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PERFORMANCE AND ROBUSTNESS DESIGN OF CONTROL SYSTEMS VIA GENETIC-SEX MULTI-OBJECTIVE OPTIMIZATION

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Abstract. In this paper new evolutionary method of solving multi-objective optimization problems is presented. This method utilizes an information about an individual sex for the purpose of distinction between different groups of objectives. In particular, this information is extracted from the fitness of individuals and applied during the parental crossovers in a multi-objective optimization process. Characteristics of this mechanism are discussed and its performance in an exemplary multi-objective PID control optimization task is presented. Copyright© 2002 *IFAC*

Key Words. Genetic algorithms, multi-objective optimization, Pareto-optimality

1. INTRODUCTION

In many practical decision processes it is necessary to optimize several objective functions at the same time (Goldberg, 1989; Michalewicz, 1996; Viennet, *et al.*, 1996; Man, *et al.*, 1997). In order to integrate those objectives into one, it is necessary to determine the relations (weights) among the partial objectives. On the other hand, with multi-objective optimization in mind, the notion of optimality is not obvious. To solve the above optimality problems, various methods can be applied, such as: weighted profits (Michalewicz, 1996), distance function (Michalewicz, 1996), sequential inequalities (Zakian and Al-naib, 1973), or ranking with reference to Pareto-optimality (Michalewicz, 1996; Man, *et al.*, 1997; Kowalczuk, *et al.*, 1999a; 1999b; Kowalczuk and Białaszewski, 1999; 2000a; 2000b). An essence of the first two methods lies in integration of many objectives into one. In the method of weighted profits, all objectives are combined using a suitable vector of weights. The method of distance consists in calculation of a norm of the difference between the objective vector and a demand vector. The method of sequential inequalities is based on transformation of all the objectives into a set of inequality constraints that can not be violated by the optimized parameters. The above methods are simple and useful, but they have the disadvantage of relaying on an arbitrary choice of the weighting vector, demand vector or limit values for the objective function. Moreover for the purpose of integration of all the objectives into one, it is necessary to determine the relations among the objectives, what is not always possible. A different approach is applied in the ranking method using the notion of Pareto-optimality. This method avoids any arbitrary weighting of objectives. Instead, a useful classification of the solutions is proposed that takes into account all the multiple objectives. This concept of optimality does not give any directions as to the choice of a single solution from amongst various Pareto-optimal solutions. The designer has therefore a chance to make an independent judgement of the offers.

All the above mentioned methods of finding multi-objective solutions can be applied in evolutionary algorithms (Goldberg, 1989; Michalewicz, 1996). This paper presents a new method of solving multi-objective optimization problems based on the evolutionary search with the ranking method referencing to Pareto-optimality. In our approach the information about a degree of membership to a given sex (Kowalczuk and Białaszewski, 2001) is attributed to each solution obtained. This information is utilized in the (crossover) process of mating, in which only individuals of different sex (possible multiple) are allowed to create their offspring's.

1.1. Multi-objective optimization problem

Consider the following n-dimensional vector of an objective function

$$\boldsymbol{f}(\boldsymbol{x}) = [f_1(\boldsymbol{x}) \quad f_2(\boldsymbol{x}) \quad \dots \quad f_m(\boldsymbol{x})]^{\mathrm{T}} \in \Re^m \quad (1)$$

where

$$\boldsymbol{x} = [x_1 \quad x_2 \quad \dots \quad x_n]^{\mathrm{T}} \in \Re^n \quad (2)$$

denotes n-dimensional vector of searched parameters, while $f_j(\boldsymbol{x})$, $j = 1,2,...,m$, is a partial objective function. Assuming that all the co-ordinates are profit functions, the multi-objective optimization task can be formulated as follows

$$\max_{\boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}) \quad (3)$$

At this stage, the formula (3) describes a multi-profit maximization task without constrains.

