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T-DOF GENERALIZED PREDICTIVE CONTROL AND AN APPLICATION FOR A LARGE DEAD TIME PROCESS OF A THERMAL POWER PLANT

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Abstract: In this paper we describe a two-degree of freedom (T-DOF) configuration of a generalized predictive control (GPC) and an application for an NOx decomposition process of a combined cycle power plant. T-DOF GPC that is designed with feedforward signals, measurable disturbance signals and a reference signal is able to prevent an undesirable action that is caused by the feedforward control. We also describe that the present T-DOF GPC is a similar to the controller with the minimal order observer. We show experimental results accomplished on a commercial power plant.

Keywords: Two-degree of freedom control, model predictive control, dead time process, thermal power plant

1. INTRODUCTION

It is well known that there is a difficulty to regulate a dead time process with a disturbance. A feedforward control which uses a measurable disturbance signal is an effective method to reduce the error caused by the disturbance. It is also effective to use the reference signal for a quick response to a setpoint change. These methods are used widely in process control fields with a proportional and integral controller (PI controller) (Seborg *et al.* 1989) and are regarded as a kind of T-DOF control (Skogestad and Postlethwaite 1996).

Generalized predictive control (GPC) (Bitmead *et al.* 1990)(Clarke *et al.* 1987) (Clarke and Mohtad 1989) has several applications in process control. A GPC design is able to make a suitable feedback property for a process with a dead time, since it retains a process model. The conventional GPC design does not include explicitly a disturbance signal, because it regards a disturbance as an unmesurable noise.

In this paper we describe a two-degree of freedom (T-DOF) configuration of a generalized predictive control (GPC). T-DOF GPC that is designed with feedforward signals, measurable disturbance signals and a reference signal has good properties described above and also is able to prevent an undesirable action which is caused by the feedforward control. We also describe that the present T-DOF GPC is a similar to the state feedback control with a minimal order observer.

Finally we show experimental results accomplished on a commercial power plant.

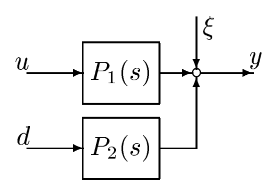


Fig. 1. Plant

2. CONTROL DESIGN

We consider a SISO plant with a measurable and unmeasurable disturbance shown as Fig.1, where y is an output, u is an input, d is a measurable disturbance and ξ is an unmeasurable disturbance. A conventional GPC is designed by using only the plant $P_1(s)$ as Fig.2, where r is a reference.

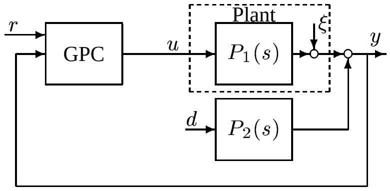


Fig. 2. Conventional GPC design

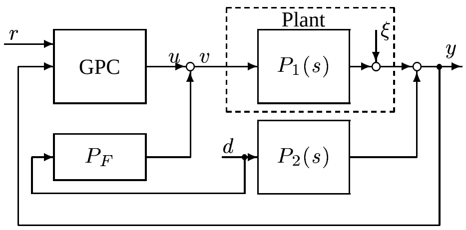


Fig. 3. GPC with a feedforward control

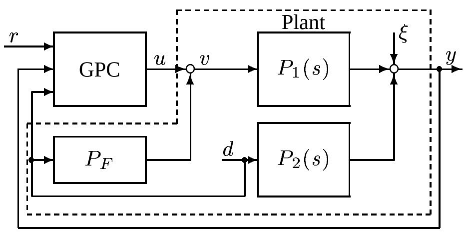


Fig. 4. GPC design with a extended model

In order to use a disturbance feedforward, a usual configuration of a controller is shown as Fig.3, where P_F is a feedforward control. In this configuration GPC does not contain the disturbance signal d, and therefore GPC and the feedforward control work independently. Figure 4 shows a GPC design with a disturbance feedforward. In this case the GPC is designed based on the extended plant which contains P_1, P_2 and P_F.

We write a plant model of Fig.4 by using the relation $v = u + P_f d$,

$$\begin{aligned} y &= P_1(s)(u + P_F(s)d) + P_2(s)d + \xi \\ &= P_1(s)u + (P_1(s)P_F(s) + P_2(s))d + \xi. \end{aligned} \quad (1)$$

The discrete time model of equation (1) is written by

$$\begin{aligned} A(z^{-1})y(t) = B(z^{-1})\Delta u(t-1) \\ +D(z^{-1})d(t). \end{aligned} \quad (2)$$

where we use the same symbols y, w and d for discrete time signals for simplicity. $A(z^{-1}),\ B(z^{-1}),\ D(z^{-1})$ are polynomials of z^{-1},

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + \cdots + a_{n+1} z^{-(n+1)} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} + \cdots + b_{n-1} z^{-(n-1)} \\ D(z^{-1}) &= d_0 + d_1 z^{-1} + \cdots + d_n z^{-(n)}. \end{aligned}$$

Δ is $1 - z^{-1}$.

