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SMITH PREDICTOR BASED NEURAL CONTROLLER WITH TIME-DELAY ESTIMATION

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Abstract: A neural control strategy for nonlinear processes with time-variant time-delay is proposed in this paper. In this strategy, a dynamic neural network based nonlinear Smith predictor is constructed to compensate for the effect of time-delay of a class of nonlinear processes. An on-line optimizing controller is illustrated based on the neural Smith predictor. It is known that the performance of the Smith predictor may be deteriorated if the time-delay of the process changes with time. In order to improve the performance of the Smith predictor, a time-delay adaptation mechanism is introduced into the control structure to track the variation of the time-delay. The simulation, comparing with the classical Smith predictive control, on a continuous-stirred-tank-reactor (CSTR), where the time-delay of the manipulating flow changes with time, is used for the test. *Copyright © 2002 IFAC*

Keywords: neural networks, optimization, time-delay, adaptation, prediction.

1. INTRODUCTION

Many chemical or bio-chemical processes contain time-delays. When time-delay is larger than or equal to the time constant of the process, the closed-loop control of the system will be very difficult. In this case, one of the alternatives to handle the large time-delay is to use prediction technique to compensate for the influence of the time-delay. Smith predictor is one of the simple and often used strategies to compensate large time-delay in industries. Usually, the Smith predictor is applied to linear systems. As most of chemical or biochemical processes involve not only large time-delay but also high non-linearity, developing of a nonlinear Smith predictor for a process, in which the non-linearity can not be ignored, is necessary. Some literatures respectively proposed a Smith predictor structures using nonlinear models for nonlinear processes (Wong and Seborg 1988; Nahas, Henson, Seborg 1992; Tan and De Keyser 1994). In this paper, we also construct a nonlinear Smith predictor using dynamic neural networks. Then, an on-line optimizing predictive control strategy based on the neural Smith predictor is illustrated.

Although there are some strategies of using nonlinear model based Smith predictors, most of them only considered the case where the time-delay is prior known or the case where the time-delay is time-invariant. However, in chemical or biochemical industries, one can, sometimes, encounter the situation that some processes may contain time-varying time-delays. For instance, a manipulated flow rate in a continuous-stirred-tank-reactor (CSTR) process may change with time, therefore, it may lead to time-varying manipulated time-delay. It is also known that the Smith predictor is sensitive to time-delay mismatch. If the effect of time variation of the time-delay is significant, the dynamic performance of the Smith predictor may be deteriorated. However, if the on-line time-delay estimation is applied, in this case, the Smith predictor can be still used. In this paper, an on-line time-delay estimation mechanism is introduced into the neural Smith predictor to track the variation of time-delay so as to improve the dynamic performance of the nonlinear Smith predictor.

It should be noted that there have been some literatures about time-delay estimation (Reed, Feintuch, and N. Bershad 1981; Lim, and Macleod 1995; Balestrino, Verona, and Landi 1998). However, these results only considered the case of linear systems. In order to handle the time-varying time-delay in nonlinear bio-chemical processes, this paper proposes a time-delay estimation strategy for the neural network based nonlinear Smith predictor. A nonlinear CSTR process with time-varying coolant flow rate, which can be considered as a time-varying time-delay in manipulated variable, is used for simulation test of the proposed approach.

This paper is organized as follows: in section 2, a neural network based Smith predictor is illustrated. Then, in the 3rd section, an on-line time-delay estimator for the dynamic neural network based model is proposed to handle the time-delay tracking problem. In section 4, the corresponding neural predictive control strategy is developed. The simulation on a CSTR process with time-varying time-delay and the comparison between the proposed approach and the conventional Smith predictive control method are presented in section 5. Finally, the 6th section gives the conclusions of this paper.