A solution (individual) in evolutionary algorithms can have the following form

$$\boldsymbol{x}_i = [x_{i_1} \quad x_{i_2} \quad \dots \quad x_{i_n}]^{\mathrm{T}} \in \Re^n \quad (4)$$

in which the co-ordinate x_{i_k} ($i = 1,2,\dots,N$, $k = 1,2,\dots,n$) embrace the k-th searched parameter of the i-th individual, and N is the number of individuals in the population

$$\boldsymbol{X} = [\boldsymbol{x}_1 \quad \boldsymbol{x}_2 \quad \dots \quad \boldsymbol{x}_N] \quad (5)$$

1.2. Pareto-optimality

The conditions of Pareto optimality (Goldberg, 1989) for the maximization task (3) can be formulated as follows. Let $\boldsymbol{f}(\boldsymbol{x}_p), \boldsymbol{f}(\boldsymbol{x}_r) \in \Re^m$, where $\boldsymbol{x}_p$ and $\boldsymbol{x}_r$ represent two individuals (possible solutions). Vector $\boldsymbol{f}(\boldsymbol{x}_p)$ is partially smaller than vector $\boldsymbol{f}(\boldsymbol{x}_r)$ if and only if for all their co-ordinates j=1,2,..., m

$$\left(\underset{j}{\forall} f_j(\boldsymbol{x}_p) \le f_j(\boldsymbol{x}_r) \right) \wedge \left(\underset{j}{\exists} f_j(\boldsymbol{x}_p) < f_j(\boldsymbol{x}_r) \right) \quad (6)$$

Thus, in the Pareto sense, individual $\boldsymbol{x}_p$ is dominated, if there is an individual $\boldsymbol{x}_r$ partially 'better' than $\boldsymbol{x}_p$ in terms of definition (6). If an individual is not dominated then it is called a Pareto-optimal (P-optimal) one.

1.3. Pareto-optimality ranking method

The ranking method according to Pareto-optimality is applied to assess the obtained individuals. In this procedure a rank $r(\boldsymbol{x}_i)$ of each individual $\boldsymbol{x}_i$ is determined by the following mapping (Man, *et al.*, 1997)

$$r(\boldsymbol{x}_i) = \mu_{\max} - \mu(\boldsymbol{x}_i) + 1, \; \mu_{\max} = \max_{i=1,2,\dots,N} \mu(\boldsymbol{x}_i) \quad (8)$$

where $\mu(\boldsymbol{x}_i)$ denotes the amount of all the individuals by which the individual $\boldsymbol{x}_i$ is dominated in the Pareto sense, while $\mu_{\max}$ stands for the maximum value among all $\mu(\boldsymbol{x}_i)$, $i = 1,2,\dots,N$.

2. RECOGNITION OF GENETIC SEX

In nature the sex division of a species appears to differentiate individuals with reference to reproductive function. According to this idea, our concept of a genetic sex consists in dividing the objective functions into several subsets, which have attributed individual genetic sexes. This sex division may result from various distinguishable characteristics of the considered objectives.

Therefore, one sex set can be constituted by objectives of a 'similar' character, which are in a kind of 'internal' rivalry (in terms of an approximately equal meaning to the designer from some point of view). Such an assortment can thus be used to discharge the designer from a cumbersome task of final isolation of a single solution from amongst all the Pareto-optimal individuals obtained in the course of multi-objective optimization.

On the other hand, different sex sets can express various groups of 'interests' that are difficult to be judged by the user in advance. In general, this division can be employed to represent an 'external' rivalry, which is not simple to be resolved. At such cases the best way can be to use the notion of Pareto-optimality as the last resort.

In a consequent approach, we propose to apply the mechanism of sex allotment during the whole computational evolution for the purpose of creating parental pools of different sex and generating new offspring.

