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AUTONOMOUS CONTROLLER DESIGN USING GENETIC ALGORITHMS IN A TWO-INERTIA MOTOR SYSTEM

Gloria Suh, Jung-il Park

School of Electrical Engineering and Computer Science, Yeungnam University, 214-1 Daedong, Gyongsan, Kyungbuk, 712-749, Korea, email : jipark@yu.ac.kr

Abstract: In a two-inertia motor system with flexible shaft, a torsional vibration is often generated, as a quick speed response close to the primary resonant frequency is required. This vibration makes it difficult to achieve a quick response of speed and disturbance rejection. This paper provides an autonomous pole assignment technique for three kinds of speed controllers (I-P, I-PD, and State feedback) using GAs(Genetic Algorithms) for a two-inertia motor system. Firstly, the optimal parameters are chosen using GAs in view of reducing overshoot and settling time, then those are used in computing the gains of each controller. Some simulation results verify the effectiveness of the proposed design. The proposed controller is expected to be the standard for controlling a two-inertia motor system with flexible shaft. *Copyright © 2002 IFAC*

Keywords : two-inertia motor system, pole assignment controller design, genetic algorithms

1. INTRODUCTION

A two-inertia motor system, such as an industrial rolling machine with a flexible shaft, has very low natural resonant frequency because of the long shaft and low stiffness between the motor and load. This makes it difficult to achieve the precise speed control due to torsional vibration. Hence, many engineers and scientists have focused attention on the reducing oscillation and the settling time in a two-inertia motor system. For example, a speed control using a PI or PID controller without an observer to estimate load torque was developed (Zhang and Furusho, 2000). The Kalman filter and LQ-based speed controller for torsional vibration suppression was also developed (Zi and Sul, 1995). Vibration suppression, which used feedback from the imperfect derivative of the estimated torsion torque, was also studied (Sugiura and Hori, 1996). The auto-tuning of controller and observer parameters of a 2-DOF control system using genetic algorithms was developed (Ito, *et al.,* 2001).

In the authors' previous work (Park, *et al.,* 2001), the systematic analysis and speed controller design technique for a two-inertia motor system was described. Also included was a description of how to assign closed-loop poles of three controllers (I-P, I-PD and State feedback) by using the new weighted ITAE(Integral of Time multiplied by the Absolute Error) performance index put a weight on overshoot, considering the fact that the overshoot easily causes vibration in a two-inertia motor system. However, numerous trials were necessary in order to choose the optimal parameters of a pole assignment controller. In order to overcome this problem, the auto-tuning technique of controller gains using genetic algorithms is presented in this paper. Some simulation results verify the effectiveness of the proposed design.

2. TWO-INERTIA MOTOR SYSTEM

In this section, we describe a model of a two-inertia motor system and the derivation of optimal controller gains by utilising a pole assigning technique for three kinds of controllers. The design criterion of each controller is to reduce the property that produces overshoot and oscillation by using a weighted ITAE performance index.

2.1 Model of two-inertia motor system

A motor and load coupled by a shaft with a finite stiffness is shown in Fig. 1, in which

J_M motor inertia; ω_M motor speed;
T_M motor torque; T_L load torque;
K_{sh} torsion stiffness of the drive shaft;
J_L load inertia; ω_L load speed;
T_{sh} shaft torque; θ_M motor angle;
θ_L load angle;

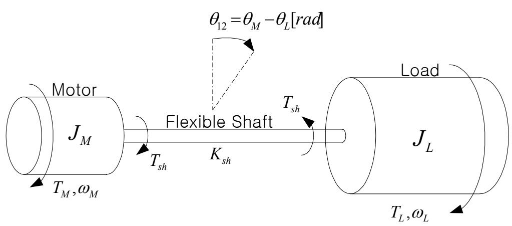


Fig. 1. Two-inertia motor system model coupled by flexible shaft

Figure 2 is a simple block diagram representation of a two-inertia motor system. The friction term, which does not effect analysis accuracy, is neglected.

