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STABILITY ANALYSIS AND SYNTHESIS OF FUZZY SYSTEMS USING INTERVAL ARITHMETIC

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Abstract: This paper deals with the design of stable and robust rule-based fuzzy control systems. The interval analysis is applied to design a stable fuzzy controller using a robust condition to assure the stability. An example with a fuzzy controller for a non-linear system is presented to illustrate the design procedure. *Copyright © 2002 IFAC*

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1. INTRODUCTION

From the control engineering point of view, the major effort in fuzzy knowledge based control has been devoted to the development of particular applications, rather than to general analysis and design methodologies for coping with the dynamic behavior of control loops, see (Driankov, *et al.*, 1993; Andújar, *et al.*, 2000). Some authors has been using concepts taken from the qualitative theory of nonlinear dynamical systems to interpret the unstability unstableness at equilibrium points, and to give a global insight into the stability problem, see (Aracil, et al., 1989; Cook, 1994). There are algorithms in the literature that they allow the calculation of the Jacobian matrix if a system of multivariable fuzzy control in closed loop (Andújar 2001). A strategy to improve the design of fuzzy rule-based controllers using stability indices based on (Aracil, et al., 1989) was proposed in (Ollero, et al., 1995) and developed in (Sánchez, et al., 2000), but the strategy based on an elective actuation on the rules that affect the stability indices requires expert knowledge.

Lyapunov theory can be used for stability analysis of Takagi-Sugeno (TS) fuzzy systems that can be extended to a TS fuzzy system with affine term, including stable adaptive fuzzy control systems (Tanaka, 1995; Johanson, et al., 1998; Wang 1997) using quadratic Lyapunov function. The stabilization and tracking problems using Lyapunov functions has been applied in different types of TS fuzzy controllers (Johansen, 1994).

The stability analysis of a non-linear system aims at as assuring an attractor around the equilibrium point. It can do adapting controller parameters. This problem can be formulated like a parametric constrained minimization problem. The function to be minimized is the non-linear closed loop equivalent function. Interval analysis has exhibited a degree of success in the resolution of this type of problem. Interval mathematics is a generalization of real mathematics in which intervals replace real numbers, interval arithmetic replaces real arithmetic, and interval analysis replaces real analysis (Moore, 1966).

This paper deals with the application of interval analysis to design a controller that assures the stability in the presence of uncertainties of a first order nonlinear system. The system has been modeled by a TS fuzzy system using input-output data and an identification method.

In the second section the mathematical background of a fuzzy model plant and a fuzzy controller is described. The third section deals with the application of interval mathematics to find the equilibrium points of a first order system. The next section is devoted to the extension of the application of interval arithmetic to design a robust fuzzy controller. Finally, the proposed methods on sections III and IV are illustrated by an example.

2. THE NONLINEAR FUZZY MODEL

Consider a plant to be controlled, described by a first order dynamical system of the form

$$\dot{x} = f(x,u) \tag{1}$$

where $x \in \Re$ is a signal of the plant that allows to be measured, and $u \in \Re$ is the input of the plant.

To control the plant it will connect to their entrance (see Fig. 1) a controller represented by means of the function $u = g(x)$.

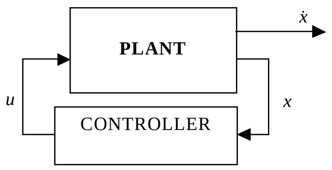

Fig. 1. Non-linear control system

An equivalent single-input-single-output first order non-linear dynamic fuzzy model to the process shown in Fig. 1 can be represented by the following group of rules:

$$R^{(l)}: IF\ x\ is\ A^l\ and\ u\ is\ B^l\ THEN\ \dot{x}^l\ is\ h^l\left(x;\theta^l\right) \tag{2}$$

where $l = 1...M$ is the number of rules, x is the state variable, A^l and B^l are fuzzy sets defined on the universe of discourse of x state variable and u is the control law. Finally, h^l (x; $\boldsymbol{\theta}^l$) is a non-linear consequent or a TS consequent with affine term:

$$h^l(x;\boldsymbol{\theta}^l) = a_0^l + a_1^l x + b^l u \tag{3}$$

where the vector $\boldsymbol{\theta}^l \in \Re^n$ represents the group of adaptive parameters.

