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POSITIVE POLYNOMIAL MATRICES AND IMPROVED LMI ROBUSTNESS CONDITIONS

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Abstract: Recently several new LMI conditions for stability of linear systems have been proposed, introducing additional slack variables to reduce the gap between conservative convex quadratic stability conditions and intractable non-convex robust stability conditions. In this paper we show that these improved LMI conditions can be derived with the help of some basic results on positive polynomial matrices, providing a clear interpretation of the role of the additional variables. The approach allows to derive in a unifying way results in the state-space and polynomial frameworks. Applications to robust stability analysis and robust stabilization of systems with multi-linear parametric uncertainty are fully described.

Keywords: Linear Systems, Robustness, LMI, State-space Methods, Polynomial Methods

1. INTRODUCTION

The main criticism formulated by control engineers against modern robust analysis and design methods for linear systems concerns the lack of efficient, easy-to-use and systematic numerical tools. Indeed, a lot of analysis techniques and most of the design techniques for uncertain systems boil down to non-convex bilinear matrix inequality (BMI) problems for which no polynomial-time algorithm has been proposed so far. This is especially true when analyzing robust stability of systems affected by highly structured (parametric) uncertainty, or when seeking a robust control law of low complexity (fixed- or low-order controller design).

In the control literature, one can distinguish several fundamentally different approaches to circumvent these difficulties. One can either resort to (a) global optimization, with a guarantee of convergence, at the price of a considerable computational burden that may prove prohibitive; (b) randomized or probabilistic methods that allow to quantify the amount of confidence one can have in the result; (c) iterative heuristics based on convex optimization, without guarantee of global convergence, but that can prove very efficient in practice and can solve problems at a low computational cost; (d) non-iterative sufficient robustness conditions, that rely on convex optimization too, but can prove overly conservative. In this paper, we will focus on the latter approach.

Within this scope, new LMI conditions for stability of linear systems have recently been introduced that partially alleviates some degrees of conservatism. The conditions, that can be traced back to (Geromel, 1998), were originally proposed for discrete-time systems only (de Oliveira, 1999). Afterwards, then have been extended to more general stability regions (Peaucelle, 2000) and other problems in robust control. The improved LMI conditions feature additional slack variables allowing some decoupling between the system state-space matrices and the Lyapunov matrix proving robust stability. Based on the same idea, similar conditions have also been used to assess robust stability of polynomial matrices (Henrion, 2001), an important step when studying uncertain systems in the polynomial framework (Kučera, 1979). Numerical experiments reported in (Bachelier, 1999) seem to illustrate that the improved LMI conditions significantly narrow the gap between conservative LMI quadratic stability conditions and intractable non-convex robust stability conditions. If most of the people agree on the useful decoupling structure in the new LMI conditions, no satisfying interpretation of these additional slack variables has been found so far, up to our knowledge.

The purpose of this paper is to provide such an interpretation, and to show how the improved LMI conditions can be readily derived from basic results on strictly positive real functions and positive polynomial matrices. The approach is sufficiently general to cover in a unified fashion continuous-time and discrete-time systems in the state-space and polynomial frameworks.

Note that because of space limitation we had to remove all the numerical examples motivating and illustrating our results. The interested reader is referred to the full version of our paper, available at

`www.laas.fr/˜henrion/Papers/varadd.ps.gz`

2. POSITIVITY AND STABILITY OF POLYNOMIAL MATRICES

Let

$$\mathcal{D} = \{s \in \mathbb{C} : \begin{bmatrix} 1 \\ s \end{bmatrix}^\star \underbrace{\begin{bmatrix} a & b \\ b^\star & c \end{bmatrix}}_{H} \begin{bmatrix} 1 \\ s \end{bmatrix} < 0\}$$

be a stability region in the complex plane, where the star denotes transpose conjugate and Hermitian matrix $H = H^\star$ has inertia $(1, 0, 1)$. Standard choices for $\mathcal{D}$ are the left half-plane ($a = 0$,$b = 1$,$c = 0$) and the unit disk ($a = -1$,$b = 0$,$c = 1$). Other choices of scalars a, b and c correspond to arbitrary half-planes and disks. Let $\partial\mathcal{D}$ denote the one-dimensional closure of $\mathcal{D}$, i.e. the set $\{s \in \mathbb{C} : a + bs + b^\star s^\star + css^\star = 0\}$. In the sequel we say that a polynomial matrix is stable when all the roots of its determinant belong to $\mathcal{D}$. Similarly, we say that a rational matrix is strictly positive real (SPR) when its real part is positive definite when evaluated along $\partial\mathcal{D}$.