We define the polynomials $E_j(z^{-1}),\ F_j(z^{-1}),\ G_j(z^{-1}),\ S_j(z^{-1}),\ M_j(z^{-1}),\ N_j(z^{-1})$ from the following Diophantine equations.

$$\begin{aligned} T(z^{-1}) &= E_j(z^{-1})A(z^{-1}) + z^{-j}F_j(z^{-1}) \\ E_j(z^{-1})B(z^{-1}) &= T(z^{-1})G_j(z^{-1}) + z^{-j}S_j(z^{-1}) \\ E_j(z^{-1})D(z^{-1}) &= T(z^{-1})M_j(z^{-1}) + z^{-j}N_j(z^{-1}) \end{aligned}$$

where $T(z^{-1})$ is a filter polynomial with coefficients $t_0, t_1, \cdots, t_n$. The coefficients of these polynomials can be explicitly written (Masuda *et al.* 1999). We define a lower triangular toeplitz matrix T_A,

$$T_A = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ a_1 & 1 & & \\ \vdots & \ddots & \ddots & \\ a_N & \cdots & a_1 & 1 \end{bmatrix} \quad (a_i = 0 \text{ if } i \geq n+2).$$

Then the coefficients of the polynomials $E_j(z^{-1})$, $G_j(z^{-1})$, $F_j(z^{-1})$, $S_j(z^{-1})$ are obtained,

$$\begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ e_N \end{bmatrix} = T_A^{-1} \begin{bmatrix} 1 \\ t_1 \\ \vdots \\ t_N \end{bmatrix} \quad (t_i = 0 \text{ if } i \geq n+1)$$

$$\begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_N \end{bmatrix} = T_A^{-1} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_N \end{bmatrix} \quad (b_i = 0 \text{ if } i \geq n)$$

$$\begin{bmatrix} f_0^j \\ f_1^j \\ \vdots \\ f_n^j \end{bmatrix} = T_A \begin{bmatrix} e_j \\ e_{j+1} \\ \vdots \\ e_{j+n} \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} s_0^j \\ s_1^j \\ \vdots \\ s_{n-1}^j \end{bmatrix} &= \begin{bmatrix} b_j & \cdots & b_1 \\ b_{j+1} & & b_2 \\ \vdots & & \\ b_{j+n-1} & & b_n \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ e_{j-1} \end{bmatrix} \\ &\quad - \begin{bmatrix} t_j & \cdots & t_1 \\ t_{j+1} & & t_2 \\ \vdots & & \\ t_{j+n-1} & & t_n \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{j-1} \end{bmatrix} \end{aligned}$$

The coefficients of the polynomials $E_j(z^{-1})$ and $G_j(z^{-1})$ are independent of j. Similar expressions for polynomials $M_j(z^{-1})$ and $N_j(z^{-1})$ can be derived.

The j-step ahead prediction $\hat{y}(t+j)$ of $y(t)$ is,

$$\hat{y}(t+j) = G_j(z^{-1})\Delta u(t+j-1) + h_j(t) \quad (3)$$

where $h_j(t)$ is,

$$\begin{aligned} h_j(t) = \frac{1}{T(z^{-1})} \big(& E_j(z^{-1})y(t) + S_j(z^{-1})\Delta u(t-1) \\ & + N_j(z^{-1})d(t) \big) . \end{aligned} \quad (4)$$

Performance index is defined by,

$$J = \sum_{j=1}^{N_P} \left\{ (\hat{y}(t+j) - r(t+j))^2 + \lambda \Delta u(t+j-1)^2 \right\} \quad (5)$$

where $r(t+j)$ is the j-steps ahead setpoint. The future control action $\Delta u(t+j-1),\ \ j=1,\cdots,N_P$ which minimizes the performance index is obtained by the equation $\partial J/\partial \Delta \hat{u} = 0$.

$$\Delta \hat{u} = -(\lambda I + G^{\mathrm{T}} G)^{-1} G^{\mathrm{T}} (h - \hat{r}) \quad (6)$$

where $h = [h_1(t)\ \cdots\ h_{N_P}(t)]^{\mathrm{T}}$, $\hat{u} = [u(t)\ \cdots\ u(t+N_P-1)]^{\mathrm{T}}$, $\hat{r} = [r(t)\ \cdots\ r(t+N_P-1)]^{\mathrm{T}}$, G is a $N_P \times N_P$ lower triangular toeplitz matrix and it's i,k element is the $z^{-(i-k)}$th coefficient of the polynomials G_j.