The open loop model with input signal (2) is obtained using a control signal u selected by considering the operating range of the system, that is, all combinations of frequencies and amplitudes that characterize the system dynamic behavior from these data should be represented. The control signal is applied on plant and input-output data, of the form $[x, u, \dot{x}]$, are collected. A gradient descendent training algorithm or another identification method is using to obtain the fuzzy rules.
For a fuzzy system with product inference engine, TS fuzzifier, and center average defuzzifier, the rules (2) respond to the following nonlinear equation:

$$\dot{x} = \frac{\sum_{l=1}^{M} w^l h^l\left(x;\boldsymbol{\theta}^l\right)}{\sum_{l=1}^{M} w^l} \tag{4}$$

where w^l are the degree of firing of the rules.

If the consequent of the rule is as the type (3), then:

$$\dot{x} = \frac{1}{\sum_{l=1}^{M} w^l}\left[\sum_{l=1}^{M} w^l a_0^l + \sum_{l=1}^{M} w^l a_1^l x + \sum_{l=1}^{M} w^l b^l u\right] \tag{5}$$

this is

$$\dot{x} = a_0^t + a_1^t x + b^t u \tag{6}$$

where a_0^t, a_1^t and b^t are variable coefficients computed as:

$$a_0^t = \frac{\sum_{l=1}^{M} w^l a_0^l}{\sum_{l=1}^{M} w^l};\quad a_1^t = \frac{\sum_{l=1}^{M} w^l a_1^l}{\sum_{l=1}^{M} w^l};\quad b^t = \frac{\sum_{l=1}^{M} w^l b^l}{\sum_{l=1}^{M} w^l} \tag{7}$$

Note that the open loop model without controller can be represented making $u = 0$ in (6), then:

$$\dot{x} = a_o^t + a_1^t x \tag{8}$$

In a similar way, the fuzzy controller of the process (see Fig.1) can be represented by the following group of rules:

$$IF\ x\ is\ C^r\ THEN\ u^r = c_0^r + c_0^r x \tag{9}$$

where $r = 1\ldots N$ is the number of rules, C^r are fuzzy sets defined on the universe of discourse of x, c_0^r and c_1^r are parameters of the consequent part.

Proceeding like in (4) and (5), for a fuzzy system with product inference engine, TS fuzzifier, and center average defuzzifier, the rules (9) can be represented by:

$$u = c_0^t + c_1^t x \tag{10}$$

where the variable coefficients are given by:

$$c_0^t = \frac{\sum_{r=1}^{N} \omega^r c_0^r}{\sum_{r=1}^{N} \omega^r}; \qquad c_1^t = \frac{\sum_{r=1}^{N} \omega^r c_1^r}{\sum_{r=1}^{N} \omega^r} \tag{11}$$

where ω^r are the degree of firing of the rules of the controller.

Substituting (10) in (6) and using (7) and (11), the closed loop output can be represented by the following non-linear function:

$$\dot{x} = \frac{\sum_{l=1}^{M} w^l a_0^l}{\sum_{l=1}^{M} w^l} + \frac{\sum_{l=1}^{M} w^l b^l}{\sum_{l=1}^{M} w^l}\frac{\sum_{r=1}^{M} \omega^r c_0^r}{\sum_{r=1}^{N} \omega^r} + \left[\frac{\sum_{l=1}^{M} w^l a_1^l}{\sum_{l=1}^{M} w^l} + \frac{\sum_{l=1}^{M} w^l b^l}{\sum_{l=1}^{M} w^l}\frac{\sum_{r=1}^{M} \omega^r c_1^r}{\sum_{r=1}^{N} \omega^r}\right] x \tag{12}$$

3. INTERVAL MATHEMATICS AND THE OPEN LOOP STABILITY PROBLEM

An interval number $X = [a,b]$ is the set $\{x : a \le x \le b\}$ of real numbers between and including the endpoints *a* and *b*. Interval arithmetic is an arithmetic defined on sets of intervals, rather than sets of real numbers.

Let be *I* the set of real compact intervals [*a*,*b*] $a,b \in \Re$. Operations in *I* satisfy the expression:

$$A\, op\, B = \{a\; op\; b : a \in A, b \in B\} \quad for \quad A, B \in I \tag{13}$$

The equation (13) characterizes the four basic interval operations (Moore, 1966):

$$\begin{aligned} &[a,b][c,\,d] = [a+c,\,b+d] \\ &[a,\,b]-[c,\,d] = [a-d,\,b-c] \\ &[a,\,b]*[c,\,d] = \begin{bmatrix} \min\left(a*c,\,a*d,\,b*c,\,b*d\right), \\ \max\left(a*c,\,a*d,\,b*c,\,b*d\right) \end{bmatrix} \\ &[a,\,b]/[c,\,d] = [a,\,b]*[1/d,\,1/c], \;\; if\;\; 0 \notin\; [c,\,d] \end{aligned} \tag{14}$$

This is, the ranges of the four elementary interval arithmetic operations are exactly the ranges of the corresponding real operations. Extension of the interval arithmetic to include 0 in division can be found in (Hansen, 1992).