The vector of the profit functions (1) can therefore be divided into s-subvectors

$$\boldsymbol{f}(\boldsymbol{x}) = [\boldsymbol{f}_1(\boldsymbol{x}) \quad \boldsymbol{f}_2(\boldsymbol{x}) \quad \dots \quad \boldsymbol{f}_s(\boldsymbol{x})]^{\mathrm{T}} \in \Re^m \tag{9}$$

where

$$\boldsymbol{f}_j(\boldsymbol{x})^{\mathrm{T}} \in \Re^{m_i}, \quad m = \sum_{i=1}^{s} m_i \tag{10}$$

denotes the j-th subvector ($j = 1,2,\dots,s$) describing the j-th genetic sex set of individuals. Within each of these sets, Pareto-optimality-based ranking of individuals is applied. In effect, each of the individuals is assigned a vector of ranks

$$\boldsymbol{r}(\boldsymbol{x}_i) = [r_1(\boldsymbol{x}_i) \quad r_2(\boldsymbol{x}_i) \quad \dots \quad r_s(\boldsymbol{x}_i)]^{\mathrm{T}} \in \Re^s \tag{11}$$

where $r_j(\boldsymbol{x}_i)$ ($j = 1,2,\dots,s$) represents a degree of membership of $\boldsymbol{x}_i$ to the j-th set of genetic sex. The sex assignment for an individual $\boldsymbol{x}_i$ is performed as follows

$$\varphi_i = \max_{l=1,2,\dots,s} \frac{r_l(\boldsymbol{x}_i)}{r_{l_{\max}}} \tag{12}$$

where

$$r_{l_{\max}} = \max_{i=1,2,\dots,N} \{r_l(\boldsymbol{x}_i)\} \tag{13}$$

Thus φ_i is a degree of membership to the l-th sex set, while $r_{l_{\max}}$ denotes a maximum rank within the l-th sex set of individuals.

The population of each of the sex set is monitored in terms of an assumed minimal number of individuals (for instance, $N/(3s)$). Lacking positions can be filled up with individuals from the lowest (waived) Pareto front of another sex set.

It is assumed that only individuals of different sex can create their offspring in the crossover process. The procedure of selecting the parental pool is carried out according to a stochastic-remainder method (Kowalczuk, *et al.*, 1999a) based on the degree of membership to a given sex set.

3. SYNTHESIS OF PID CONTROLLER

As an example of the application of the proposed approach to a multi-objective synthesis problem, a PID controller is designed. Such controller can be described in the frequency domain as

$$G_c(s) = K_p + \frac{1}{sT_i} + sT_d \tag{14}$$

where K_p, T_i, T_d are the searched parameters. In our example

$$\boldsymbol{x} = [x_1 \quad x_2 \quad x_3]^{\mathrm{T}} \in \Re^3 \tag{15}$$

denotes the searched vector of the PID parameters ($x_1 = K_p$, $x_2 = T_i$, $x_3 = T_d$). In the multi-objective optimization of the controller (14) for a given linear plant, the following objective functions are considered

$$IMSE = f_1(\boldsymbol{x}) = \int_0^{\infty} [\dot{e}(\boldsymbol{x},t) + \lambda e(\boldsymbol{x},t)]^2 dt \tag{16}$$

$$ITSE = f_2(\boldsymbol{x}) = \int_0^{\infty} t^2 e^2(\boldsymbol{x},t) dt \tag{17}$$

$$ISC = f_3(\boldsymbol{x}) = \int_0^{\infty} u^2(\boldsymbol{x},t) dt \tag{18}$$

$$g_m(\boldsymbol{x}) = f_4(\boldsymbol{x}) = \textit{gain margin} \tag{19}$$

$$p_m(\boldsymbol{x}) = f_5(\boldsymbol{x}) = \textit{phase margin} \tag{20}$$

The integral objectives (16)-(18) can be computed by solving suitable continuous Lyapunov equations.

