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INTEGRAL EVALUATIONS FOR A CLASS OF MULTIVARIABLE FUZZY CONTROL SYSTEMS

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Abstract: The problem of the integral evaluations of the state and control vectors in continuous multi-input-multi-output fuzzy control systems is investigated. The fuzzy controller is assumed to be nonlinearity in a bounded sector and the controlled plant - a linear system described by the differential or integral equations. The stability problem is investigated and the problem of how to use the integral evaluations for the fuzzy control system design is proposed.

Keywords: Fuzzy control, Nonlinear systems; Performance; Absolute stability

1. INTRODUCTION

Despite the increasing number of industrial applications of fuzzy control, there are some gaps in the analytical methods of fuzzy control quality assessment and stability analysis of the closed-loop systems containing nonlinear fuzzy controllers. Still lagging behind fuzzy controllers applications seem to be the systematic methods of analysis and design of fuzzy controllers, especially in the case of multi-input-multi-output systems, even if the controlled plants are linear.

In this work continuous closed-loop systems containing multivariable fuzzy controller and the linear or at least linearizable stationary dynamical plant are investigated. The plant can be asymptotically stable or not, but has to be stabilizable by the linear feedback. The fuzzy controller is regarded as a nonlinear function satisfying a sector condition, similarly as in (Ray and Majumder, 1984), (Calcev *et al.*, 1998) and the other works. Although the sector condition is a restriction of the class of investigated fuzzy controllers, it applies to a great deal of fuzzy expert systems as direct control devices in the closed-loop. Most often such systems either exactly are, or resemble the sector bounded nonlinearities in the sense of functions approximation.

The main goal of this article is to give an analytical method of the state and control vector norms evaluations. It seems to be a novelty in the fuzzy control literature. The controlled plant can be described by a differential or integral equations. The obtained integral evaluations one can use to quality assessment of the fuzzy control systems. The way of how to use these evaluations for fuzzy controller design is briefly explained.

2. ASSUMPTIONS AND PROBLEM STATEMENT

We consider the multi-input-multi-output (MIMO) closed-loop system as in Fig. 1. There are n state variables and m control signals, which constitute the vectors $\mathbf{x}(t) = [x_1(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ and $\mathbf{u}(t) = [u_1(t), \ldots, u_m(t)]^T \in \mathbb{R}^m$ in the system. We assume that the controlled plant is *linear* one or linearizable and described by the state equation

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \qquad \mathbf{x}(0) \in \mathbb{R}^n \qquad (1)$$

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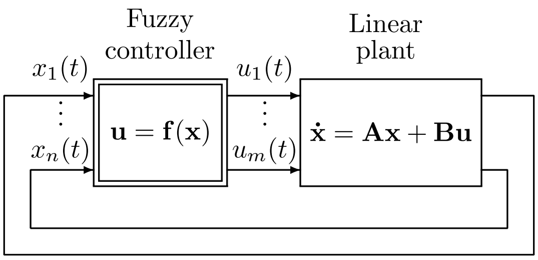

Fig. 1. Closed-loop MIMO fuzzy control system

where $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathbf{B} \in \mathbb{R}^{n \times m}$. The controller is regarded as *nonlinear part* of the system, which describes the vector function $\mathbf{u}(t) = \mathbf{f}[\mathbf{x}(t)]$. We consider such a class of the fuzzy controllers, for which at least one output $u_j = f_j[\mathbf{x}(t)]$ *essentially* depends on variables from the set $\{x_1, \ldots, x_n\}$ (Kluska, 2000, p. 35) and the following *sector condition*

$$\beta_j \leq \frac{u_j}{\sigma_j} \leq \beta_j + k_j, \qquad \forall \sigma_j \neq 0 \tag{2}$$
$$\sigma_j = 0 \Rightarrow u_j = 0$$
$$\sigma_j = \sum_{i=1}^{n} h_{ji} x_i, \quad j = 1, \ldots, m$$

is satisfied. This means that for each controller output u_j there exist nonnegative numbers β_j and k_j, $(j = 1, \ldots, m)$ and linear combination of the state vector components, that the inequalities (2) hold. The variables σ_j constitute an *artificial vector* $\boldsymbol{\sigma} = [\sigma_1, \ldots, \sigma_m]^T = \mathbf{H}\mathbf{x} \in \mathbb{R}^m$, where $\mathbf{H} \in \mathbb{R}^{m \times n}$. The elements of the matrix $\mathbf{H}$ follow only from the fuzzy controller features but not from the features of the plant (Kluska, 2000). It should be stressed that $\boldsymbol{\sigma}$ is *not* the plant output and does not occur in the system in Fig. 1.