In the paper we will consider square polynomial matrices $N(s) = N_0 + N_1 s + \cdots + N_d s^d$ and $D(s) = D_0 + D_1 s + \cdots + D_d s^d$ of size n and degree d.

Lemma 1. Polynomial matrix $N(s)$ is stable if and only if there exists a stable polynomial matrix $D(s)$ such that rational matrix $N(s)D^{-1}(s)$ is SPR.

Proof: From the definition of an SPR rational matrix, $N(s)D^{-1}(s)$ SPR with $D(s)$ stable implies $N(s)$ stable. Conversely, if $N(s)$ is stable then the choice $D(s) = N(s)$ makes the rational matrix $N(s)D^{-1}(s) = I$ obviously SPR. □

When $D(s)$ is a given stable polynomial matrix, Lemma 1 is a sufficient condition for stability of matrix polynomial $N(s)$. Let $\mathcal{N}$ denote the set of coefficients of all polynomial matrices $N(s)$ of degree d, let $\mathcal{S}$ denote the subset of $\mathcal{N}$ corresponding to stable polynomial matrices only, and let $\mathcal{S}_D$ denote the subset of $\mathcal{N}$ such that $N(s)D^{-1}(s)$ is SPR. Then it holds $\mathcal{S}_D \subset \mathcal{S} \subset \mathcal{N}$. It is well-known that $\mathcal{S}$ is generally not a convex set. In contrast, it turns out that $\mathcal{S}_D$ is a convex set (an inner approximation of the actual stability domain) that can be described by a linear matrix inequality (LMI), as captured by the following

Lemma 2. Let $N = [N_0 N_1 \cdots N_d]$, $D = [D_0 D_1 \cdots D_d]$ and

$$\Pi = \begin{bmatrix} I_n & & & 0 \\ & \ddots & & \vdots \\ & & I_n & 0 \\ 0 & I_n & & \\ \vdots & & \ddots & \\ 0 & & & I_n \end{bmatrix}$$

be a projection matrix of size $2dn \times (d+1)n$. Given a stable polynomial matrix $D(s)$, polynomial matrix $N(s)$ ensures SPRness of rational matrix $N(s)D^{-1}(s)$ if and only if there exists a matrix $P = P^T$ of dimension dn such that

$$D^T N + N^T D - H(P) \succ 0 \qquad (1)$$

where

$$H(P) = \Pi^T (H \otimes P)\Pi = \Pi^T \begin{bmatrix} aP & bP \\ b^\star P & cP \end{bmatrix} \Pi.$$

Proof: SPRness of $N(s)D^{-1}(s)$ is equivalent to the infinite dimensional matrix inequality $N(s)D^{-1}(s) + D^{-\star}(s)N^\star(s) \succ 0$ for all s in $\partial\mathcal{D}$ where the star denotes transpose conjugate. Polynomial matrix $D(s)$ being non-singular along $\partial\mathcal{D}$, we can equivalently write $D^\star(s)N(s) + N^\star(s)D(s) \succ 0$ for all s in $\partial\mathcal{D}$. Let $\Pi_d(s) = [I_n \; sI_n \; \cdots \; s^d I_n]^T$ be a basis for polynomial matrices of size n and degree d. It holds $\Pi_d^\star(s)\Pi_d(s) = (1 + ss^\star + \cdots + (ss^\star)^d)I_n \succ 0$ for all s in $\partial\mathcal{D}$ so that there always exists a strictly positive scalar γ such that $R(s) = D^\star(s)N(s) + N^\star(s)D(s) - \gamma\Pi_d^\star(s)\Pi_d(s) \succeq 0$ for all s in $\partial\mathcal{D}$. Therefore, SPRness of the original rational matrix is equivalent to positivity of polynomial matrix $R(s)$ along the stability boundary. This idea has already been pursued e.g. in (Stipanović, 2000).