The above criterion functions have been divided into two sex sets represented by the following vector

$$\boldsymbol{f}(\boldsymbol{x}) = [\boldsymbol{f}_1(\boldsymbol{x}) \quad \boldsymbol{f}_2(\boldsymbol{x})]^{\mathrm{T}} \in \Re^5 \tag{21}$$

The first sex set is composed of the three performance objectives of integral type

$$\boldsymbol{f}_1(\boldsymbol{x}) = [f_1(\boldsymbol{x}) \quad f_2(\boldsymbol{x}) \quad f_3(\boldsymbol{x})]^{\mathrm{T}} \in \Re^3 \tag{22}$$

while the second sex set embraces two robustness measures: the gain margin and the phase margin

$$\boldsymbol{f}_2(\boldsymbol{x}) = [f_4(\boldsymbol{x}) \quad f_5(\boldsymbol{x})]^{\mathrm{T}} \in \Re^2 \tag{23}$$

To find the P-optimal PID parameters the proposed evolutionary procedure has been applied, assuming that the i-th individual has the following form

$$\boldsymbol{x}_i = [x_{1_i} \quad x_{2_i} \quad x_{3_i}]^{\mathrm{T}} \in \Re^3 \tag{24}$$

In the evolutionary multi-optimization process two kinds of the genetic sex of individuals (performance and robustness) are iteratively assigned and applied in generation of new populations of solutions.

3.1. An exemplary multi-objective optimization

Consider the following unstable non-minimum-phase linear plant

$$G_p(s) = \frac{(-0.5s+1)(s+4)}{s(s+2)(s^2+6s+10)}$$

The synthesis of the PID controller boils down to the issue of multi-objective optimization of the vector (15), represented by (24), with regard to the goal expressed by the objective functions (16)-(20).

The following cube of the parameters of the PID controller is to be searched

$$x_1 = K_p \in [0,10],\ x_2 = T_i \in [0,3],\ x_2 = T_d \in [0,1]$$

The resulting evolutionary optimization algorithm can be described as follows:

Program EA
Initiation of *N* individuals in population *X*;
while $t \le t_{max}$
Computation of fitness of each individual;
Pareto-optimality genetic-sex ranking of individuals;
Genetic-sex recognition of individuals;
Selection of individuals to each sex set;
Creation of new population *X'* by:
- process of mating of different-sex individuals;
- mutation;
Replacing old population ($X \leftarrow X'$);
$t \leftarrow t+1$;
end

3.2. Results of evolutionary search

As the result of the evolutionary Pareto-optimal search with genetic sex recognition a set of 21 P-optimal individuals have been obtained. This set is composed of two sex sets. In the first (performance)

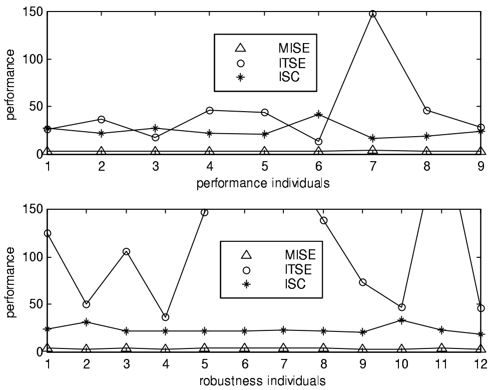

Fig. 1. P-optimal performance and robustness individuals in terms of performance.

sex set, 9 individuals have been found, while in the second (robustness) sex set, 12 individuals have been selected. Figs. 1 and 2 depict the obtained P-optimal performance and robustness individuals. Fig. 3 shows a distribution of two selected integral objectives for the P-optimal individuals belonging to the performance sex set ('triangles') and to the robustness sex set ('circles'). Fig. 4 illustrates a similar comparison of the obtained individuals in terms of the two robustness measures.