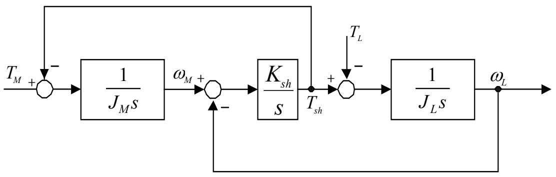


Fig. 2. Block diagram of a two-inertia system

The transfer function from T_M to ω_M in Fig. 2 can be calculated as follows:

$$T(s) = \frac{s^2 + \omega_a^2}{J_M s(s^2 + \omega_0^2)} \tag{1}$$

where ω_a and ω_0 represent the anti-resonant frequency and the resonant frequency, respectively. The inertia ratio of load to motor, K , and the resonance ratio, R , are defined as follows:

$$\omega_0 = \omega_a\sqrt{1+K}\ ,\ K = \frac{J_L}{J_M}\ ,\ \omega_a = \sqrt{\frac{K_{sh}}{J_L}}\ ,\ R = \frac{\omega_0}{\omega_a} = \sqrt{1+K} \tag{2}$$

2.2 Analysis and design of controllers for a two-inertia motor system

Figures 3 through 5 represent the structure of each speed control system using I-P, I-PD, and state feedback controllers, respectively. The closed-loop transfer function for I-P controller of Fig. 3 is given by

$$\frac{\omega_L}{\omega_r} = \frac{K_I\omega_a^2}{J_M s^2(s^2 + \frac{K_P}{J_M}s + \omega_0^2) + K_P\omega_a^2 s + K_I(s^2 + \omega_a^2)} \tag{3}$$

The closed-loop transfer function for the system shown in Fig. 4 using an I-PD controller is obtained as follows:

$$\frac{\omega_L}{\omega_r} = \frac{K_I\omega_a^2}{\tilde{J}_M s^2(s^2 + \frac{K_P}{\tilde{J}_M}s + \tilde{\omega}_0^2) + K_P\omega_a^2 s + K_I(s^2 + \omega_a^2)} \tag{4}$$

where $\tilde{J}_M = J_M + K_D$, $\tilde{\omega}_0 = \omega_a\sqrt{1+\tilde{K}}$, $\tilde{K} = \frac{J_L}{\tilde{J}_M}$ (5)

The closed loop transfer function for a state feedback controller as shown in Fig. 5 is given by

$$\frac{\omega_L}{\omega_r} = \frac{K_I\omega_a^2}{J_M s^2(s^2 + \frac{K_1}{J_M}s + \hat{\omega}_0^2) + (K_1 + K_2)\omega_a^2 s + K_I(s^2 + \omega_a^2)} \tag{6}$$

where $\hat{\omega}_0 = \sqrt{\omega_0^2 + \frac{K_3}{J_M}}$ (7)

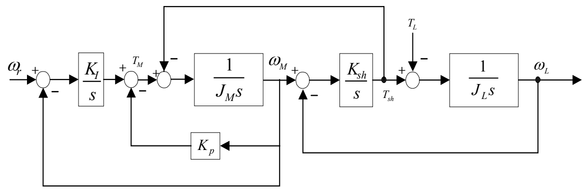


Fig. 3. Speed control system with I-P controller

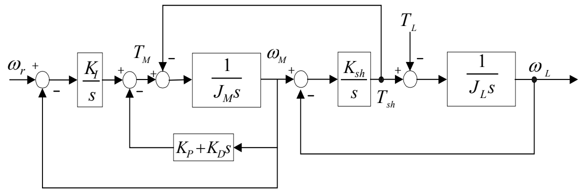


Fig. 4. Speed control system with I-PD controller

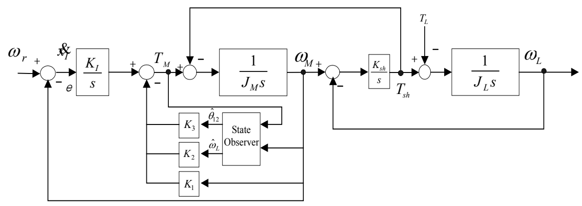


Fig. 5. Speed controller system with state feedback controller

The weighted ITAE performance index, which is used in this paper, is given by

IF $e(t) > 0$ THEN $I = \int_0^{\tau} te(t)dt$

ELSE $I = \int_0^{\tau} t|e(t)|^{0.7}dt$

END IF (8)

where $e(t) = \omega_r - \omega_M$.