2. PREDICTOR USING NEURAL NETWORK

It is known that the Smith predictor (Smith 1957, 1959) is a simple and often used strategy for large time-delay compensation. The principle of the Smith predictor is to construct both of a model with time-delay and a model free of time-delay as well. It is assumed that the models that are used to describe the process are precise enough. By subtracting the output of the model with time-delay from the output of the process, in this case, the model without time-delay will play a dominant role in the architecture of the Smith predictor. Therefore, the effect of the time-delay of the process will be eliminated by this compensation. As the Smith predictor is usually used for linear systems, therefore, it will be necessary to develop nonlinear Smith predictor for nonlinear processes with large time-delay if the non-linearity involved in the process can not be ignored. Wong and Seborg (1988) proposed an affine nonlinear model based nonlinear Smith predictor for nonlinear system time-delay compensation. During the recent decade, neural networks have been explored to model and control some complex nonlinear systems. Nahas, Henson et. al. (1992) proposed a Smith predictor structure using neural networks. Tan and De Kesyer (1994) used diagonal recurrent neural network to construct nonlinear Smith predictor and proposed on-line optimizing predictive PID control strategy for nonlinear systems with large time-delay. Huang, Lewis and Liu (2000) applied recurrent neural networks to the Smith predictive control structure for a tele-robot system. In this paper, the nonlinear Smith predictor is built by using neural networks with auto-regressive connection that is motivated by nonlinear auto-regressive and moving average (NARMA) model.

Suppose the process is described by the mapping f: $R^{n+m} \to R$, i.e.

$$y_k = f(Y_{k-1}, U_{k-d}) + \varepsilon_k \qquad (1)$$

where ε_k is the disturbance and noise sequence, $f(.) \in C^2$, $Y_{k-1} = [y_{k-1},...,y_{k-n}]^T \in R^n$ and $U_{k-d} = [u_{k-d},...,u_{k-d-m}]^T \in R^m$ are respectively the output and input vectors, and d is the time-delay. The neural network based models used to construct Smith predictors are respectively described by

$$yd_k = W^T S(x), \qquad (2)$$

where yd_k is the output of the neural model with time-delay, $W = [w_1,...,w_h]^T \in R^h$ is the weight vector connecting the outputs of the hidden layer and the output of the model, $S(x) = [s(x_1),...,s(x_h)]^T \in R^h$ is the output vector of the hidden layer, $s(x) = (1-e^{-x})/(1+e^{-x})$ is the sigmoid function, and the inputs of the sigmoid function in the hidden layer are of the form, i.e.

$$x_i = \sum_{j=1}^{na} w_{ij} yd_{k-j} + \sum_{j=1}^{nb} w_{i,na+j} u_{k-d-j}, \quad i = 1,...,h \qquad (3)$$

where na and nb are respectively the lags of the output and input of the neural model, w_{ij} s are the weights. The introduction of the auto-regression of the model output into the network can be useful to simulate the dynamics of the process. The corresponding neural model that is free of time-delay can be described by

$$y0_k = W^T S(z), \qquad (4)$$

and

$$z_i = \sum_{j=1}^{na} w_{ij} y0_{k-j} + \sum_{j=1}^{nb} w_{i,na+j} u_{k-j}. \quad i = 1,...,h \qquad (5)$$

Comparing (2)-(3) with (4)-(5), both of the models have the same architecture and parameters. The only difference between the two neural models is in their inputs. It is assumed that the neural models are well trained so that they can have accurate descriptions for the cases of the process with and without time-delay. Based upon this assumption, the following formula

$$yp_k = y_k - yd_k + y0_k = y_k - W^T S(x) + W^T S(z) \qquad (6)$$

where yp_k is the output of the Smith predictor, will lead to

$$yp_k = y0_k = W^T S(z). \qquad (7)$$

It implies that the effect of the time-delay of the process has been eliminated or the Smith predictor has compensated for the effect of time-delay.