From the structure of the linear map $H(P)$, one can check that $\Pi_d^\star(s)H(P)\Pi_d(s) = (a + bs + b^\star s^\star + css^\star)\Pi_{d-1}^\star(s)P\Pi_{d-1}(s) = 0$ for all s in $\partial\mathcal{D}$ and it is easy to see that matrix $P = P^T$ captures the whole degrees of freedom one has when representing polynomial matrix $R(s)$ in the basis $\Pi_d(s)$, see e.g. (Genin, 2000). In other words, $R(s) = \Pi_d^\star(s)R\Pi_d(s)$ if and only if there exists $P = P^T$ such that $R = D^TN + N^TD - H(P) - \gamma I_{(d+1)n}$.

The remaining of the proof then consists in showing that $R(s) \succeq 0$ along $\partial\mathcal{D}$ if and only $R \succeq 0$ for some $P = P^T$ and some $\gamma > 0$, or equivalently, if and only if LMI (1) holds for some $P = P^T$. This follows from the decomposition as a sum of squares of $R(s) = \sum_i Q_i^\star(s)Q_i(s)$, see e.g. (Nesterov, 2000; Lasserre, 2001). □

Combining Lemma 1 with Lemma 2, we deduce the following result which will be instrumental to the derivation of the LMI stability conditions throughout the sequel of the paper.

Lemma 3. Polynomial matrix $N(s)$ is stable if and only if there exists a stable polynomial matrix $D(s)$ and a matrix $P = P^T$ satisfying LMI (1).

3. CONNECTION WITH EXISTING LMI STABILITY CONDITIONS

In this section, we show that we can easily recover the improved LMI stability conditions of (de Oliveira, 1999; Peaucelle, 2000) with the help of Lemma 3. For notational simplicity, throughout the paper we will denote symmetric blocks in LMIs by a star.

3.1 *State-space Framework*

Stability of the pencil matrix $N(s) = sI - A$ is equivalent to stability of the constant matrix A, and the LMI condition of Lemma 3 yields the following

Lemma 4. Matrix A is stable if and only if there exists a stable matrix F and a matrix $P = P^T$ satisfying

$$\begin{bmatrix} F^TA + A^TF - aP & \star \\ -A - F - b^\star P & 2I - cP \end{bmatrix} = \begin{bmatrix} -F^T \\ I \end{bmatrix} \begin{bmatrix} -A & I \end{bmatrix} + \begin{bmatrix} -A^T \\ I \end{bmatrix} \begin{bmatrix} -F & I \end{bmatrix} - \begin{bmatrix} aP & bP \\ b^\star P & cP \end{bmatrix} \succ 0. \quad (2)$$

Proof: Just let $D(s) = sI - F$ in Lemma 3. □

So far we have made little progress since we have shown that checking stability of some matrix A amounts to checking stability of some other matrix F plus checking some LMI condition in a matrix $P = P^T$. The next result shows that, provided P is positive definite, one can relax the stability constraint on F.

Lemma 5. Matrix A is stable if and only if there exists a matrix F and a matrix $P = P^T \succ 0$ satisfying LMI (2).

Proof: Assume that LMI (2) holds for some $P = P^T \succ 0$. Then

$$\begin{aligned} & -\begin{bmatrix} I \\ A \end{bmatrix}^T \left(\begin{bmatrix} -F^T \\ I \end{bmatrix} \begin{bmatrix} -A & I \end{bmatrix} + \begin{bmatrix} -A^T \\ I \end{bmatrix} \begin{bmatrix} -F & I \end{bmatrix} \right. \\ & \quad \left. - \begin{bmatrix} aP & bP \\ b^\star P & cP \end{bmatrix} \right) \begin{bmatrix} I \\ A \end{bmatrix} \\ & \quad = aP + bPA + b^\star A^TP + cA^TPA \prec 0. \end{aligned}$$