Bounds on the ranges of real functions can be obtained easily by interval arithmetic. A function $F : I(D) \to I$ is called an inclusion function for *f* if $f(x) \in F(X), \;\; \forall x \in X \in I$. Natural interval extension of real functions *f*(*x*), can be used to build interval inclusion functions *F*(*X*).

The interval arithmetic can be applied to find the equilibrium points of a dynamic system. Consider the non-linear first order system of the form (8):

$$\dot{x} = f(x), \quad x \in X_0 \in I \tag{15}$$

where *x* is the state variable defined in an interval X_0. To find the equilibrium points of (15) requires solving the following equation:

$$f(x) = 0, \quad x \in X_0 \in I \tag{16}$$

A basic branch and bound interval arithmetic algorithm can be used to find deterministically all the equilibrium points (see algorithm (1)) .The algorithmic is based on (Kearfott, 1996).

```
S = EquilibriumPoints (f, X0, ξ)

Insert X0 in L
while (NoEmpty(L))
    X ← first(L)
    if 0 ∉ F(X) then reject X
    else
        if (max(|F(X)|) < ξ) then insert X in S
        else
                X1, X2 = bisec X
                insert X1, X2 in L
        endif
    endif
endwhile
return S
end
```

Algorithm (1).- Interval Algorithm to find equilibrium points

Note that algorithm (1) has three arguments: the system *f*, the initial interval search space X_0 and a parameter ξ that indicates the precision of solutions. The interval subspaces solution **S** is the output. The list $\mathbf{S} = \{X \mid \forall x \in X, (|f(x)| - 0) < \xi\}$ and the list **L** (that contains the interval subspaces to process in the future) are used to carry out the search process.

The algorithm is based on getting first subspace *X* of **L**. If the condition 0 ∉ *F*(*X*) is reached, then the interval subspace *X* does not contain an equilibrium point, so it is rejected . If *F*(*X*) contains zero and precision is not reached then, *X* is divided into two subspaces (X_1,X_2) that are inserted in **L** . The algorithm ends when **L** is empty and returns **S** (a set of intervals that contain equilibrium points).

The above algorithm can be improved using the Interval Newton method that provides quadratically convergence.

4. INTERVAL ARITHMETIC AND THE CLOSED LOOP STABILITY PROBLEM

The interval arithmetic can be applied to assure a robust and stable closed loop non-linear system.

Consider the closed loop first order system of the form (12):

$$\dot{x} = f_{cl}(x, \boldsymbol{\theta}_c), \ \ x \in X_0 \in I, \ \ \boldsymbol{\theta}_c \in \Re^n \qquad (17)$$

where $\boldsymbol{\theta}_c$ are the controller parameters. The purpose is to design a robust regulator adapting the controller parameters of nonlinear equivalent closed loop system defined on the state space.

To assure a stable closed loop, a positive definite scalar function is defined: $V(x) = x^2$. Its first order derivate is $\dot{V}(x) = 2x\dot{x} = 2xf_{cl}(x, \boldsymbol{\theta}_c)$. If $\dot{V}(x) < 0$ $\forall x \in X_0 \in I \ \ x \neq 0$ then $V(x)$ is a Lyapunov function and the closed loop is a stable system. The rest of section proposes search methods to find controller parameters that assure a stable closed loop in X_0.

1.1 Assuring one equilibrium point at origin.

Assuring one equilibrium point at origin and stabilizing the closed loop system is a problem that can be formulated like a constrained optimization problem, that is:

$$\begin{aligned} & \underset{\boldsymbol{\theta}_c, s}{Min}\ s \\ & subject\ to: \\ & \qquad -s \leq f_{cl}(0, \boldsymbol{\theta}_c) \leq s \qquad (18) \\ & \qquad x_i f_{cl}(x_i, \boldsymbol{\theta}_c) < 0 \\ & with: \\ & \qquad 0 \leq s \leq \xi; \ \ i = 1 \ldots n; \ \ x_i < x_{i+1}; \\ & \qquad x_i \in X_0 \in I; \ \ s, \xi \in \Re \end{aligned}$$

where s is a slack variable and ξ is a positive constant. The optimization problem (18) can be resolved using interval arithmetic methods or a more efficient method like SQP local search for a large number of parameters.