Let us denote by $\lambda[\mathbf{A}]$ eigenvalues of the matrix $\mathbf{A}$. The matrix $\mathbf{A}$ in the equation (1) can be stable, i.e. $\operatorname{Re}\lambda[\mathbf{A}] < 0$ or not, but we assume that the linear part of the system viewed "from $\mathbf{u}$ to $\boldsymbol{\sigma}$" is stabilizable. If the matrix $\mathbf{A}$ is not stable, then one can apply the well known method of eigenvalues shifting, see e.g. (Desoer and Vidyasagar, 1975), to obtain the following system

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t) + \mathbf{B}\mathbf{u}_1(t) \\ \mathbf{u}_1(t) = \mathbf{u}(t) - \mathbf{L}\boldsymbol{\sigma}(t) \\ \mathbf{u}(t) = \mathbf{f}[\mathbf{x}(t)] \\ \boldsymbol{\sigma}(t) = \mathbf{H}\mathbf{x}(t) \\ \mathbf{A}_1 = \mathbf{A} + \mathbf{B}\mathbf{L}\mathbf{H}, \quad \mathbf{L} = \operatorname{diag}[\beta_1, \ldots, \beta_m] \\ (k_j\sigma_j - u_{1j})\, u_{1j} \geq 0, \quad j = 1, \ldots, m \end{cases} \tag{3}$$

where $\mathbf{u}_1(t) = [u_{11}, \ldots, u_{1m}]^T$. For the function $\mathbf{u}_1(t)$ the sector condition (2) holds with $\beta_j = 0$ for $j = 1, \ldots, m$. In the sequel we will assume that either the matrix $\mathbf{A}$ is stable, or the matrix $\mathbf{A}_1$ is stable. If $\operatorname{Re}\lambda[\mathbf{A}] < 0$, then $\beta_j = 0$ for $j = 1, \ldots, m$ and the system (1)-(2) will be considered. If $\operatorname{Re}\lambda[\mathbf{A}_1] < 0$, then $\beta_j > 0$ for some $j \in \{1, \ldots, m\}$ and the system (3) will be considered.

The scalar product of the vector functions $\mathbf{u}(t)$ and $\mathbf{y}(t)$ in the square integrable vector space in the time interval $[0, T]$ will be denoted by

$$\langle \mathbf{u}, \mathbf{y} \rangle_T = \int_0^T \sum_{j=1}^{m} u_j(t)\, y_j(t)\, dt$$

and the norm of the vector $\mathbf{x}(t)$ in the space $L_2(0, T)$ by $||\mathbf{x}||_T = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle_T}$, where $T \leq \infty$. For $T = \infty$ the index T will be omitted.

Beside the system (3), another more general system described by the integral equation

$$\begin{cases} \boldsymbol{\sigma}(t) = \mathbf{z}_1(t) + (\mathbf{G}_1 * \mathbf{u}_1)(t) \\ \mathbf{u}_1(t) = \mathbf{u}(t) - \mathbf{L}\boldsymbol{\sigma}(t) \\ \mathbf{u}(t) = \mathbf{f}[\mathbf{x}(t)] \\ \boldsymbol{\sigma}(t) = \mathbf{H}\mathbf{x}(t) \\ \langle \mathbf{u}, \boldsymbol{\sigma} - \mathbf{K}^{-1}\mathbf{u} \rangle_T \geq 0, \quad \forall T \leq \infty \end{cases} \tag{4}$$

will be considered, where $\mathbf{z}_1(t)$ is an external force with $||\mathbf{z}_1|| < \infty$, $\mathbf{G}_1$ the impulse response with $||\mathbf{G}_1|| < \infty$, and $(\mathbf{G}_1 * \mathbf{u}_1)(t)$ - the convolution integral, i.e. $\int_0^t \mathbf{G}_1(t-\tau) * \mathbf{u}_1(\tau)\, d\tau$. In the particular case, when the linear part is described by the differential equation (1), then

$$\mathbf{z}_1(t) = \mathbf{H}e^{\mathbf{A}_1 t}\mathbf{x}(0) \tag{5}$$

and

$$\mathbf{G}_1(t) = \mathbf{H}e^{\mathbf{A}_1 t}\mathbf{B} \tag{6}$$

However, the Laplace transform of the matrix function $\mathbf{G}_1(t)$ can be or not a rational function of the complex variable s. This allows one to consider more general linear systems than those described by differential equations (1).

