \quad (17b)$$

Condition 17b, together with eqns.3 and 16 define an alternative controller parametrization for the minimization of J_{y} subject to optimal J_{track}. Instead of minimizing over the vector of transfer functions $\mathbf{q}$ we minimize over the scalar transfer function m' subject to (17b) and recover m from eqn.16 and $\mathbf{n}$ as a solution to the Bezout identity $\mathbf{b}^T\mathbf{n} = p - am$. The solution to the latter is non-unique but, as explained earlier, this has no bearing in the minimization of J_{y}.

Moreover, note that p and $\mathbf{h}$ which minimize J_{track} (see eqns.2, 3) do not appear in problem 17. This implies that it is possible to simultaneously optimize both J_{track} and J_{y}. That is, the pair m_{opt}, $\mathbf{n}_{\mathbf{opt}}$ minimizing J_{y} subject to the optimal J_{track} actually attain the minimum J_{y} over the whole class of stabilizing controllers. This fact is summarized in the theorem below.

Theorem 3.1. Let m_{opt} be the minimizer of problem 15 and $\mathbf{n}_{opt}$ be such that:

$$\mathbf{n}_{opt} \in l_1 \qquad am_{opt} + \mathbf{b}^T\mathbf{n}_{opt} = p. \quad (18)$$

Then the pair m_{opt}, $\mathbf{n}_{opt}$ not only minimizes J_{track} but also achieves the minimum J_{y} over the whole class of stabilizing controllers.

In general m_{opt} and $\mathbf{n}_{opt}$ will not be polynomial. However, an arbitrarily good approximation to the optimal can be obtained by taking m' to be a polynomial of arbitrarily high order. Let $\underline{\gamma}$ be a vector of length n_γ, V_β be the $n_\gamma \times n_\beta$ Vandermonde matrix for the set $\{\beta_i\}$, and

$$\underline{v}_\beta = \begin{bmatrix} 1/a(\beta_1) \\ \vdots \\ 1/a(\beta_{n_\beta}) \end{bmatrix}.$$

Moreover, let n_a be the number of elements of the polynomial a, and C_{b_i}, C_a denote the first n_γ columns and $n_\gamma + n_a - 1$ rows of the Toeplitz convolution matrix for $\mathbf{b}_i$ and a respectively (see convention in §1). Then Problem 17 can be solved within any desired accuracy using the following LP:

$$\mu_{opt} = \min_{\underline{\gamma}, \underline{\tau}_+, \underline{\tau}_-} \begin{bmatrix} 0^T & s_V^T & s_V^T \end{bmatrix} \begin{bmatrix} \underline{\gamma} \\ \underline{\tau}_+ \\ \underline{\tau}_- \end{bmatrix} \quad (19a)$$

subject to:

$$\begin{bmatrix} V_\beta^T & \underline{0}^T & \underline{0}^T \\ C_{b_1} & & \\ \vdots & -I & I \\ C_{b_l} & & \\ -C_a & & \end{bmatrix} \begin{bmatrix} \underline{\gamma} \\ \underline{\tau}_+ \\ \underline{\tau}_- \end{bmatrix} = \begin{bmatrix} \underline{v}_\beta \\ \underline{0} \\ \vdots \\ \underline{0} \\ -1 \\ \underline{0} \end{bmatrix}, \quad \underline{\tau}_+, \underline{\tau}_- \geq 0, \quad (19b)$$

where:

$$\underline{s}_V^T = \left[s_1^d \, \underline{1}^T \; \dots \; s_l^d \, \underline{1}^T \; s^x \, \underline{1}^T \right]. \quad (19c)$$

As $n_\gamma \to \infty$, μ_{opt} tends to the solution of Problem 17 and the minimizer γ_{opt} tends to the vector of the coefficients of m'_{opt} such that $m_{opt} = pm'_{opt}$. The computational advantage of using the LP of (19) over the equivalent LP using truncated $\mathbf{q}$ is two-fold. First of all for a certain order of the closed loop polynomials (given by $\underline{\tau}_+ - \underline{\tau}_-$) there are fewer variables since we are effectively minimizing over the polynomial m' rather than the vector of polynomials $\mathbf{q}$. Moreover, the equality constraints defined by the Toeplitz matrices are finite and need not be truncated like they would be if we used truncated $\mathbf{q}$. The reduction in the number of equality constraints and the fact that they are exact makes the LP of (19) faster and more robust to numerical errors.