In virtue of Lyapunov's Theorem (see e.g. (Henrion, 2001c)), this implies that A is stable. Conversely, if A is stable then by Lemma 4 there exists a stable matrix F and a matrix $P = P^T$ satisfying LMI (2). Then

$$\begin{aligned} & -\begin{bmatrix} I \\ F \end{bmatrix}^T \left(\begin{bmatrix} -F^T \\ I \end{bmatrix} \begin{bmatrix} -A & I \end{bmatrix} + \begin{bmatrix} -A^T \\ I \end{bmatrix} \begin{bmatrix} -F & I \end{bmatrix} \right. \\ & \quad \left. - \begin{bmatrix} aP & bP \\ b^\star P & cP \end{bmatrix} \right) \begin{bmatrix} I \\ F \end{bmatrix} \\ & \quad = a + bPF + b^\star F^TP + cF^TPF \prec 0. \end{aligned}$$

Since F is stable, we can invoke Theorem 2.2.3 in (Horn, 1991) to conclude that $P \succ 0$. □

3.2 *Polynomial Framework*

Now if we assume that $N(s)$ is an arbitrary polynomial matrix, Lemma 5 becomes

Lemma 6. Polynomial matrix $N(s)$ is stable if and only if there exists a polynomial matrix $D(s)$ and a matrix $P = P^T \succ 0$ satisfying

$$D^TN + N^TD - H(P) \succ 0. \quad (3)$$

Proof: We proceed as in the proof of Lemma 5, showing that the requirement that $D(s)$ be stable in Lemma 3 amounts to enforcing positivity of P. The projection of LMI (3) must be done on the subspace spanned by the columns of matrix

$$\begin{bmatrix} I_n & & \\ & \ddots & \\ & & I_n \\ -D_0 & \cdots & -D_{d-1} \end{bmatrix}$$

and stability is proved with Lyapunov's Theorem applied to the companion matrix associated with polynomial matrix $D(s)$, which is here assumed to be monic for simplicity. The non-monic case can be treated similarly. □

A slightly different version of Lemma 6 appeared in (Henrion, 2001b), with a different proof based on quadratic programming and semidefinite programming duality.

Lemma 7. (Henrion, 2001b) Polynomial matrix $N(s)$ is stable if and only if there exists a matrix $P = P^T \succ 0$ satisfying $N^T N - H(P) \succ 0$.

Proof: It is an application of the projection lemma, an algebraic result well-known to the control community, see e.g. (Skelton, 1998, Theorem 2.3.12). □

4. APPLICATION TO ROBUST STABILITY ANALYSIS

4.1 *State-space Framework*

The nice feature in Lemma 5 is that system matrix A and Lyapunov matrix P are decoupled in the sense that, contrary to the standard Lyapunov inequality, there is no cross-product between A and P. When A is affected by parametric uncertainty, this allows for the construction of a parameter-dependent Lyapunov matrix significantly narrowing the gap between conservative standard quadratic stability results (corresponding to a parameter-independent Lyapunov matrix) and intractable robust stability results.

We assume that system matrix is affected by multi-linear (or multi-affine) uncertainty, i.e. $A(\lambda)$ depends on a m-dimensional vector of uncertain scalar parameters λ_i living in some hyper-rectangle Λ, and $A(\lambda)$ is linear in the parameter λ_i when all the remaining parameters λ_j, $j \neq i$ are fixed. For notational simplicity, we denote by A_i for $i = 1, \ldots, m$ the vertices of $A(\lambda)$ obtained by enumerating each vertex of Λ. The following robust stability analysis result is a straightforward application of Lemma 5.

Lemma 8. (de Oliveira, 1999; Peaucelle, 2000) Multi-linear matrix $A(\lambda)$ with vertices A_i is robustly stable if there exists a matrix F and matrices $P_i = P_i^T \succ 0$ satisfying the LMI

$$\begin{bmatrix} F^T A_i + A_i^T F - aP_i & \star \\ -A_i - F - b^\star P_i & 2I - cP_i \end{bmatrix} \succ 0, \quad i = 1, \ldots, m. \tag{4}$$

Proof: Since LMI (4) is simultaneously linear in A_i and in P_i, parameter λ enters multi-linearly in $A(\lambda)$, and Λ is a hyper-rectangle, it follows that

$$\begin{bmatrix} F^T A(\lambda) + A^T(\lambda) F - aP(\lambda) & \star \\ -A(\lambda) - F - b^\star P(\lambda) & 2I - cP(\lambda) \end{bmatrix} \succ 0$$

where $P(\lambda) \succ 0$ is a parameter-dependent multi-linear Lyapunov matrix proving robust stability of matrix $A(\lambda)$ in virtue of Lemma 5. □

4.2 *Polynomial Framework*

Similarly, the LMI condition of Lemma 6 is simultaneously linear in coefficients of $N(s)$ and in Lyapunov matrix P. This property can be exploited to provide less conservative convex conditions for assessing robust stability of polynomial matrices.