In the case of a setpoint feedforward, we write a plant model by using a relation $v = u + P_F r$,

$$\begin{aligned} y &= P_1(s)(u + P_F(s)r) + \xi \\ &= P_1(s)u + P_1(s)P_F(s)r + \xi. \end{aligned} \quad (7)$$

We can express a similar formula as equation (2) from equation (7). It is a same procedure as above to derive a control law in the case of the setpoint feedforward.

3. INTERPRETATION OF THE DESIGN METHOD

In this section we describe that the proposed GPC design is equivalent to the state feedback with a minimal order observer with the disturbance signal or the setpoint signal.

Interpretation of GPC by a state space design has been discussed (Lee *et al.* 1994)(Ling and Lim 1996). Reference (Masuda *et al.* 1999) states the equivalence between a state space design and the polynomial design of the usual GPC. We summarize the result shortly.

In this case the plant model does not include the disturbance,

$$A(z^{-1})y(t) = B(z^{-1})\Delta u(t-1). \quad (8)$$

Then equation (4) becomes,

$$h_j(t) = \frac{1}{T(z^{-1})} \left(E_j(z^{-1})y(t) + S_j(z^{-1})\Delta u(t-1) \right). \quad (9)$$

Equations (3)(5)(6) are same as in the previous section. A state space model corresponding to equation (8) is

$$\begin{aligned} x(t+1) &= Px(t) + K\Delta u(t) \\ y(t) &= Cx(t). \end{aligned} \quad (10)$$

A minimal order observer (Kailath 1980) is written by

$$\begin{aligned} w(t+1) &= Lw(t) + Ey(t) + F\Delta u(t) \\ \hat{x}(t) &= Rw(t) + Vy(t). \end{aligned} \quad (11)$$

where $\hat{x}(t)$ is an estimation of the state $x(t)$. The j-step ahead prediction $\hat{y}(t+j)$ of $y(t)$ from the state space model (10) is,

$$\hat{y}(t+j) = CP^j \hat{x}(t) + \sum_{i=1}^{j} CP^{j-i} K \Delta u(t+i-1). \quad (12)$$

Then the control $\Delta \hat{u} = [\Delta u(t) \cdots \Delta u(t+N_P-1)]^T$ which minimize the performance index (5) is,

$$\Delta \hat{u} = -(\lambda I + G_S^{\mathrm{T}} G_S)^{-1} G_S^{\mathrm{T}} (h_S - \hat{r}) \quad (13)$$

where G_S is an $N_P \times N_P$ lower triangular toeplitz matrix and it's i,j element is $CP^{i-j}K$,

$$G_S = \begin{bmatrix} CK & 0 & \cdots & 0 \\ CPK & CK & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CP^{N_P-1} & \cdots & CPK & CK \end{bmatrix}. \quad (14)$$

h_S is a N_P vector and the i th element is $CP^i \hat{x}(t)$,

$$h_S = \begin{bmatrix} CP \\ CP^2 \\ \vdots \\ CP^{N_P} \end{bmatrix} \hat{x}(t). \quad (15)$$

Theorem 1. (Masuda *et al.* 1999) G_S is equal to G. The j th element of h_S is equal to h_j of equation (9).

This result can be extended to the GPC design considered in the previous section. We write a state space model corresponding to equation (2),

$$\begin{aligned} x(t+1) &= Px(t) + K\Delta u(t) + Rd(t) \\ y(t) &= Cx(t). \end{aligned} \quad (16)$$

A minimal order observer becomes,

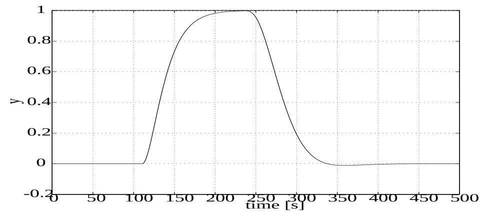
$$\begin{aligned} w(t+1) &= Lw(t) + Ey(t) + F\Delta u(t) + Vd(t) \\ \hat{x}(t) &= Rw(t) + Vy(t). \end{aligned} \quad (17)$$

Then $\hat{x}(t)$ in equation (15) is exchanged by the output of the observer (17).