3.3 *Minimizing J_u subject to optimal J_{track} and optimal J_y*

The solution for the controller minimizing simultaneously J_{y} and J_{track} is not unique since given an m_{opt} there is a class of $\mathbf{n}_{opt}$ satisfying the Bezout identity of (18). This class is easily obtainable as:

$$\mathbf{n}_{opt} = \mathbf{n}_p + \mathbf{K}_b\mathbf{q}_p, \; am_{opt} + \mathbf{b}^T\mathbf{n}_p = p, \; \mathbf{n}_p, \mathbf{q}_p \in l_1 \quad (20)$$

where $\mathbf{K}_b$ has already been defined in Lemma 2.1 as the kernel of $\mathbf{b}^T$. Moreover from eqns.9 and 10 it is clear that J_{u} depends on $\mathbf{n}$. Having minimized J_{y} and J_{track} it would therefore be sensible to use the remaining degrees of freedom in the choice of $\mathbf{n}$ by minimizing J_{u} over the class of (20). Again this leads to a "bad-rank" l_1 problem that can be solved by any of the methods quoted in §3.1. Using the Q-design method to minimize J_{u} leads to an LP with truncated constraints. Numerical examples show that the minimum J_{u} achievable is affected by the minimization of J_{track} and J_{y}. Therefore performing the successive minimization of the three indices J_{track}, J_{y},J_{u} gives the global minimum for the first two but not for the third. On the other hand, minimizing J_{y} and then J_{u} in succession is computationally more efficient than minimizing a linear combination of them.

4. FAULT ACCOMMODATION: REDUCING THE COMPUTATIONAL BURDEN

The fault-accommodation algorithm works by monitoring the indices of sensor and actuator performance s^x and $\underline{s}^d$. Each time these change the optimal controller parameters need to be re-computed. Therefore the LP's optimizing fault-tolerance need to be performed on-line, as the faults develop. The computational cost of the fault-accommodation algorithm then becomes of paramount importance. As explained in §3 one way to reduce the computational burden is to minimize J_{y} and J_{u} in succession. Moreover, the optimization can be made simpler by reducing the numbers of terms in the controllers. However both these measures are likely to yield heavily sub-optimal solutions.

An alternative is to constrain the polynomial vector $\mathbf{q}$ to be given by a mix of n_ρ stable pre-defined polynomial vectors $\mathbf{q}_i$ such that:

$$\mathbf{q} = \sum_{i=1}^{n_\rho} \rho_i \mathbf{q}_i \qquad \mathbf{q}_i \in l_1, \tag{21}$$

and defining the controller $m, \mathbf{n}$ from the parametrization of eqn.4. Fault-tolerance can then be achieved by optimizing J_{mft} over the coefficients ρ_i rather than the polynomial vector $\mathbf{q}$. For transparency and efficiency purposes it would be desirable to deal optimally with the complete failure of any actuator or of the sensor by setting one of the variables ρ_i equal to 1 and all the others equal to zero. This can be done by choosing the polynomial vectors $\mathbf{q}_i$ as the solutions to the optimal fault-accommodation problem (§3) for s^x and for each one of the elements of $\underline{s}^d$ going to infinity in turn. Different criteria for the choice of the set of $\mathbf{q}_i$ can of course be used, so that optimality is recovered for a variety of fault scenarios. This is illustrated through an example in §5. Using more functions $\mathbf{q}_i$ would obviously improve fault-tolerance as well as the number of variables of the on-line optimization, leading to a trade-off between optimality and computation time. Note that the $\mathbf{q}_i$ can be computed off-line.

The foremost advantage of the "mixed" scheme proposed in this section compared to the optimal methods of §3 is that, for a given order of controller, we have fewer variables in the optimization and hence a faster algorithm. For the minimization over all stabilizing controllers to have a comparable computational cost we would need to keep the controller order much lower, leading to suboptimality for all operating conditions. On the other hand, the scheme proposed in this section can provide optimal controllers in the case of complete failure of the sensor or of one of the actuators, while being far more transparent to the operator than the schemes of §3.

5. NUMERICAL EXAMPLES

In this section the benefits of fault-accommodation will be illustrated using numerical examples. The plant under consideration has two inputs and one output. The open loop pole and zero polynomials are taken to be:

$$a = (1 - 1.5z^{-1})(1 - 0.99z^{-1})(1 + 1.1z^{-1}) \tag{22}$$

$$\mathbf{b} = \begin{bmatrix} 1.5z^{-1}(1 - 1.3z^{-1})(1 + 0.5z^{-2}) \\ 2z^{-1}(1 - 1.2z^{-1})(1 + 0.6z^{-2}) \end{bmatrix}. \tag{23}$$

For simplicity throughout this section we assume that $J_{\mathrm{mft}} = J_{\mathrm{y}}$.

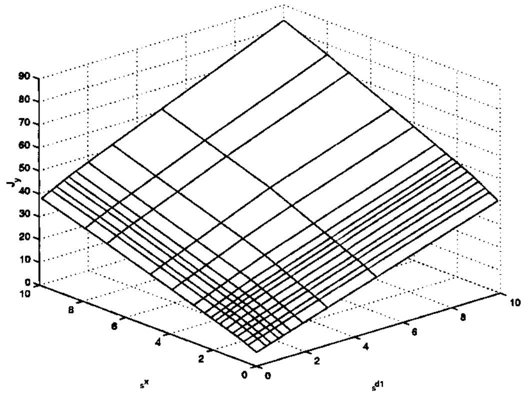

Fig. 2. Variation of J_{y}, optimal fault-accommodation.