A controller designed by using this weighted ITAE index reduces the overshoot or oscillation because the closed-loop system has a large damping property by weighting for overshoot. This technique assists us in selecting optimal location of poles without oscillation. Also, the minimum values of this ITAE index can easily be derived compared to those of the conventional ITAE index, which does not have 0.7 in an exponent (Park, *et al.*, 2001).

The closed loop transfer function can be arranged as follows:

$$\frac{\omega_L}{\omega_r} = \frac{\omega_1^2 \omega_2^2}{(s^2 + 2\varsigma_1 \omega_1 s + \omega_1^2)(s^2 + 2\varsigma_2 \omega_2 s + \omega_2^2)} \tag{9}$$

where ω_i and ς_i (for i=1,2) are the natural frequency and the damping ratio, respectively. Comparing (9) and each closed-loop transfer function, the gains of each controller and relation equation are obtained as follows:

I-P speed controller

$$K_P = 2(\varsigma_1 \omega_1 + \varsigma_2 \omega_2) J_M \tag{10}$$

$$K_I = \frac{\omega_1^2 \omega_2^2}{\omega_a^2} J_M \tag{11}$$

$$\omega_a^2(\omega_1^2 + \omega_2^2 + 4\varsigma_1\varsigma_2\omega_1\omega_2) - \omega_1^2\omega_2^2 = \omega_a^4(K+1) = \omega_a^2\omega_0^2 \tag{12}$$

$$\omega_1\varsigma_1(\omega_2^2 - \omega_a^2) = \omega_2\varsigma_2(\omega_a^2 - \omega_1^2) \tag{13}$$

I-PD speed controller

$$K_P = 2(\varsigma_1 \omega_1 + \varsigma_2 \omega_2) \tilde{J}_M \tag{14}$$

$$K_I = \frac{\omega_1^2 \omega_2^2}{\omega_a^2} \tilde{J}_M \tag{15}$$

$$K_D = \frac{\omega_a^4 J_L}{\omega_a^2(\omega_1^2 + \omega_2^2 + 4\varsigma_1\varsigma_2\omega_1\omega_2) - \omega_1^2\omega_2^2 - \omega_a^4} - J_M \tag{16}$$

$$\omega_1\varsigma_1(\omega_2^2 - \omega_a^2) = \omega_2\varsigma_2(\omega_a^2 - \omega_1^2) \tag{17}$$

$$\tilde{\omega}_0^2 = \omega_a^2(1 + \tilde{K}) = \omega_1^2 + \omega_2^2 + 4\varsigma_1\varsigma_2\omega_1\omega_2 - \frac{\omega_1^2\omega_2^2}{\omega_a^2} \tag{18}$$

State feedback speed controller with integral

$$K_1 = 2(\varsigma_1 \omega_1 + \varsigma_2 \omega_2) J_M \tag{19}$$

$$K_I = \frac{\omega_1^2 \omega_2^2}{\omega_a^2} J_M \tag{20}$$

$$K_2 = \frac{2J_M}{\omega_a^2}(\omega_1\varsigma_1(\omega_2^2 - \omega_a^2) - \omega_2\varsigma_2(\omega_a^2 - \omega_1^2)) \tag{21}$$

$$K_3 = J_M(\omega_1^2 + \omega_2^2 + 4\varsigma_1\varsigma_2\omega_1\omega_2 - \frac{\omega_1^2\omega_2^2}{\omega_a^2} - \omega_0^2) \tag{22}$$

$$\hat{\omega}_0^2 = \omega_1^2 + \omega_2^2 + 4\varsigma_1\varsigma_2\omega_1\omega_2 - \frac{\omega_1^2\omega_2^2}{\omega_a^2} \tag{23}$$