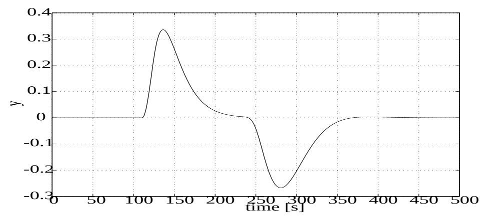
Theorem 2. G_S is equal to G. The j-th element of h_S is equal to h_j of equation (4).

4. NUMERICAL SIMULATION

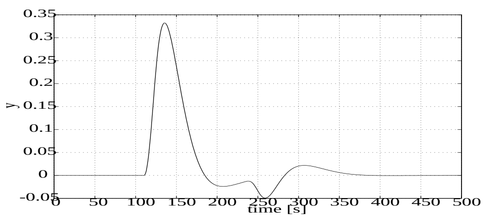
We show simulation results in order to demonstrate the difference by design methods. Plant model used in this simulation is derived from an actual process described in the next section.



(a) conventional GPC design

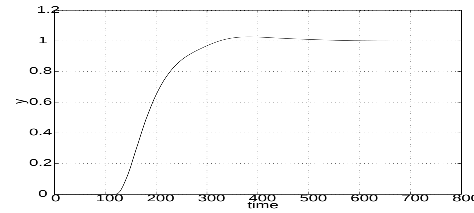


(b) GPC with a feedforward control

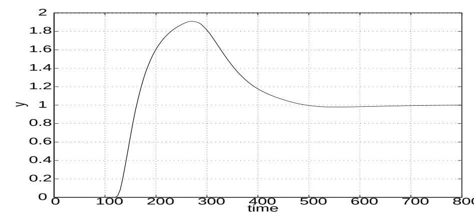


(c) GPC design with a feedforward model

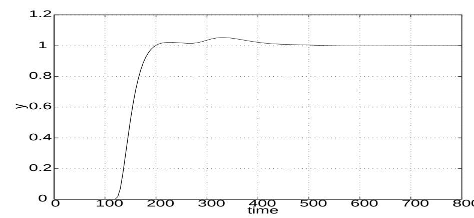
Fig. 5. Simulation results for a disturbance rejection



(a) conventional GPC design



(b) GPC with a feedforward control



(c) GPC design with a feedforward model

Fig. 6. Simulation results for a setpoint change

$$P_1(s) = \frac{e^{-120s}}{(1+30s)(1+5s)}$$
$$P_2(s) = \frac{e^{-110s}}{1+30s}.$$

(1) Disturbance rejection (Fig.5)
- Conventional GPC design
- GPC with disturbance feedforward
- GPC design with a feedforward model

(2) Setpoint change (Fig.6)
- conventional GPC design
- GPC with setpoint feedforward
- GPC design with a feedforward model

Figure 5 shows that a disturbance feedforward reduces a deviation. In Fig.5(b) an undesirable undershoot appears but in Fig.5(c) this response disappears. Figure 6 shows a setpoint feedforward make a quick tracking. In Fig.6(b) a large overshoot appears but in Fig.6(c) this response disappears same as the disturbance feedforward.

5. EXPERIMENTAL RESULTS

We present experimental results that have been accomplished a thermal power plant of which the rated power output is 165MW.

Nitrogen oxides (NOx) gas is generated by fuel combustion in a boiler or a gas turbine used in a thermal power plant. An NOx decomposition process is installed in order to reduce an amount of NOx discharge.

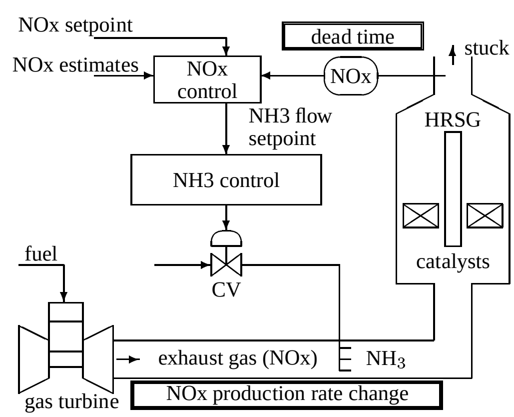


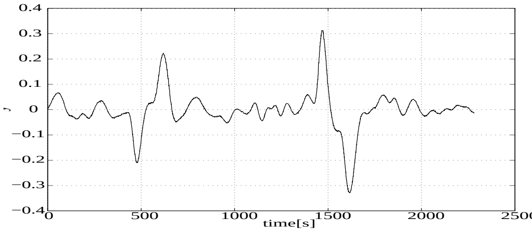
Fig. 7. Combined cycle power plant

An amount of NOx discharge flow can be manipulated by ammonia (NH_3) injection to catalysts by the reaction, $4NO+4NH_3+O_2 \rightarrow 4N_2+6H_2O$. The NOx decomposition process has a large dead time that is caused by gas analyzing and NH_3 flow delay.