3. ON-LINE TIME-DELAY ESTIMATOR

The neural Smith predictor shown in above section only considers the case where time-delay is prior known and time-invariant. However, in chemical or biochemical industries, one, sometimes, may encounter the case where the time-delay varies with time. For example, the variation of the manipulated coolant flow rate in a CSTR process may result in time-varying time-delay. If the neural model with constant time-delay used in Smith predictor, in this case, it will not match the time-delay of the process and the effect of the time-delay will not be compensated. Hence, the time-delay mismatch will deteriorate the performance of the Smith predictor (Balestrino et. al.,1998). Lim and Macleod (1995) developed a time-delay estimate approach for linear filter. Balestrino et. al. (1998) proposed a method to estimate the steady state value of time-delay. In this section, an approach using the technique of on-line nonlinear programming to estimate the time-varying time-delay for neural network based model is proposed. Then, the resulted time-delay estimate mechanism is embedded into the neural Smith predictor so as to make the neural model in the predictor track the change of the time-delay.

Consider the process with time-variant time-delay

$$y_k = f(Y_{k-1}, U_{k-\tau_k}) + \varepsilon_k . \qquad (8)$$

where τ_k is the time-variant time-delay. It is supposed to be separated as

$$\tau_k = d_k + \delta\tau_k , \qquad (9)$$

where d_k is the integer part of the time-delay whilst $\delta\tau_k$ denotes the fractal part of the time-delay, which is constrained within the range of one sample period, i.e. [0,1]. The corresponding neural model with time-delay has the similar formulae as (2) and (3) but (3) will be represented as

$$x_i = \sum_{j=1}^{na} w_{ij} yd_{k-j} + \sum_{j=1}^{nb} w_{i,na+j} u_{k-\hat{\tau}_k-j} , \qquad (10)$$

in this formula, $\hat{\tau}_k$ is the estimate of the time-delay. The estimation of time-delay will be partitioned as integer and fractional parts as well. In order to estimate the time-delay, the estimator of the fractional part of time-delay is proposed as follows:

$$\delta\hat{\tau}_k = g(V_k, I_k) , \qquad (11)$$

where $g(.)\in C^2$ realizes the mapping $g: R^q \rightarrow R$, where $q=q_1+q_2+q_3$; V_k is the weight matrix; $I_k = [\delta\hat{\tau}_{k-1},...,\delta\hat{\tau}_{k-q_1}, \rho_{k-1},...,\rho_{k-q_2}, e_{k-1},...,e_{k-q_3}]^T$, where $\rho = \partial yd/\partial u$; and $e_k = y_k - yd_k$. The formula (11) can be realized using a neural network, i.e.

$$\delta\hat{\tau}_k = \sum_{i=1}^{H} v_i s(z_i) = \sum_{i=1}^{H} v_i s[\sum_{j=1}^{q} v_{ij} I_{k-j}] . \qquad (12)$$

Then based on the estimated result of the fractional part, the integer part is adjusted separately.

Suppose that the time-delay changes slowly so that it means that the time-delay can be considered as constant during one sample period. For the estimation of the fractional part of time-delay, the gradient of the output of the neural model with time-delay respect to $\delta\hat{\tau}$, i.e. $\partial yd_k/\partial\delta\hat{\tau}$ should be calculated. Considering (2) and (10), it yields

$$\frac{\partial yd_k}{\partial\delta\hat{\tau}} = \sum_{i=1}^{h} w_i s'(x_i)[\sum_{j=1}^{na} w_{ij}\frac{\partial yd_{k-j}}{\partial\delta\hat{\tau}} + \sum_{j=1}^{nb} w_{i,na+j}\frac{\partial u_{k-\hat{\tau}_k-j}}{\partial\delta\hat{\tau}}] \qquad (13)$$

where $s'(x) = \frac{ds(x)}{dx} = 0.5(1-s(x)^2)$. In (11), the effect of the recurrent connection to the gradient has been considered. Using a method of first-order interpolation can derive the gradient $\frac{\partial u_{k-\hat{\tau}_k-j}}{\partial\delta\hat{\tau}}$.