If we choose K_2 as a positive constant and $\omega_2 \rangle \omega_1$, $\varsigma_1\omega_1 = \varsigma_2\omega_2$, then the relation between ω_1 and ω_2 in a closed-loop system is given by

$$\omega_2 = \sqrt{2\omega_a^2\left(1 + \frac{K_2}{K_1}\right) - \omega_1^2} > \sqrt{2\omega_a^2 - \omega_1^2} \tag{24}$$

3. CONTROLLER DESIGN USING GENETIC ALGORITHMS

In order to find a minimum ITAE index value as described in the authors' previous work (Park, *et al.,* 2001), the ITAE calculation for many cases must be done. To overcome this calculation burden, we are introducing an autonomous method which can be used to find optimal parameters of a controller using genetic algorithms.

In this paper, poles are assigned to have identical real part as shown in (25) that gives optimal performance in terms of the settling time of transient response (Zhang and Furusho, 2000). The condition for these poles is given by

$$-\omega_1\varsigma_1 = -\omega_2\varsigma_2 \tag{25}$$

3.1 Outline of the controller design

In the proposed autonomous design, two individuals, that is, ς_1 and ω_1 / ω_a, are optimised by using genetic algorithms. These are selected at random at first, then vary with values between 0.6 and 1.0 according to the genetic operation. In the genetic operation, an inverse of the ITAE value is evaluated as the fitness value, where the higher fitness results in the better solution. The best solution at each generation is successively reflected in the controller gains.

The overall sequence of steps needed to choose optimal parameters using genetic algorithms is shown in Fig. 6.

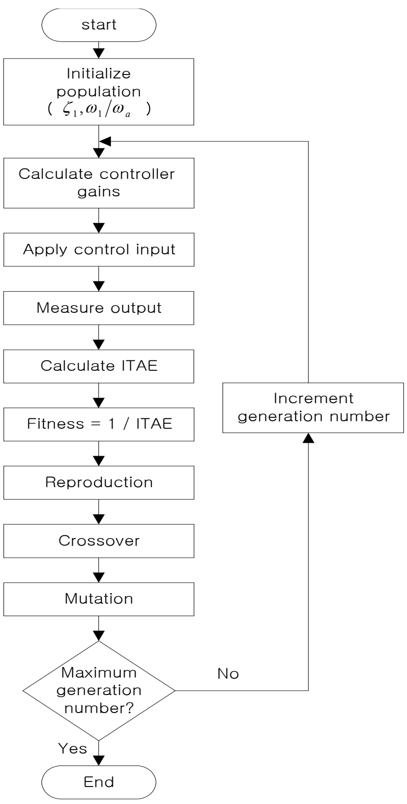


Fig. 6 The sequence of steps needed to choose optimal parameters

The parameters of genetic algorithms used are shown in Table 1.

Table 1 Parameters of genetic algorithms

Item	Condition
Population size	8
Number of individuals	2
Crossover probability	0.9
Mutation probability	0.012
Maximum generation number	30

3.2 Choosing optimal parameters and their verification

The specifications of the two-inertia motor system used in this study are shown in Table 2.

Table 2 Mechanical parameters of a two-inertia motor system

Item	Value
Motor inertia $[Kgm^2]$	7.455×10^{-5}
Torsion stiffness $[Nm/rad]$	0.05
Resonant frequency $[rad/s]$	39.69
Anti-resonant frequency $[rad/s]$	30
Inertia ratio (K)	0.75
Sampling time $[ms]$	2

In order to verify the performance of auto-tuning using GAs, we compare the responses before training with responses after training for each controller. And we also compare the optimal parameters obtained by numerous trials for many ς_1 and ω_1/ω_a with the optimal parameters obtained by using GAs.