Figure 2 shows the variation of J_{y} as fault-accommodation is performed using the optimal LP of (19). The values shown are for $\underline{s}_2^d = 1$.

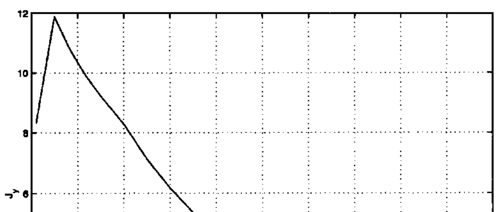

Fig. 3. Faulty actuator 1. Comparing % suboptimality of sets 1 (continuous line) and 2 (dashed line).

The plots of figure 3 show the suboptimality of the "mixed" strategy (§4) as the first actuator fails, namely as $\underline{s}_1^d$ increases and $\underline{s}_2^d$, s^x are held fixed at 1. In order to show the flexibility of this strategy the results obtained using two different sets of $\mathbf{q}_i$ are displayed. The continuous line is given by using "set 1" and the dashed line by using "set 2". These are defined in Tables 1 and 2 respectively.

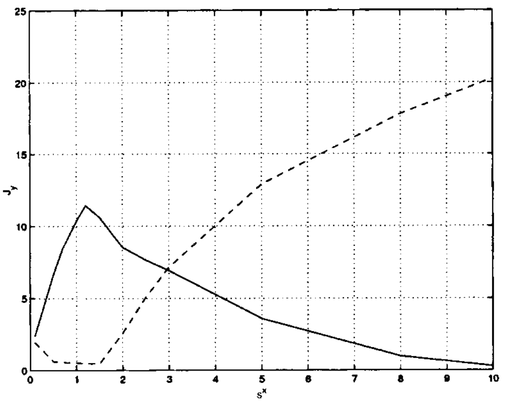

Fig. 4. Faulty sensor . Comparing % suboptimality of sets 1 (continuous line) and 2 (dashed line).

Table 1. Set 1

	Optimal solution for	$\underline{s}^d$	s^x
$\mathbf{q}_1$	Faulty act. 1	$[10\ 0.1]$	0.1
$\mathbf{q}_2$	Faulty act. 2	$[0.1\ 10]$	0.1
$\mathbf{q}_3$	Faulty sensor	$[0.1\ 0.1]$	10

Table 2. Set 2

	Optimal solution for	$\underline{s}^d$	s^x
$\mathbf{q}_1$	Faulty act. 1	$[10\ 0.1]$	0.1
$\mathbf{q}_2$	Faulty act. 2	$[0.1\ 10]$	0.1
$\mathbf{q}_3$	All working equally well	$[1\ 1]$	1

As expected both sets are optimal as $\underline{s}_1^d \to \infty$ (of course the same property will pertain with $\underline{s}_2^d$). Using set 1 will allow to recover the optimal controller in the case of complete failure of one of the actuators or of the sensor. Set 2 is unable to deal efficiently with sensor faults but on the other hand is also optimal when all instrumentation is working equally well. This is shown in figure 4 where the suboptimalities of the two sets are compared for s^x varying and $\underline{s}_1^d = \underline{s}_2^d = 1$ (sensor fault scenario).

6. CONCLUSIONS

This paper has treated the problem of accommodating sensor and actuator faults. The faults are modelled as an increase in the measurement noises and input disturbances respectively. The bounds on these are assumed to be known thanks to the use of intelligent instrumentation.

The control design objective is to accommodate optimally faults as they develop, in real-time while maintaining optimal performance with respect to reference tracking (measured by the l_2 cost index J_{track}). l_1 norms are used to define a suitable measure of fault-tolerance (J_{mft}) incorporating the effect that faults have on both the output (J_{y}) and the inputs (J_{u}). It is shown that this problem can be solved using norm minimization methods which are extensively treated in the literature. Moreover, it is shown that the minimizations of J_{track} and J_{y} alone have the property of being independent of each other, i.e. that there exist controllers that simultaneously achieve the optimal J_{y} and the optimal J_{track}. Also, this optimization can be performed using a controller parametrization alternative to the Youla, leading to a simpler algorithm. Furthermore, the optimal controller is not unique so that the remaining degrees of freedom can be used to minimize J_{u}.

The optimal controller parameters have to be computed on-line as faults develop prompting the need for a computationally inexpensive fault-accommodation algorithm. This motivates the alternative fault-accommodation scheme described in §4. This scheme reduces the on-line computational burden and still yields optimal reference tracking while optimally accommodating faults in given fault scenarios. The reduction in on-line computation is achieved at the cost of some suboptimality in fault-tolerance for other reliability levels of the instruments. This alternative scheme has the added advantage of great transparency to the user since it can deal with the failure of a single instrument by setting one coefficient equal to 1 and all the others equal to 0. Finally, the numerical examples of §5 suggest that the degree of suboptimality can be rather small.

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