From the first-order Taylor's series expansion, it leads to

$$u_{k-\hat{\tau}_k} \approx u_{k-\hat{d}_k-1} + \delta\tau\frac{u_{k-\hat{d}_k} - u_{k-\hat{d}_k-1}}{k-\hat{d}_k-(k-\hat{d}_k-1)} = u_{k-\hat{d}_k-1} + \delta\tau(u_{k-\hat{d}_k} - u_{k-\hat{d}_k-1}) . \qquad (14)$$

Hence, the gradient of $u_{k-\hat{\tau}_k}$ with respect to $\delta\tau$ can be derived as

$$\frac{\partial u_{k-\hat{\tau}_k-j}}{\partial\delta\hat{\tau}} = u_{k-\hat{d}_k-j} - u_{k-\hat{d}_k-j-1} . \qquad (15)$$

To determine the weights of the neural network used for the modeling of fractional time-delay, the gradients of $\delta\hat{\tau}$ with respect to v_i and v_{ij} are calculated respectively by:

$$\frac{\partial\delta\hat{\tau}}{\partial v_i} = s(z_i) \quad i=1,...,H , \qquad (16)$$

and

$$\frac{\partial\delta\hat{\tau}}{\partial v_{ij}} = \sum_{i=1}^{H} v_i s'(z_i)(I_{k-j} + \sum_{j=1}^{q_1} v_{ij}\frac{\partial\delta\tau_{k-j}}{\partial v_{ij}}] . \quad j=1,...,q \qquad (17)$$

Moreover, the gradients of the output of the neural model with respect to the weights are obtained as

$$\frac{\partial yd_k}{\partial w_i} = s(x_i), \quad i=1,...,h ; \qquad (18)$$

and

$$\frac{\partial yd_k}{\partial w_{ij}} = \sum_{i=1}^{h} w_i s'(x_i)[\sum_{j=1}^{na} w_{ij}\frac{\partial yd_{k-j}}{\partial w_{ij}} + yd_{k-j}] , \quad j=1,...,na \qquad (19)$$

as well as

$$\frac{\partial yd_k}{\partial w_{ij}} = \sum_{i=1}^{h} w_i s'(x_i)[\sum_{j=1}^{na} w_{ij}\frac{\partial yd_{k-j}}{\partial w_{ij}} + u_{k-\hat{\tau}(k)-j}] . \quad j=1,...,nb \qquad (20)$$

Define the index

$$Q = 0.5e_k^2 = 0.5(y_k - yd_k)^2 , \qquad (21)$$

and the parameter matrices, i.e.

$$\theta = [w_1,...,w_h \mid v_1,...,v_H]^T ; \quad \omega = [w_{ij} \mid vij]^T_{(h+H)\times(na+nb+q)} . \qquad (22)$$

Then, the estimate of these matrices will be

$$\theta = \theta - \lambda_1\frac{\partial Q}{\partial\theta}, \text{and} \quad \omega = \omega - \lambda_2\frac{\partial Q}{\partial\omega} . \qquad (23)$$

where $\lambda_i > 0, \quad (i=1,2)$ are the optimizing step-sizes. If an optimizing algorithm with second-order convergence, e.g. a modified Levenberg-Marquardt method, is applied, the update of matrices, i.e. θ and ω, becomes

$$\psi(k) = \psi(k-1) - \lambda[H(k) + \alpha I]^{-1} \frac{\partial Q}{\partial \psi} \quad , \qquad (24)$$
$$+\beta(\psi(k-1) - \psi(k-2)), \quad \psi = \theta, \omega$$

where $H = (\frac{\partial yd_k}{\partial \psi})(\frac{\partial yd_k}{\partial \psi})^T$, $\alpha > 0$ and is adjustable factor. In order to increase the possibility to escape from some local minimums, a momentum term is embedded into this algorithm and $\beta > 0$ is the momentum factor.