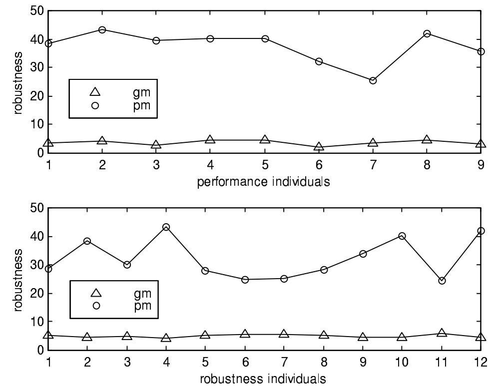

Fig. 2. P-optimal performance and robustness individuals in terms of robustness.

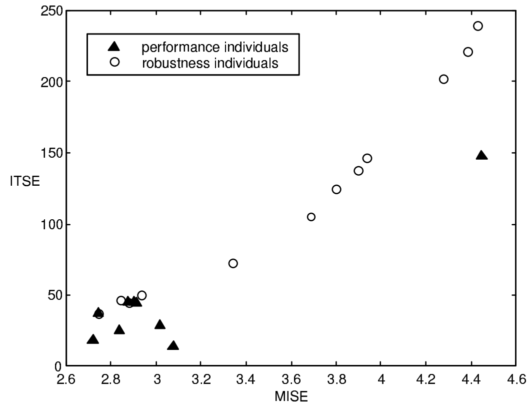

Fig. 3. P-optimal solutions against selected performance plane.

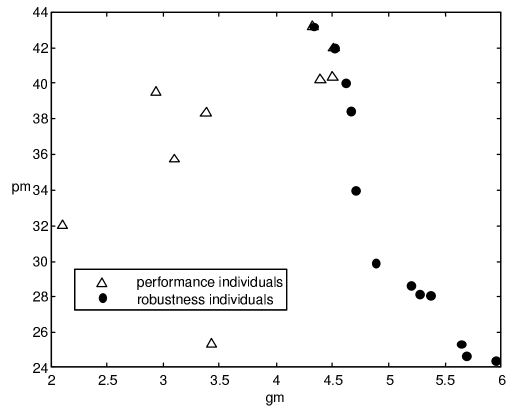

Fig. 4. P-optimal solutions against robustness plane.

P-optimal solutions obtained both by global Pareto-maximization ('squares') and by Pareto-optimization with the performance-sex recognition ('dots') are compared in Fig. 5 in terms of two chosen performance objectives. The P-optimal individuals gained by global Pareto-minimization ('squares') and by the robustness-sex Pareto-optimization ('dots') are also characterized in Fig. 6 in terms of robustness measures. As can be seen from Figs. 5 and 6, the

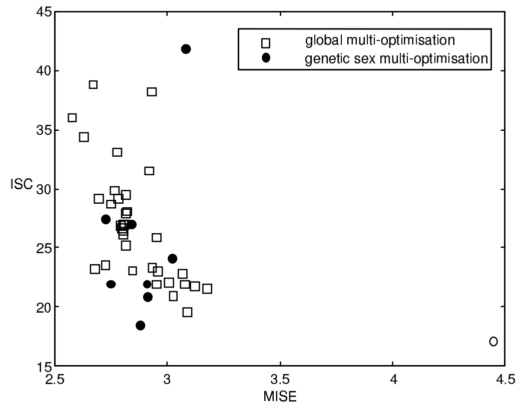

Fig. 5. Two types of P-optimization in terms of performance.

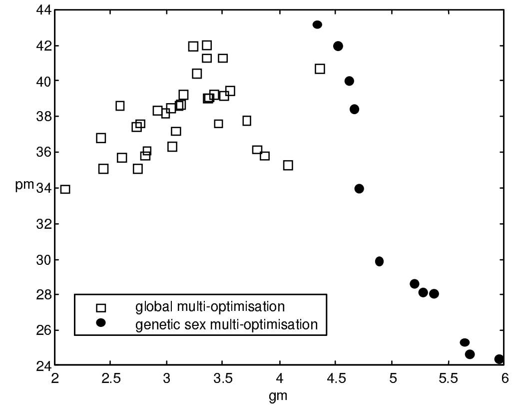

Fig. 6. Two types of P-optimization in terms of robustness.

presented approach with genetic-sex recognition is generally more efficient and informative as compared to the global Pareto-optimization method. The achieved individuals with a definite sex property can easily dominate (in the Pareto sense) over the individuals obtained by means of a global Pareto-optimization procedure.