As in the previous section, we assume that $N(s, \lambda)$ is a polynomial matrix with multi-linear dependence in a parameter vector λ that lives in a hyper-rectangle. When λ describes the vertices of Λ we will denote the vertices of $N(s, \lambda)$ by $N_i(s)$ for $i = 1, \ldots, m$. With these notations, the polynomial counterpart of Lemma 8 can be formulated as follows.

Lemma 9. (Henrion, 2001) Multi-linear polynomial matrix $N(s, \lambda)$ with vertices $N_i(s)$ is robustly stable if there exists a matrix D and some matrices $P_i = P_i^T \succ 0$ satisfying the LMI

$$D^T N_i + N_i^T D - H(P_i) \succ 0, \; i = 1, \ldots, m. \tag{5}$$

Proof: See the proof of Lemma 8, where we use Lemma 6. □

5. APPLICATION TO ROBUST DESIGN

5.1 *State-space Framework*

In this section we will consider the problem of robust structured static state feedback of a system whose matrices $A(\lambda)$ and $B(\lambda)$ are multi-linear functions of the parameters λ_i living in a hyper-rectangle Λ accounting for the uncertainty affecting the system. We denote by A_i and B_i for $i = 1, \ldots, m$ the respective vertices of $A(\lambda)$ and $B(\lambda)$ when λ describes the vertices of Λ.

We assume that the sought static state feedback matrix K is subject to convex structural constraints that can be captured by an LMI

$$G(K) \succeq 0. \tag{6}$$

For example, one can impose a block diagonal structure on K to address problems of decentralized control. We can also enforce a quadratic norm constraint $K^T K \preceq I$ to ensure a low-gain feedback. Finally, the problem of robust static output feedback can also fit this scope, the static state feedback matrix being linearly constrained to $K = K_0 C$ for some static output feedback matrix K_0 to be found.

In Lemma 4, LMI (2) is simultaneously linear in system matrix A and Lyapunov matrix P, so we can easily derive the following

Lemma 10. System $A(\lambda)$, $B(\lambda)$ with multi-linear uncertainty and vertices A_i and B_i is robustly stabilizable by a constrained static state feedback K if, given a stable matrix F, there exists some matrices $P_i = P_i^T$ satisfying the LMI

$$\begin{bmatrix} \begin{pmatrix} F^T(A_i + B_i K) \\ +(A_i + B_i K)^T F \\ -aP_i \end{pmatrix} & \star \\ \begin{pmatrix} -(A_i + B_i K) \\ -F - b^\star P_i \end{pmatrix} & 2I - cP_i \end{bmatrix} \succ 0 \quad (7)$$
$$i = 1, \ldots, m$$

with the additional LMI constraint (6).

5.2 *Polynomial Framework*

Now we will extend Lemma 3 to perform robust design of linear systems described by polynomial matrices. The transfer function of the uncertain linear system is $B(s,\lambda)A^{-1}(s,\lambda)$ where both $A(s,\lambda)$ and $B(s,\lambda)$ are polynomial matrices that are multi-linear in a parameter vector λ that lives in a hyper-rectangle Λ. The design problem amounts to finding a dynamical output-feedback controller with transfer function $X^{-1}(s)Y(s)$ such that the closed-loop denominator matrix $N(s,\lambda) = A(s,\lambda)X(s) + B(s,\lambda)Y(s)$ is robustly stable for all admissible uncertainty vector λ. As usual, when λ describes the vertices of Λ we will denote the vertices of $A(s,\lambda)$, $B(s,\lambda)$ and $N(s,\lambda)$ by A_i, B_i and N_i respectively, for $i = 1, \ldots, m$.