When the estimated $\delta\hat{\tau}_k$ is obtained, both the integer part will be updated by (Lim and Macleod, 1995)

$$\begin{cases} \hat{d}_k = \hat{d}_k - 1, \\ \delta\hat{\tau}_k = \delta\hat{\tau}_k + 1, \end{cases} \delta\hat{\tau}_k \in (-\infty, k+\eta] \quad \text{or}$$
$$\begin{cases} \hat{d}_k = \hat{d}_k + 1, \\ \delta\hat{\tau}_k = \delta\hat{\tau}_k - 1, \end{cases} \delta\hat{\tau}_k \in [k+1+\eta, \infty) \quad , \qquad (26)$$

and

$$\begin{cases} \hat{d}_k = \hat{d}_k, \\ \delta\hat{\tau}_k = \delta\hat{\tau}_k, \end{cases} \quad \delta\hat{\tau}_k \in (k+\eta, k+1+\eta) \qquad (27)$$

where $0 < \eta < 1$ is a very small number. To simplify the procedure of on-line computation, one can pre-train the neural model with unit time-delay off-line. Then, the on-line update is implemented just for time-delay estimation. The architecture of the neural Smith predictor with on-line time-delay estimator is shown in Fig.1. In this architecture, NMD denotes the neural model with time-delay and NM represents the neural model free of time-delay; the time-delay estimator feeds the on-line estimated the time-delay into the neural model with time-delay to compensate for the variation of time-delay.

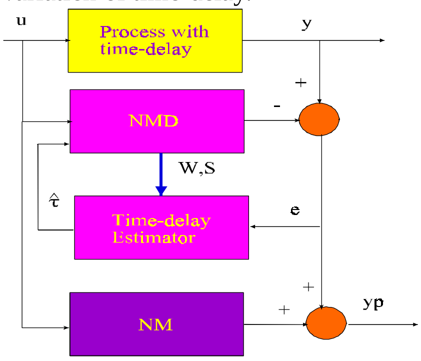

Fig. 1 Neural Smith predictor with time-delay estimator

4. NEURAL PREDICTIVE CONTROL

The neural Smith control is developed based on the neural Smith predictor with time-delay estimator illustrated in above section. The control strategy is considered as a on-line programming problem so that the control approach can be described as

$$u_k = \arg\min_u J(e_p) \quad \text{and} \quad e_p = r - yp \qquad (28)$$

where J(.) is the performance index for predictive control, and r is the set-point of the control system. The gradient algorithm with momentum term is applied to this procedure of optimization. Thus the control rule is as follows:

$$u_k = u_{k-1} - \gamma \frac{\partial J}{\partial u_{k-1}} + \pi(u_{k-1} - u_{k-2}), \qquad (29)$$

where $\gamma > 0$ is the optimizing step size, and $\pi > 0$ is the momentum factor. Notice that this structure is equivalent to a low-pass filter which can suppress the high frequency noise and oscillation. Also, the momentum factor π is considered as the pole of the low pass filter. Therefore its value should be constrained within (0,1) to guarantee the stability. From another point of view, the momentum term may increase the possibility to jump out of some local minimums in the case where $\frac{\partial J}{\partial u_{k-1}}$ becomes zero but $u_{k-1} - u_{k-2} \neq 0$.

5. SIMULATION TEST

The proposed approach of neural Smith control is tested on a simulated continuous-stirred-tank-reactor (CSTR) process. The process considered here is assumed that reactive species S flows into a perfectly mixed tank where it undergoes an irreversible exothermic reaction $S \rightarrow P$.