1.2 Verifying the stability in all domain.

A solution of (18) does not guarantee the stability of closed loop system represented by (17). To verify stability of closed loop system, algorithm 1 can be used. If there is only one equilibrium point in $x = 0$ the equivalent closed loop system is stable.

1.3 Assuring the stability in all domain

The interval arithmetic can be used to find the parameters $\boldsymbol{\theta}_c$ where (17) is stable with just one parametric search. In this case, (18) can be reformulated dividing the domain X_0 of state variable into n intervals X_i. The interval evaluation of F_{cl} in each X_i, is used to define interval constraints that provides a sufficient condition to guarantee stability in all X_0. The new formulation is showed in (19) where $X_i F_{cl}(X_i, \boldsymbol{\theta}_c)$ is an interval evaluation of $\dot{V}(x)$ and "sup" returns the maximum value of an interval, so $\forall x \in X_i \ \ xf_{cl}(x, \boldsymbol{\theta}_c) \leq \sup\left(X_i F_{cl}(X_i, \boldsymbol{\theta}_c)\right)$. Note that the solution of (19) guarantees the stability in all the state space X_0.

$$\begin{aligned} & \underset{\boldsymbol{\theta}_c, s}{Min}\ s \\ & subject\ to: \\ & \qquad -s \leq f_{cl}(0, \boldsymbol{\theta}_c) \leq s \\ & \qquad \sup\left(X_i F_{cl}(X_i, \boldsymbol{\theta}_c)\right) < 0 \qquad (19) \\ & with: \\ & \qquad 0 \leq s \leq \xi; \ \ i = 1 \ldots n; \ \ X_i \in I; \\ & \qquad \bigcup_{i=1}^{n} X_i = X_0 - \{0\}; \ \ s, \xi \in \Re \end{aligned}$$

1.4 Assuring robust stability.

The condition to assure a stable system is $xf_{cl}(x, \boldsymbol{\theta}_c) < 0 \ \ \forall \ \ x \in X_0 \ \ x \neq 0$, but the model uncertainties and plant perturbations can convert a stable system into an unstable system when $f_{cl}(x)$ is close to zero for $x \neq 0$. Interval constraints can be added to find robust controllers. It is possible to assure a feasible region for the closed loop system where $\dot{V}(x) = xf_{cl}(x, \boldsymbol{\theta}_c)$ remains far from zero for $x \neq 0$. The conditions $|m_1 x| \leq |f_{cl}(x, \boldsymbol{\theta}_c)|$ and $|m_2 x| \geq |f_{cl}(x, \boldsymbol{\theta}_c)| \ \ \forall x \in X_0 \ \ m_1, m_2 \in \Re \ \ 0 > m_1 > m_2$ can be imposes using interval constraints. In this case, (19) can be reformulated adding new interval constraints (20). Fig. 2 shows this feasible region.

$$\begin{aligned} & \underset{\boldsymbol{\theta}_c, s}{Min}\ s \\ & subject\ to: \\ & \qquad -s \leq f_{cl}(0, \boldsymbol{\theta}_c) \leq s \\ & \qquad \sup\left(X_i F_{cl}(X_i, \boldsymbol{\theta}_c) < 0\right) \\ & \qquad \sup\left(|m_1 X_i| - |f_{cl}(X_i, \boldsymbol{\theta}_c)|\right) \leq 0 \\ & \qquad \sup\left(-|m_2 X_i| + |f_{cl}(X_i, \boldsymbol{\theta}_c)|\right) \leq 0 \qquad (20) \\ & with: \\ & \qquad 0 \leq s \leq \xi; \ \ i = 1 \ldots n; \ \ j = 1..m; \\ & \qquad X_i \in I; \bigcup_{i=1}^{n} X_i = X_0 - \{0\}; \\ & \qquad s, \xi, m_1, m_2 \in \Re; \ \ m_2 < m_1 < 0 \end{aligned}$$

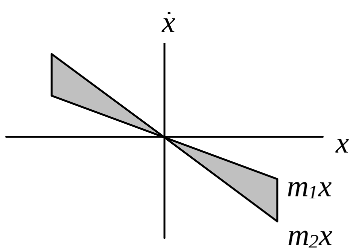

Fig. 2. Feasible conic region of closed loop system

Using the formulation presented in this section, stable fuzzy controller can be designed using the following steps:

Method I:
1.Obtain the fuzzy model (2) from input-output data
2.Find a candidate controller using (18).
3.Verify the stability of closed loop fuzzy model by algorithm (1).