I-P speed controller

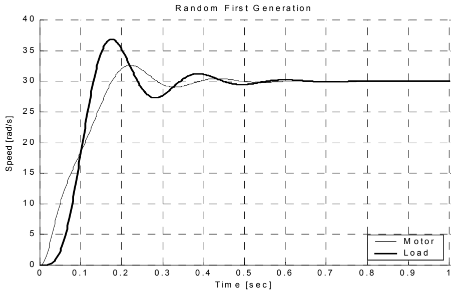
Table 3 shows optimal parameters, ς_1 and ω_1/ω_a, obtained with GAs(values without parentheses) and those(values within parentheses) obtained by numerous trials for several ς_1 and ω_1/ω_a in an I-P control. Both cases have almost same values. We found the optimal parameters for several inertia ratios, respectively.

Table 3 Optimal parameters for I-P control

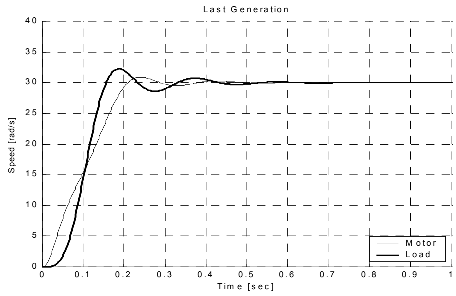
Design conditions	K	0.5	0.75	1.0	1.5	2.0
	R	1.23	1.32	1.41	1.58	1.73
Selected para-Meters	ITAE	6.132	5.046	4.570	4.435	4.108
	Fitness	0.163	0.198	0.219	0.226	0.243
	ς_1	0.70 (0.73)	0.72 (0.75)	0.81 (0.79)	0.84 (0.84)	0.87 (0.84)
	ω_1/ω_a	0.60 (0.60)	0.61 (0.60)	0.66 (0.63)	0.68 (0.74)	0.84 (0.91)

Using optimal parameters obtained by genetic operation, I/P gains are calculated using (10), (11).

Figure 7 shows the responses before and after training with GAs, respectively. The overshoot and oscillation of the case after training when using GAs is less than that of the case before training. However, even though we selected optimal parameters for the proposed method, oscillation still occurred in the transient response in the I-P control.



(a) ς_1=0.62, ω_1/ω_a=0.79(using random parameters before training)



(b) ς_1=0.72, ω_1/ω_a=0.61(using optimal parameters after training)

Fig. 7. Speed responses for I-P controller

I-PD speed controller

Table 4 shows optimal parameters obtained with GAs(values without parentheses) and those(values within parentheses) obtained by numerous trials for several ς_1 and ω_1/ω_a in an I-PD control. Using optimal parameters obtained by genetic operation, I/P/D gains are calculated using (14), (15), (16).

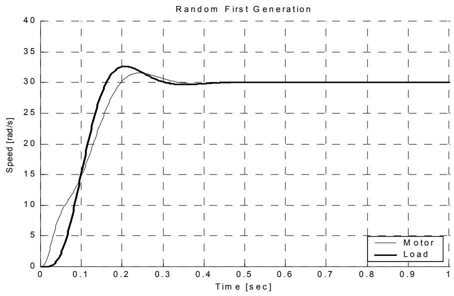
Table 4 Optimal parameters for I-PD control

Design conditions	K	0.5	0.75	1.0	1.5	2.0
	R	1.23	1.32	1.41	1.58	1.73
Selected para-Meters	ITAE	4.573	4.539	4.397	4.387	4.391
	Fitness	0.220	0.227	0.228	0.228	0.243
	ς_1	0.926 (0.89)	0.903 (0.90)	0.892 (0.90)	0.906 (0.91)	0.912 (0.91)
	ω_1/ω_a	0.812 (0.70)	0.826 (0.72)	0.754 (0.73)	0.766 (0.75)	0.760 (0.76)

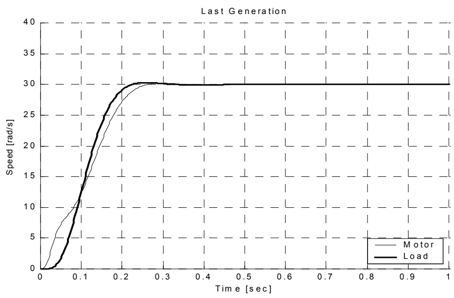
In the case of the I-PD controller, the optimal values appear at almost the same values of ς_1 and ω_1/ω_a irrespective of the inertia ratio (Park, *et al.*, 2001). For the inertia ratio of 0.75, ς_1 will be 0.903 and ω_1/ω_a will be 0.826. Then from (24) and (25), ω_2 will be 1.15 ω_a and ς_2 will be 0.65. The I/P/D gains are obtained as follows:

$$K_P = 1.2825\omega_a J_L \,,\ K_I = 0.3845\omega_a^2 J_L \,,\ K_D = 0.43 J_L - J_M$$

Similarly, both cases using the I-PD controller are plotted as shown in Fig. 8.