3.3. Simulation results

Let us pick up four solutions of different kinds: (1) $\boldsymbol{x}_1 = [2.4549 \quad 2.2021 \quad 0.6307]^{\mathrm{T}}$ (performance), (2) $\boldsymbol{x}_2 = [2.0564 \quad 2.865 \quad 0.6992]^{\mathrm{T}}$ (robustness), (3) $\boldsymbol{x}_3 = [3.6681 \quad 2.4691 \quad 0.5757]^{\mathrm{T}}$ (traditional), (4) $\boldsymbol{x}_4 = [4.1484 \quad 2.9842 \quad 0.7460]^{\mathrm{T}}$ (Ziegler-Nichols); with their corresponding values of the objectives: $\boldsymbol{f}(\boldsymbol{x}_1) = [2.83 \quad 25.88 \quad 27.01 \quad 3.37 \quad 38.39]^{\mathrm{T}}$, $\boldsymbol{f}(\boldsymbol{x}_2) = [2.74 \quad 37.14 \quad 22.05 \quad 4.32 \quad 43.25]^{\mathrm{T}}$, $\boldsymbol{f}(\boldsymbol{x}_3) = [2.93 \quad 12.94 \quad 38.20 \quad 2.09 \quad 33.92]^{\mathrm{T}}$, and $\boldsymbol{f}(\boldsymbol{x}_4) = [2.32 \quad 24.48 \quad 48.37 \quad 1.68 \quad 27.80]^{\mathrm{T}}$.

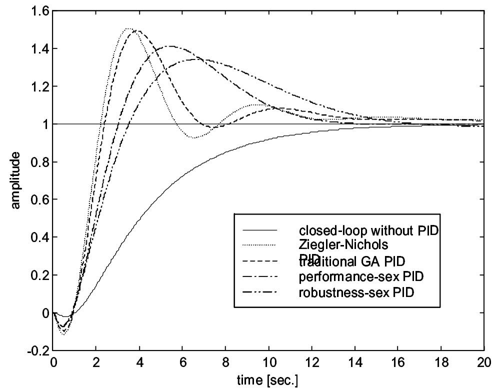

Fig. 7. Step responses of the closed-loop control systems for the exact plant model.

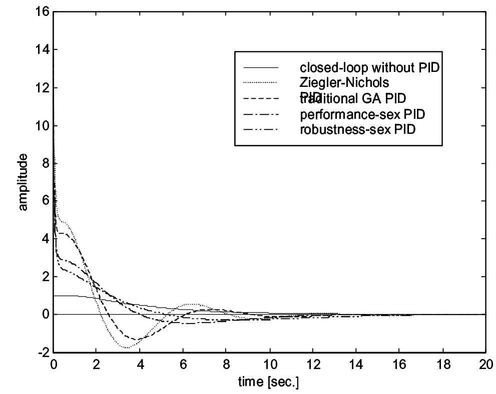

Fig. 8. Control signals for the exact-model plant.

The performance of these solutions has been verified by simulation taking into account the exact-model plant. Fig. 7 presents step responses of the closed-loop control systems governed by the four controllers defined by the vectors $\boldsymbol{x}_i$, i=1,...,4 and the feedback. Evidently, the solution $\boldsymbol{x}_1$ outperforms the other ones (not only in terms of $\boldsymbol{f}(\boldsymbol{x})$, but also in the settling time), while $\boldsymbol{x}_2$ results in a most robust PID system.