Method II:
1.Obtain the fuzzy model (2) from input-output data
2.Find a stable controller using (19), or a robust controller using (20) .

5. EXAMPLES

Suppose a single input-output first order non-linear dynamic TS fuzzy model constructed from the following rules:

$$
\begin{aligned}
&\text{IF } x \text{ is } A^1 \text{ and } u \text{ is } B^1 \text{ THEN } \dot{x}^1 = -3.1124 - 0.7231x + 1.4355u\\
&\text{IF } x \text{ is } A^2 \text{ and } u \text{ is } B^2 \text{ THEN } \dot{x}^2 = -5.0208 - 0.6083x - 2.2965u\\
&\text{IF } x \text{ is } A^3 \text{ and } u \text{ is } B^3 \text{ THEN } \dot{x}^3 = 1.9320 + 1.1651x + 1.9223u\\
&\text{IF } x \text{ is } A^4 \text{ and } u \text{ is } B^4 \text{ THEN } \dot{x}^4 = 1.8074 - 0.1921x + 2.0752u\\
&\text{IF } x \text{ is } A^5 \text{ and } u \text{ is } B^5 \text{ THEN } \dot{x}^5 = 3.7876 - 2.0436x + 1.5483u\\
&\text{IF } x \text{ is } A^6 \text{ and } u \text{ is } B^6 \text{ THEN } \dot{x}^6 = -1.3423 + 0.0885x + 0.3792u\\
&\text{IF } x \text{ is } A^7 \text{ and } u \text{ is } B^7 \text{ THEN } \dot{x}^7 = 0.3757 + 0.9975x + 0.0210u\\
&\text{IF } x \text{ is } A^8 \text{ and } u \text{ is } B^8 \text{ THEN } \dot{x}^8 = 4.9533 + 2.3863x + 1.1502u \quad (21)\\
&\text{IF } x \text{ is } A^9 \text{ and } u \text{ is } B^9 \text{ THEN } \dot{x}^9 = -5.1089 + 0.7818x - 2.5545u\\
&\text{IF } x \text{ is } A^{10} \text{ and } u \text{ is } B^{10} \text{ THEN } \dot{x}^{10} = -4.6787 - 7.8010x - 0.9931u\\
&\text{IF } x \text{ is } A^{11} \text{ and } u \text{ is } B^{11} \text{ THEN } \dot{x}^{11} = 0.0866 - 3.9195x + 0.9690u\\
&\text{IF } x \text{ is } A^{12} \text{ and } u \text{ is } B^{12} \text{ THEN } \dot{x}^{12} = 1.5365 + 2.0129x + 0.7611u\\
&\text{IF } x \text{ is } A^{13} \text{ and } u \text{ is } B^{13} \text{ THEN } \dot{x}^{13} = -5.1570 + 0.8969x - 1.7535u\\
&\text{IF } x \text{ is } A^{14} \text{ and } u \text{ is } B^{14} \text{ THEN } \dot{x}^{14} = -1.2878 - 1.5183x - 1.5695u\\
&\text{IF } x \text{ is } A^{15} \text{ and } u \text{ is } B^{15} \text{ THEN } \dot{x}^{15} = -4.7647 + 0.2327x - 2.3819u
\end{aligned}
$$

The open loop fuzzy model (u = 0) has three equilibrium points as is showed in Fig.4: one unstable and two stables.

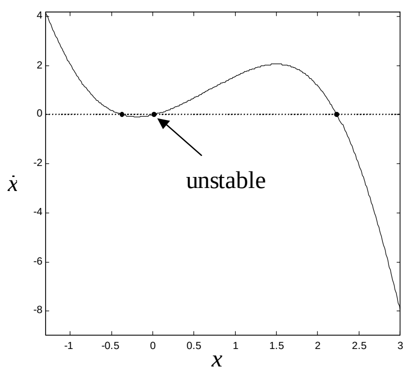

Fig. 4. The open loop fuzzy model

Applying the algorithm (1) where the input f is the fuzzy model (21), the state space is defined on $X_0 = [-3, 3]$ and a precision factor ξ = 0.000001 results in:

[2.236934, 2.236937]
[-4.768371e-006, 4.768371e-006]
[-0.372862, -0.372851]

The above results are the intervals that contain the equilibrium points of (21), (see Fig.4).