3. STABILITY CONDITIONS

In this Section three theorems will be given - see (Kudrewicz, 1970), (Desoer and Vidyasagar, 1975), (Vidyasagar, 1978) and (Junger, 1989), whose more detailed implementations and proofs of stability problems were given in (Kluska, 2000). The theorems will be applicable to stability analysis of the continuous fuzzy control systems described by (1)-(2) or (4). We will try to use the lemmas which give sufficient stability conditions not very far from necessary ones.

3.1 *Small gain theorem*

Theorem 1 *If in the system (1)-(2) the condition*

$$p^2 \cdot \sup_{\omega} \max_{j} \lambda_j(\omega) < 1 \tag{7}$$

is satisfied, where

$$p^2 = \sum_{j=1}^{m} \left| \frac{1}{2} k_j \right|^2 \sum_{i=1}^{n} |h_{ji}|^2 > 0, \quad \{h_{ji}\}_{m \times n} = \mathbf{H} \tag{8}$$

$\lambda_1(\omega), \ldots, \lambda_m(\omega)$ are eigenvalues of the Hermitian matrix $\mathbf{V}^*(j\omega) \cdot \mathbf{V}(j\omega)$, and

$$\mathbf{V}(j\omega) = \left(\mathbf{I} - \mathbf{G}_0(j\omega) \frac{1}{2} \mathbf{KH} \right)^{-1} \mathbf{G}_0(j\omega) \tag{9}$$

$$\mathbf{G}_0(j\omega) = (j\omega \mathbf{I} - \mathbf{A})^{-1} \mathbf{B} \tag{10}$$

then $x_i \in L_2(0, \infty)$, $x_i(t)$ is a bounded function and $\lim_{t \to \infty} x_i(t) = 0$ for any $i = 1, \ldots, n$.

3.2 *Theorem of Junger*

Theorem 2 *If in the system (1)-(2) the condition*

$$\operatorname{Re} \lambda [\mathbf{W}(j\omega)] < 0, \qquad \forall \omega \in [-\infty, \infty] \tag{11}$$

is satisfied, where

$$\begin{aligned} \mathbf{W}(j\omega) &= \mathbf{G}(j\omega) - \mathbf{K}^{-1} \\ \mathbf{G}(j\omega) &= \mathbf{H}(j\omega \mathbf{I} - \mathbf{A})^{-1} \mathbf{B} \\ \mathbf{K} &= \operatorname{diag}[k_1, \ldots, k_m] \end{aligned} \tag{12}$$

then there exist the norms $||\mathbf{x}||$ and $||\mathbf{u}||$.

3.3 *Theorem of Kudrewicz*

Theorem 3 *If in the system (1)-(2) the condition*

$$\delta = -\frac{1}{2} \sup_{\omega} \max_{j} \mu_j(\omega) > 0 \tag{13}$$

is satisfied, where $\mu_1(\omega), \ldots, \mu_m(\omega)$ are eigenvalues of the Hermitian matrix $\mathbf{W}(j\omega) + \mathbf{W}^*(j\omega)$ and the matrix $\mathbf{W}(j\omega)$ is given by (12), then the solutions $x(t)$ of the system are bounded and $\lim_{t \to \infty} x_i(t) = 0$ for any $i = 1, \ldots, n$.

One can prove that the sufficient stability condition for the system (4) is given by the inequality

$$\delta_1 = -\frac{1}{2} \sup_{\omega} \max_{j} \rho_j(\omega) > 0 \tag{14}$$

where $\rho_1(\omega), \ldots, \rho_m(\omega)$ are eigenvalues of the Hermitian matrix $\mathbf{W}_1(j\omega) + \mathbf{W}_1^*(j\omega)$, as follows

$$\begin{aligned} \mathbf{W}_1(j\omega) &= \mathbf{G}_1(j\omega) - \mathbf{K}^{-1} \\ \mathbf{G}_1(j\omega) &= \mathbf{H}(j\omega \mathbf{I} - \mathbf{A}_1)^{-1} \mathbf{B} \\ \mathbf{A}_1 &= \mathbf{A} + \mathbf{BLH} \\ \mathbf{L} &= \operatorname{diag}[\beta_1, \ldots, \beta_m] \\ \mathbf{K} &= \operatorname{diag}[k_1, \ldots, k_m] \end{aligned} \tag{15}$$

The proof follows from lemmas given in (Kluska, 2000).