(a) ς_1=0.70, ω_1/ω_a=0.96(using random parameters before training)



(b) ς_1=0.90, ω_1/ω_a=0.83(using optimal parameters after training)

Fig. 8. Speed responses for I-PD controller

State feedback speed controller with integral

In rhe state feedback controller, ω_2 is set as follows:

$$\omega_2 = \sqrt{2\omega_a^2 - \omega_1^2} \times \alpha \tag{26}$$

where α is the positive constant to meet inequality (24). The constant, α , is selected based on the torsion amount. If α is large, the torsion amount becomes large, and vice versa. Here, α is set at 1.5. The optimal parameters are summarised in Table 5.

Table 5 Optimal parameters for state feedback control

Design conditions	K	0.5	0.75	1.0	1.5	2.0
	R	1.23	1.32	1.41	1.58	1.73
Selected parameters	ITAE	2.976	2.964	2.937	2.917	2.902
	Fitness	0.336	0.337	0.341	0.343	0.345
	ς_1	0.914 (0.90)	0.914 (0.90)	0.906 (0.90)	0.906 (0.90)	0.906 (0.90)
	ω_1/ω_a	0.958 (0.94)	0.958 (0.94)	0.948 (0.94)	0.948 (0.94)	0.948 (0.94)

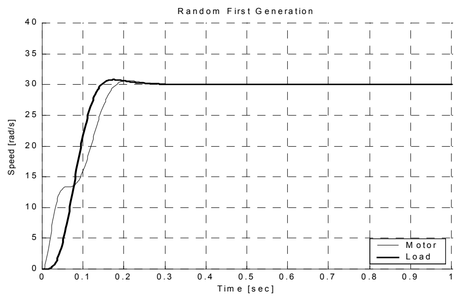
The optimal parameters are almost the same, irrespective of the inertia ratio. This result coincides with the result of the authors' previous work (Park, *et al.*, 2001). This implies that the controller can be designed irrespective of inertia ratio. The optimal parameters are obtained as follows:

$$\varsigma_1 = 0.914 \,,\ \omega_1 = 0.958\omega_a \,,\ \omega_2 = 1.04\alpha\omega_a \,,\ \varsigma_2 = 0.84/\alpha \,.$$

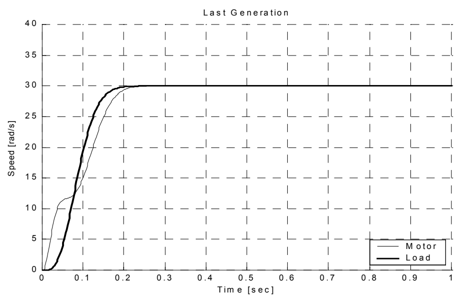
Then the gains of the state feedback controller from these values are obtained as follows:

$$K_1 = 3.502\omega_a J_M \,,$$
$$K_I = 0.993\alpha^2\omega_a^2 J_M \,,$$
$$K_2 = 1.895(\alpha^2 - 1)\omega_a J_M \,,$$
$$K_3 = (2.985 + 0.089\alpha^2)\omega_a^2 J_M - K_{sh}$$

In the state feedback controller design, it is also required to properly select observer gain, which affects the system response. The responses are shown in Fig. 9.



(a) ς_1=0.81, ω_1/ω_a=0.93(using random parameters before training)



(b) ς_1=0.91, ω_1/ω_a=0.96(using optimal parameters after training)

Fig. 9. Speed responses for state feedback controller

From figures 7 through 9, the responses which derived by using optimal parameters obtained by using GAs indicate much better performance than the one derived using parameters at random. From tables 3 through 5, the optimal values obtained by using GAs nearly coincide with those of the authors' previous work (Park, *et al.*, 2001). This seems to indicate the effectiveness of GAs.