The next step is to design a controller than can stabilize the plant (21) around one equilibrium point at origin (x = 0). There are two different methods that were described in section IV.

Method I:

Applying the constrained optimization problem (18) with X_0 = [-2.5, 3], a distance of 0.1 between two consecutive x_i and ξ = 0.00001 results in the following controller parameters:

$$
\begin{aligned}
&\text{IF } x \text{ is } C^1 \text{ THEN } u^1 = 0.12985770528635 - 0.00864347446107x\\
&\text{IF } x \text{ is } C^2 \text{ THEN } u^2 = 0.73058213383531 - 0.71784375456007x\\
&\text{IF } x \text{ is } C^3 \text{ THEN } u^3 = -0.55777780417901 - 2.72825012841125x\\
&\text{IF } x \text{ is } C^4 \text{ THEN } u^4 = -0.65189614404829 - 1.04548270823069x\\
&\text{IF } x \text{ is } C^5 \text{ THEN } u^5 = -0.03141068256311 - 0.03907693556821x
\end{aligned}
$$

The next step is to verify the global stability using the algorithm (1). In this case, there is just one equilibrium point. (See fig 5).

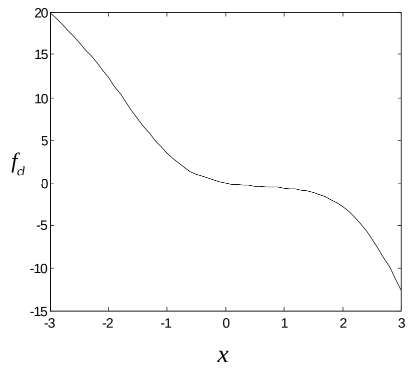

Fig. 5. Closed loop system applying (18) using a distance of 0.1

Method II:

Applying the constrained optimization problem (19) with X_0 = [-3,3] and a width of 0.1 for intervals X_i results in a controller that guarantees the system stability in X_0, so algorithm (1) is not necessary. (See Fig. 6)

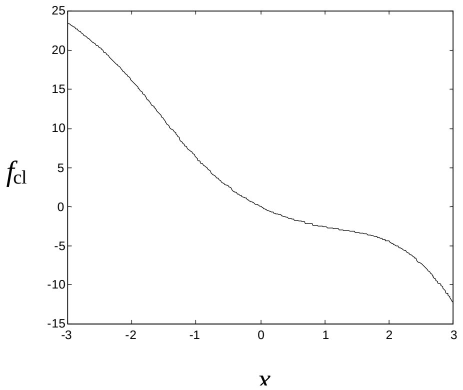

Fig. 6. Closed loop system applying (19) and using a width of 0.1

A more robust controller can be found applying the constrained optimization problem (20) with $X_0 = [-3,3]$, m_1=-3, m_2=-5 and a width of 0.05 for intervals X_i. Fig. 7 shows that f_{cl} remains between lower and upper bounds.

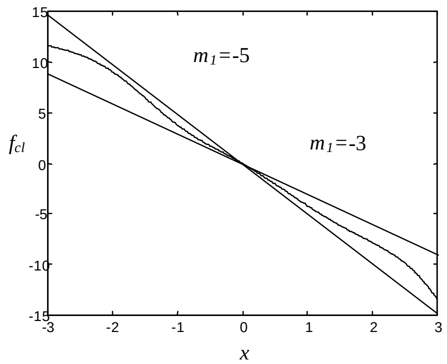

Fig. 7. Closed loop system applying (21) and using X_0 = [-3,3], m_1=-3, m_2=-5 and a width of 0.05

6. CONCLUSIONS

In this paper, a design approach that can be used to construct a stable fuzzy controller based on a fuzzy plant model has been developing. The first proposed method uses a parametric search to assure one equilibrium point at origin and an interval algorithm to verify that controller parameters conform a stable in all domain closed loop nonlinear function equivalent to the fuzzy plant model and fuzzy controller. The second method uses the interval arithmetic to assure the stability in all domain of closed loop function on just one parametric search algorithm.

The methods proposed here can be extended to find the possible presence of limit cycles on second order systems based on Pointcaré-Bendixon theorem. The interval mathematics application to high order systems will be described in a future paper.

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