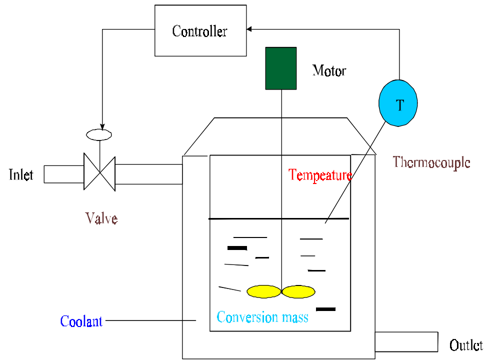

Fig. 2 The CSTR process

The response of the process relates changes in the cooling water rate in the reactor jacket to changes in the concentration of species S in the reactor. The process is described using the following dimensionless differential equations (Uppal et. al., 1974):

$$\dot{x}_1 = -x_1 + D\xi(x_1, x_2)$$
$$\dot{x}_2 = -x_2 + BD\xi(x_1, x_2) + C(u(t-\tau(t)) - x_2) \qquad (30)$$
$$\xi(x_1, x_2) = (1 - x_1)\exp(\frac{x_2}{1 + x_2/\phi})$$

where the parameters :D=0.072, C=0.3, B=8.0, and ϕ =20.0; x_1 denotes the reactor conversion, x_2 represents the dimensionless reactor temperature, u is the reactor jacket temperature, and τ is the time-delay as well. Suppose the process is sampled every 0.1 min.. Notice that the time-delay is a function of time, which is supposed to be

$$\tau(t)=\begin{cases}2+0.0005t, & t\le 250\,\text{min}.\\ 3.25+1.5\sin(0.0025(t-250)), & t>250\,\text{min}.\end{cases}$$

The neural network used to model the process, in the case where the unit time-delay is considered, has the architecture of two hidden nonlinear nodes and four inputs, i.e. $\{\hat{x}_2(k-1), \hat{x}_2(k-2), u(k-1), u(k-2)\}$. The model validation result is shown in Fig. 3. From Fig. 3, it can be seen that the neural model can have a good approximation to the process. Based on the obtained neural model, we can construct the neural Smith control system with time-delay estimator. The neural network for time-delay estimation has the structure of 12 hidden nodes and 4 auto-regressive inputs, 4 past gradient of reactor temperature with respect to control variable u, and 4 past error between the process output and the neural model. The control parameters are selected as $\gamma = 0.15$ and $\pi = 0.75$. The time-delay estimator is updated using (24). It is noted that in this case, only the time-delay, the scale variable is estimated so that computation effort is greatly reduced.

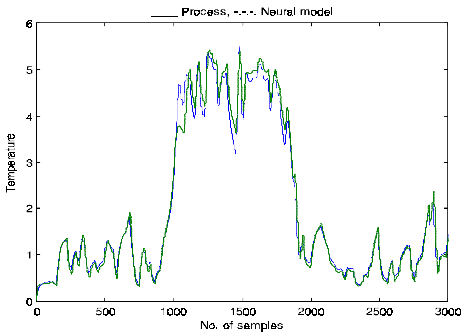

Fig. 3 The model validation of the CSTR

The corresponding neural Smith control with time-delay estimator is applied to the control of the process. Fig. 4 illustrates the control results. While the result of time-delay estimation is demonstrated in Fig.5.

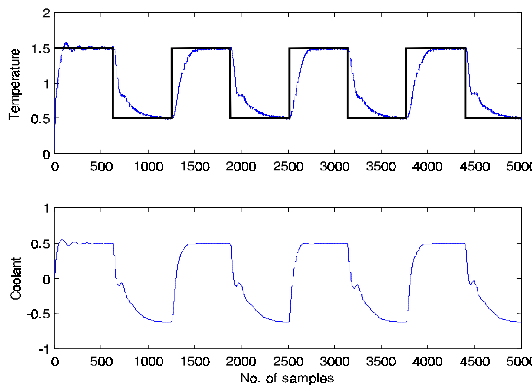

Fig. 4 Neural Smith control with time-delay estimator

For comparison, the neural Smith control, with the same control parameters as the strategy with time-delay estimator but the time-delays are fixed on constants, is also applied to the process. Fig. 6 and Fig. 7 respectively show the results of the control with estimated time-delay of 2 and 6 mins.. From these control results, we can conclude that, in the case of time-variant time-delay, the Smith control with constant time-delay will not obtain satisfactory control performance. It may lead to aggressive control if the time-delay is under estimated or conservative control suppose the time-delay is over estimated. While the neural Smith control with time-delay estimator can improve the dynamic performance of the control system. Therefore, if we want to obtain high quality control, the time-delay estimator introduced into neural Smith predictor can be one of the promising alternatives in the case where time-delay changes with time. Also, the proper selection of the parameters, i.e. γ and π, in the control algorithm shown in (29) will be important to guarantee the robustness of the control system when the time-delay mismatch exists. However, the robust design to model mismatch will often lead to some sacrifice of the performance.