Exactly as we assumed in the state-space framework that the static feedback matrix K satisfies structural LMI constraints, we assume here that the controller polynomial matrices $X(s) = X_0 + X_1 s + \cdots$ and $Y(s) = Y_0 + Y_1 s + \cdots$ entering linearly in polynomial matrix $N(s,\lambda)$ have prescribed structure, which we denote by the LMI

$$G(N) \succeq 0. \quad (8)$$

For example, we can assume that $X(s)$ and $Y(s)$ have given (presumably low) degree. Some coefficients in $X(s)$ and $Y(s)$ may be given, such as with a PID controller $\frac{Y(s)}{X(s)} = k_P + \frac{k_I}{s} + k_D s$ where $X_0 = 0$, $X_1 = 1$, $X_2 = 0$ and $Y_0 = k_I$, $Y_1 = k_P$, $Y_2 = k_D$.

Under these assumptions, the polynomial counterpart of Lemma 10 can be formulated as follows.

Lemma 11. System $A(s,\lambda)$, $B(s,\lambda)$ with multi-linear uncertainty and vertices $A_i(s)$, $B_i(s)$ is robustly stabilizable by a constrained output feedback controller $X(s)$, $Y(s)$ if, given a stable polynomial matrix $D(s)$ of the same degree as polynomial matrices $N_i(s) = A_i(s)X(s) + B_i(s)Y(s)$, there exists some matrices $P_i = P_i^T$ satisfying the LMI

$$D^T N_i + N_i^T D - H(P_i) \succ 0, \; i = 1, \ldots, m. \quad (9)$$

with the additional LMI constraint (8).

6. CONCLUSION

We have proposed a new approach to derive improved LMI robustness conditions, narrowing the gap between conservative convex quadratic stability conditions and intractable non-convex robust stability conditions. The approach is general enough to treat in a unifying way continuous-time and discrete-time systems in the state-space and polynomial frameworks. It is based on the theory of positive polynomial matrices: stability of a polynomial matrix $N(s)$ is ensured if and only if some bilinear polynomial matrix form in $N(s)$ and $D(s)$ is positive along the stability boundary for some stable polynomial matrix $D(s)$ to be found.

Usually, polynomial matrix $N(s)$ is unknown (it may depend linearly on design parameters) and polynomial matrix $D(s)$ is given (it plays the role of an additional slack variable decoupling $N(s)$ from the Lyapunov matrix). So when $N(s)$ is assumed to belong to a given set accounting for possible uncertainty affecting some dynamical system, one important step in solving the robustness problem amounts to finding a sensible choice of an additional variable $D(s)$. Based on the numerical examples described in the full version of this paper, it is tempting to set $D(s)$ to some nominal value of $N(s)$, that is to say the nominal closed-loop system denominator matrix obtained with a standard design algorithm in the absence of uncertainty. In particular, it is believed that such a choice can prove useful in heuristic iterative design schemes based on cone complementarity, where $N(s)$ and $D(s)$ are iteratively sought and an initial guess on $D(s)$ is generally required to start the overall algorithm. However, recent findings on robust SPR design with l_2 uncertainty (Bianchini, 2001) tend to show that the choice of the nominal polynomial $N(s)$ as an additional variable $D(s)$ is not always appropriate. This point needs to be studied in deeper detail.

We have studied robust stability matrices and polynomial matrices affected by (highly structured) multi-linear interval uncertainty. We are now applying the same ideas to linear systems affected by (unstructured) ellipsoidal uncertainty, also known as l_2, or rank-one uncertainty. We may also extend our approach to meet other requirements that mere stability, such as H_2 or H_∞ performance, tracking or disturbance rejection. Related results will be reported elsewhere.

Besides providing a clear, new interpretation of the additional variables featured in the improved LMI robustness conditions, the approach proposed in this paper is also interesting from the numerical point of view. Indeed, the Hankel or Toeplitz structure of the dual LMI problem associated with problems coming from positivity of polynomials can be exploited to reduce the computational cost of solving the original primal problem, as shown in (Alkire, 2001). Numerical properties of these kind of optimization problems must however be studied in further detail, since the Hankel structure is often synonymous of bad numerical conditioning, as pointed out in (Genin, 2000b).

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