Corollary 1 Let us denote by $\lambda_{\max}[\mathbf{A}]$ the maximal eigenvalue of the matrix $\mathbf{A}$. From the Hirsch theorem (Bodewig, 1956), of the form

$$\operatorname{Re} \lambda [\mathbf{W}(j\omega)] \le \frac{1}{2} \lambda_{\max} [\mathbf{W}(j\omega) + \mathbf{W}^*(j\omega)] \tag{16}$$

one can conclude that:

(1) in the case of multi-output (MIMO) fuzzy controllers, when $m > 1$, from the condition (11) one obtains less conservative results, than those from the condition (13),
(2) in the case of one-output (MISO) fuzzy controllers, when $m = 1$, the results of both criteria are the same, because the inequality (16) becomes the equality.

4. INTEGRAL EVALUATIONS FOR THE STATE AND CONTROL VECTORS

In this Section the *upper* evaluations of the norm of the state and control vectors for the system (4) will be given, which one can also use for the systems (1)-(2) or (3).

4.1 *Upper bound of the state vector norm*

When analyzing proofs of stability theorems, one can derive the following evaluations for the state vector norm (Wiktorowicz, 2001)

$$||\mathbf{x}|| \le ||\mathbf{z}_2|| + \frac{1}{\delta_1} ||\mathbf{G}_2|| \cdot ||\mathbf{z}_1|| \tag{17}$$

where

$$\mathbf{z}_2(t) = \mathrm{e}^{\mathbf{A}_1 t} \mathbf{x}(0), \quad \mathbf{G}_2(t) = \mathrm{e}^{\mathbf{A}_1 t} \mathbf{B} \tag{18}$$

$$||\mathbf{G}_2|| = \sup_{\omega} \max_{j} \sqrt{\gamma_j(\omega)} \tag{19}$$

and $\gamma_1(\omega), \ldots, \gamma_m(\omega)$ are eigenvalues of the Hermitian matrix $\mathbf{G}_2^*(j\omega) \cdot \mathbf{G}_2(j\omega)$.

4.2 *Upper bound of the control vector norm*

In the considered system $\mathbf{u}_1 = \mathbf{u} - \mathbf{L}\boldsymbol{\sigma}$, thus $||\mathbf{u}|| \le ||\mathbf{u}_1|| + \max_j \beta_j \cdot ||\boldsymbol{\sigma}||$. From the first equation of the system (4) one can conclude that

$$||\boldsymbol{\sigma}|| \le ||\mathbf{z}_1|| + ||\mathbf{G}_1|| \cdot ||\mathbf{u}_1||$$

where

$$||\mathbf{G}_1|| = \sup_{\omega} \max_{j} \sqrt{\lambda_j(\omega)} \tag{20}$$

and $\lambda_1(\omega), \ldots, \lambda_m(\omega)$ are eigenvalues of the Hermitian matrix $\mathbf{G}_1^*(j\omega) \cdot \mathbf{G}_1(j\omega)$ and $\mathbf{G}_1(t)$ is given by (6). Because of the inequality

$$||\mathbf{u}_1|| \le \frac{1}{\delta_1} ||\mathbf{z}_1|| \tag{21}$$

the following evaluation

$$||\mathbf{u}|| \leq ||\mathbf{z}_1|| \left[\frac{1}{\delta_1} + \max_j \beta_j \left(1 + \frac{1}{\delta_1} ||\mathbf{G}_1|| \right) \right] \tag{22}$$

holds where $\mathbf{z}_1$ is given by (5).

Corollary 2 The fuzzy controller can be easily designed as a linear one. A theorem about linearity of the fuzzy controller of Takagi-Sugeno type (Takagi and Sugeno, 1985) was given in (Kluska, 1995). In our case the fuzzy controller is linear one, if

$$\mathbf{u}(t) = \mathbf{LHx}(t) \tag{23}$$

holds. In this case one obtains

$$||\mathbf{x}|| = ||\mathbf{z_2}|| \tag{24}$$

$$||\mathbf{u}|| = ||\mathbf{LHz_2}|| \tag{25}$$

where $\mathbf{z}_2$ is given by (18) and $\mathbf{A}_1 = \mathbf{A} + \mathbf{BLH}$, and $\mathbf{L} = \mathrm{diag}\,[\beta_1, \ldots, \beta_m]$. The proof is rather simple and will be omitted here.