Figure 10 shows the fitness values in the genetic process when each controller is used. From Fig. 10, we can see that the fitness values increase in small increments according to an increasing generation. This means that the procedure of using GAs performs well. Comparing the three controllers, the fitness value of the state feedback control is larger than that of any other control. This means that the state feedback controller has the best performance.

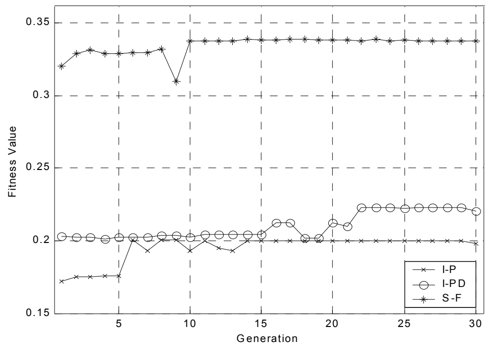
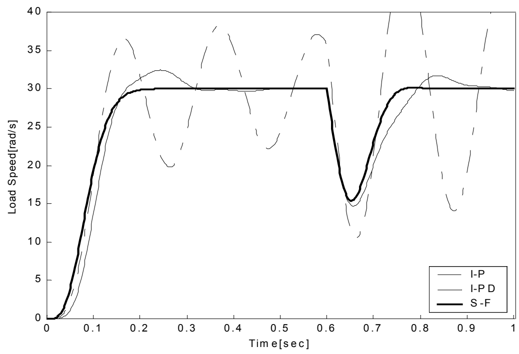


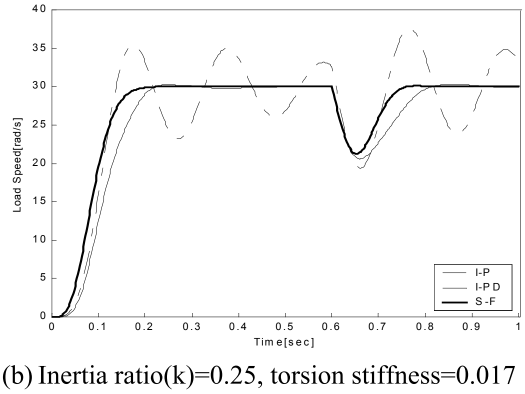
Fig. 10. Fitness values in the genetic process

3.3 Comparison of three kinds of controllers

In this section, to evaluate the three controllers for a two-inertia motor system with a very low inertia ratio, which easily causes the oscillation, the following simulations are carried out for two different inertia ratios 0.15 and 0.25, respectively. The rejection behaviour of disturbance is also evaluated. The specifications used are the same as shown in Table 2 except for inertia ratio and torsion stiffness.



(a) Inertia ratio(K)=0.15, torsion stiffness = 0.01



(b) Inertia ratio(k)=0.25, torsion stiffness=0.017

Fig. 11. Load speed responses when the inertia ratio is small. (K=0.15,0.25)

In Fig. 11, we can see that I-P controller causes oscillation and larger oscillation for disturbance particularly. However, the state feedback controller gives us a robust performance without the oscillation even though the inertia ratio is small. It also has a fast recovery compared to the I-P controller on disturbance. A state feedback controller designed in this way provides us with the best performance compared with the I-P controller and I-PD controller, and it can also be designed irrespective of inertia ratio.

4. CONCLUSION

This paper described how to find the location of poles in order to reduce oscillation and settling time by using genetic algorithms for three speed controllers, namely, an I-P, an I-PD, and a state feedback controller in a two-inertia motor system. The controller that was designed based on the genetic algorithm allowed us to obtain the best system response which reduced oscillation and torsion. With the proposed auto-tuning of controller gains using genetic algorithms, we could resolve the problem of calculating an ITAE index value for many cases in order to select optimal parameters for the controllers.

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