Figs. 8 shows the corresponding control signals for the PID systems in the case of the exact-model plant, while Fig. 10 concerns a perturbed-plant case.

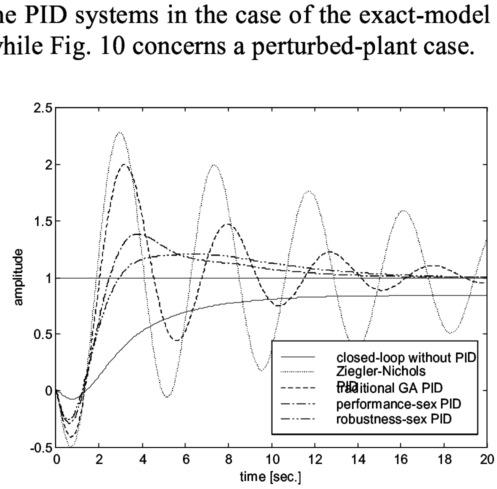

Fig. 9. Step responses of the closed-loop control systems for a perturbed plant.

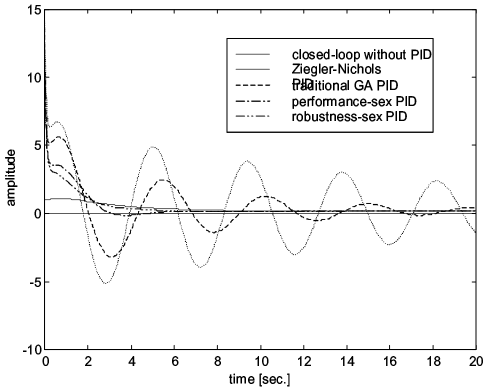

Fig. 10. Control signals for a perturbed plant.

For the perturbed case the parameters of the 'exact' plant have been changed (multiplicatively, with the respect to the nominal values of the parameters) with the use of uniformly-distributed deviations.

Fig. 9 introduces the results of simulation of the step responses of the closed-loop control systems for the perturbed plant. Clearly, the genetic sex-synthesized controllers (x_1 and x_2) have an evidently increased insensitivity to the applied deviation from the nominal plant as compared to the other controllers. It is also important that in most cases the robustness-sex solutions (Kowalczuk and Białaszewski, 2001) conquer the performance-sex-intending controllers.

4. CONCLUSION

The proposed method of solving multi-objective optimization problems is based on evolutionary search with genetic sex recognition. Information about a degree of membership to a given sex set is extracted by a suitable Pareto-optimization ranking processing of the fitness functions of the obtained solutions. This information is utilized in the (crossover) process of mating, in which only individuals of different (possible multiple) sex are allowed to create their offspring. An exemplary application of the genetic-sex recognition in an evolutionary procedure of Pareto-optimization of parameters of PID controllers confirms its usefulness and effectiveness in multi-objective optimization.

An instructive feature of the proposed optimization approach is the way of application of the Pareto-optimization method. Within the sex sets the Pareto-optimization is used as a tool of 'local' judgement of the 'internal' one-sex rivals for the purpose of their uniform estimation and selection to a new parental pool in each iteration cycle of EC (GA). What means: (1) a new mechanism of pre-selection of transient (and also final) individuals (solutions), and (2) a mutual inter-sex support in the genetic search.

The standard concept of Pareto-optimality can still be applied to the final set of solutions on a regular basis. Another way of processing, which can be proposed, is based on a notion of global optimality level (Kowalczuk and Białaszewski, 2000a) calculated with respect to either the fitness functions or the degrees of sex membership.

There are also two types of practical improvements in the performance of GA gained by the proposed approach as opposed to the traditional multi-objective evolutionary algorithms: (1) the obtained Pareto-fronts are more regular and P-optimal, and (2) the user gains a clear settlement for his/her decision upon the final-solution(s) selection.

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