5. HINTS FOR DESIGNERS

Using the integral evaluations, one can aply the following procedure for the MIMO fuzzy controller design, or when analysing the MIMO fuzzy control system.

(1) Computation the stability region of the family of linear systems in the parameter space $r_1, \ldots, r_m$, using so-called *generalized Hurwitz condition*, which has to be fulfilled - see (Kluska, 2000):

$$\mathrm{Re}\,[\mathbf{A}_r] < 0, \quad \forall\, r_j \in [\beta_j, \beta_j + k_j] \tag{26}$$
$$\mathbf{A}_r = \mathbf{A} + \mathbf{BRH}, \quad \mathbf{R} = \mathrm{diag}\,[r_1, \ldots, r_m]$$

(2) Choosing the stability criterion:
   (a) Theorem 2 - in the case of the differential equation description $\dot{\mathbf{x}}(t) = \mathbf{Ax}(t) + \mathbf{Bu}(t)$,
   (b) Theorem 3 or 1 - in the case of the integral equation $\boldsymbol{\sigma}(t) = \mathbf{z}(t) + (\mathbf{G} * \mathbf{u})(t)$. Then the stability region of the nonlinear system should be computed.

(3) Analytical computation of the norms $||\mathbf{x}||$ and $||\mathbf{u}||$ for the linear system as the function of lower sector bounds $\beta_1, \ldots, \beta_m$ - see Corollary 2. In this point the fuzzy controller is assumed to be temporarily linear one.

(4) Computation of *upper evaluations* I_x and I_u as the functions of sector parameters $\beta_1, \ldots, \beta_m$ and $k_1, \ldots, k_m$ according to inequalities (17) and (22), such that

$$||\mathbf{x}||^2 \leq I_x, \qquad ||\mathbf{u}||^2 \leq I_u$$

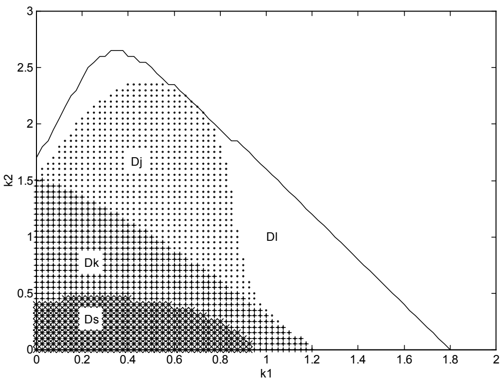

Fig. 2. Stability regions computed numerically: the linear system - D_l; the nonlinear system: from Junger theorem - D_j, from Kudrewicz theorem - D_k, from small gain theorem - D_s

(5) Choosing the sector bounds for the fuzzy controller based on the quality index I defined as the weighted sum of evaluations I_x and I_u.

Example Consider the system (1)-(2) in which

$$\mathbf{A} = \begin{bmatrix} -4 & -9 & -10 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

and the initial condition $\mathbf{x}(0) = [1, 0, 0]^T$. Since $\mathrm{Re}\,\lambda\,[\mathbf{A}] < 0$, thus $\beta_1 \geq 0$ and $\beta_2 \geq 0$ are allowed.

(1) Computing the stability region of the family of linear systems. The matrix A_r given by (26)

$$\mathbf{A}_r = \begin{bmatrix} 2r_1 - 4 & -r_2 - 9 & -10 - r_2 \\ 1 & r_2 & r_2 \\ r_1 & 1 & 0 \end{bmatrix}$$

is stable iff the parameters r_1 and r_2 belong to the region D_l as depicted in Fig. 2. In this region the generalized Hurwitz condition is satisfied.