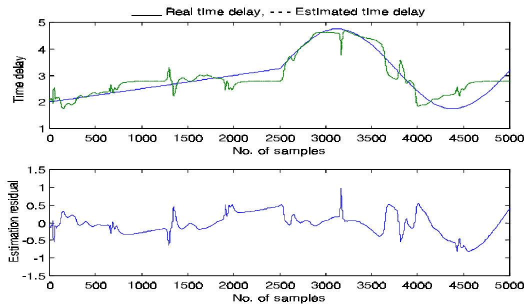

Fig. 5 Time-delay estimated result

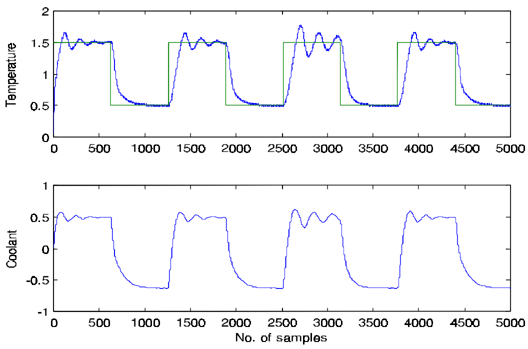

Fig. 6 Neural Smith control with under-estimated time-delay

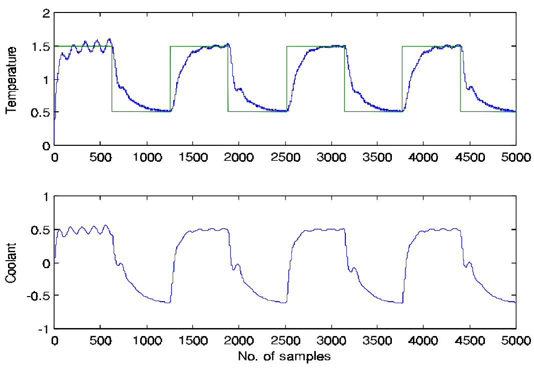

Fig. 7 Neural Smith control with over-estimated time-delay

6.CONCLUSIONS

In this paper, a strategy of nonlinear processes with time-delay is proposed. Firstly, the neural network based Smith predictor can be used for time-delay compensation for nonlinear processes. Then neural network based time-delay estimator is proposed and is introduced into the neural Smith control structure to tackle the effect of time-variant time-delay. Different from the adaptive time-delay estimators respectively proposed by Lim and Macleod (1995) as well as Balestrino et. al.,(1998), which can only handle linear processes or only estimate steady state time-delay, the proposed method can handle not only nonlinear processes but also time-variant time-delay. Moreover, the proposed method of time-delay estimation has an advantage that the optimizing procedure for weights training can be cut-off if the weight-training is converged and the time-varying region of time-delay is constrained.

The developed approach is tested on a simulated CSTR process. The simulation results show that the control performance is improved when the time-delay estimator is introduced into the neural Smith control architecture. Comparing with the neural Smith control with constant time-delay, the method proposed in this paper has obtained better dynamic response.

As the procedure of time-delay estimation is separated into integer and fractional parts. Therefore, the selection of the initial values is very important to the convergence. In this paper, the empirical method is used but sometimes time-consuming. Hence, to increase the robustness and efficiency to the selection of the initial condition of the algorithm for time-delay estimator is an interesting topic for the research in the future.

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