(2) Computation of stability region of the nonlinear system. We examine stability of the system using Theorems 1-3.
Application of Theorem 1. From (8) one obtains

$$p^2 = \frac{1}{4}k_1^2 + \frac{1}{2}k_2^2$$

and from (10) the matrix $\mathbf{G}_0(s) = \mathcal{L}\left[\mathrm{e}^{\mathbf{A}t}\mathbf{B}\right] = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$, ($\mathcal{L}$ denotes Laplace operator), next, based on (9) the matrix $\mathbf{V}(s) = \left(\mathbf{I} - \mathbf{G}_0(s)\frac{1}{2}\mathbf{KH}\right)^{-1}\mathbf{G}_0(s)$, which has the form

$$\mathbf{V}(s) = \frac{2}{M(s)} \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \\ V_{31} & V_{32} \end{bmatrix}$$

where $M(s) = m_3 s^3 + m_2 s^2 + m_1 s + m_0$, and
$m_3 = 4$,
$m_2 = 16 - 4k_1 - 2k_2$,
$m_1 = 36 + 20k_1 - 8k_2 + 3k_1 k_2$,
$m_0 = 40 - 6k_2 + k_1 k_2$,
$V_{11} = 4s^2 - (3k_2 + 20)\, s - k_2$,
$V_{12} = -2\left(s^2 + 9s + 10\right)$,
$V_{21} = (k_2 + 4)\, s + 3k_2 - 20$,
$V_{22} = 2\left[s^2 + (3 - k_1)\, s + 5k_1\right]$,
$V_{31} = 2s^2 + (8 - k_2)\, s + 22 - 3k_2$,
$V_{32} = (2 - k_1)\, s + 6 - 11k_1$.
The norm $||\mathbf{V}||$ we calculate numerically, and then, after checking the small gain condition (7) inside the region D_l we obtain the stability region of the nonlinear system depicted by D_s in Fig. 2.

Application of Theorems 2 and 3. Based on (12) the matrix $\mathbf{W}(s) = \mathcal{L}\left[\mathbf{H}e^{\mathbf{A}t}\mathbf{B}\right] - \mathbf{K}^{-1}$:

$$\mathbf{W}(s) = \begin{bmatrix} \dfrac{2s(s-5)}{Q(s)} - \dfrac{1}{k_1} & -\dfrac{s^2 + 9s + 10}{Q(s)} \\ \dfrac{s^2 + 6s + 1}{Q(s)} & \dfrac{s^2 + 4s + 3}{Q(s)} - \dfrac{1}{k_2} \end{bmatrix}$$

where $Q(s) = s^3 + 4s^2 + 9s + 10$. Checking numerically the conditions (11) and (13) inside the region D_l we obtain stability regions of the nonlinear system depicted by D_j and D_k in Fig. 2.

(3) Computation of $||\mathbf{x}||$ and $||\mathbf{u}||$ for the linear system. For the given $x(0) = [1, 0, 0]^T$ one obtains $\mathcal{L}[\mathbf{z}_2(t)] = \mathcal{L}\left[e^{\mathbf{A}_1 t}\mathbf{x}(0)\right] = (s\mathbf{I} - \mathbf{A}_1)^{-1}\mathbf{x}(0)$ and using Parseval theorem

$$||\mathbf{x}||^2 = ||\mathbf{z}_2||^2 = ||z_{21}||^2 + ||z_{22}||^2 + ||z_{23}||^2 \tag{27}$$

where

$$||z_{21}||^2 = \frac{d_1 d_0 + \beta_2^2 d_0 + 2\beta_2 d_0 + d_2 \beta_2^2}{2 d_0 \Delta}$$

$$||z_{22}||^2 = \frac{d_0 + d_2 \beta_1^2 \beta_2^2}{2 d_0 \Delta}$$

$$||z_{23}||^2 = \frac{(\beta_1 \beta_2 - 1)^2 d_2 + \beta_1^2 d_0}{2 d_0 \Delta}$$

with $\Delta = d_1 d_2 - d_0$, and
$d_2 = 4 - 2\beta_1 - \beta_2$,
$d_1 = 10\beta_1 - 4\beta_2 + 3\beta_1\beta_2 + 9$,
$d_0 = 10 - 3\beta_2 + \beta_1\beta_2$.
Based on (25) one obtains

$$||\mathbf{u}||^2 = ||\mathbf{L}\mathbf{z}_1||^2 = \beta_1^2 ||z_{11}||^2 + \beta_2^2 ||z_{12}||^2 \tag{28}$$

where

$$||z_{11}||^2 = \frac{d_1 d_0 + \beta_2^2 d_0 + 2\beta_2 d_0 + d_2 \beta_2^2}{2 d_0 \Delta}$$

$$||z_{12}||^2 = \frac{(\beta_1 + 1)^2 d_0 + d_2}{2 d_0 \Delta}$$

The norms $||\mathbf{x}||$ and $||\mathbf{u}||$ depend on sector

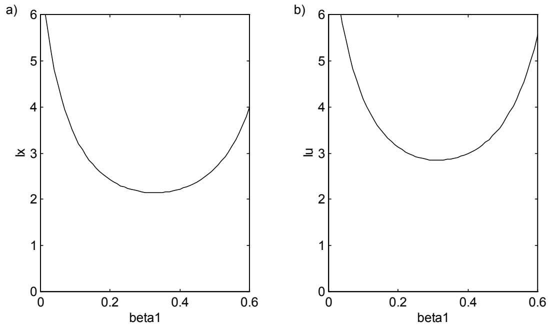

Fig. 3. Upper bounds of the norms - a): $I_x(\beta_1)$, - b): $I_u(\beta_1)$ by the assumptions: $k_1 = 0.25\beta_1$, $\beta_2 = 1$ and $k_2 = 0.2$

bounds β_1 and β_2. They can be used for optimal linear controller synthesis in the form $\mathbf{u}(t) = \mathbf{L}\mathbf{H}\mathbf{x}(t)$. However, we prefer the use of the integral evaluations for "optimal" (in some sense) nonlinear fuzzy controller design.

(4) Computation of upper evaluations for $||\mathbf{x}||$ and $||\mathbf{u}||$ of the fuzzy controller. Taking into account additional assumptions, the evaluations (17) and (22) can be computed as follows. For $\beta_1 \in [0.01, 0.6]$ by the additional assumptions:

$$k_1 = 0.25\beta_1, \quad \beta_2 = 1, \quad k_2 = 0.2 \tag{29}$$

one can compute δ_1 from the condition (14), the norms $||\mathbf{z}_2||$, $||\mathbf{z}_1||$ and the norms of matrices $||\mathbf{G}_2||$ and $||\mathbf{G}_1||$ given by (19) and (20). As a result one obtains the evaluations I_x and I_u as the functions of the parameter β_1, as shown in Fig. 3.

(5) Choosing the sector bounds for the fuzzy controller. Suppose for the global quality index

$$I = \alpha_1 I_x + \alpha_2 I_u, \qquad \alpha_1 + \alpha_2 = 1, \quad \alpha_i \geq 0$$

we assume $\alpha_1 = 0$. In this case $I = I_u$ is minimal for $\beta_{1opt} = 0.31$, (see Fig. 3b). This value corresponds with the upper sector bound $\beta_{1opt} + k_1 = 0.388$, and $k_1 = 0.25\beta_{1opt}$. In this way an "optimal" sector is obtained in the sense that the *upper evaluations of the norms* $||\mathbf{x}||$ *and* $||\mathbf{u}||$ *are minimal* (this is not equivalent with optimality in the usual sense: "$\alpha_1 ||\mathbf{x}|| + \alpha_2 ||\mathbf{u}||$ is minimal"). Thus, one can choose $\beta_{1opt} = 0.31$. This means that the function $\mathbf{u} = \mathbf{f}(\mathbf{x})$ with which the fuzzy controller is equivalent should satisfy the following sector conditions

$$0.31 \leq \frac{u_1(t)}{\sigma_1(t)} \leq 0.388, \qquad \forall \sigma_1 \neq 0$$

$$1 \leq \frac{u_2(t)}{\sigma_2(t)} \leq 1.2, \qquad \forall \sigma_2 \neq 0$$

and $\sigma_j = 0 \Rightarrow u_j = 0$, $(j = 1, \ldots, m)$, by $\sigma_1 = x_1$ and $\sigma_2 = x_2 + x_3$.

6. CONCLUSION

Stability was the basic condition for the correct design of the fuzzy control system. We tried to use lemmas concerning stability which give sufficient stability conditions not very far from necessary ones. The integral evaluations of the state and control vectors were given and the way of how to use them to fuzzy control systems design was shown, as well. We suggest the following general procedure of fuzzy controller design in the case of linear or linearizable plant.

- First, the fuzzy controller should be designed as linear one.
- Next, the controller can be modified so that the resulting input-output controller function should:
  - · remain nonlinearity in the bounded sector,
  - · guarantee that the closed-loop system will be stable for all nonlinear functions within this sector,
  - · ensure that the system responses are not worse (in the sense of $||\mathbf{x}||$ and $||\mathbf{u}||$ minimization) than in the linear system.

The modification of the controller function can be made heuristically or using one of the methods of learning or adaptation.

7. REFERENCES

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