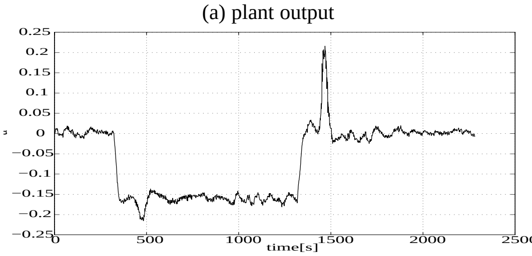
The NOx generation rate changes frequently according to a plant operation. The object of NOx control is to maintain NOx discharge rate at a setpoint against a change of NOx generation rate.

Figure 7 shows a schematic diagram of a NOx decomposition process of a combined cycle power plant(Nakamoto *et al.* 1995).

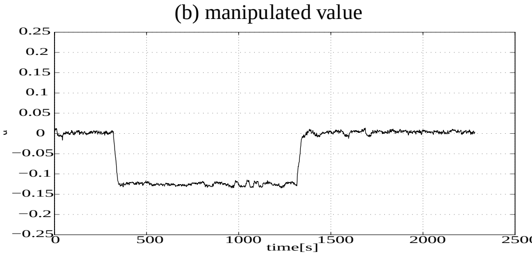
Figure 8 shows a test result of NOx generation rate change by a conventional GPC with a feedforward control as shown in Fig.3. Figure 9 shows a result for



(a) plant output

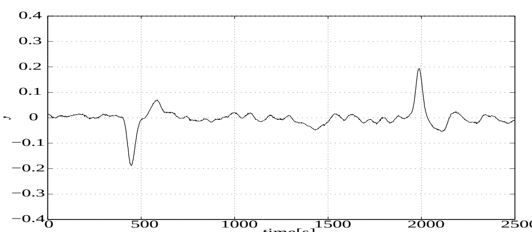


(b) manipulated value

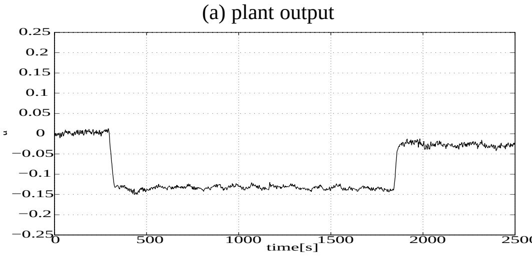


(c) disturbance value

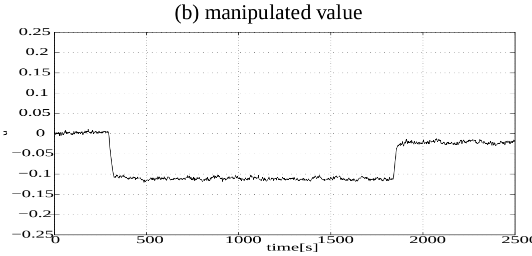
Fig. 8. GPC with a feedforward control



(a) plant output



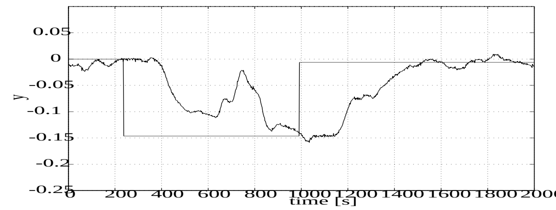
(b) manipulated value



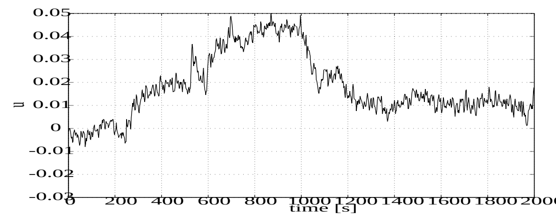
(c) disturbance value

Fig. 9. GPC desgin with a feedforward model

same condition as Fig.8 by a proposed GPC design. Same as simulation results, the proposed GPC can prevent the undesirable undershoot. Figure 10,11 and 12 show experimental results of a setpoint change. These figures show same results as the simulation.

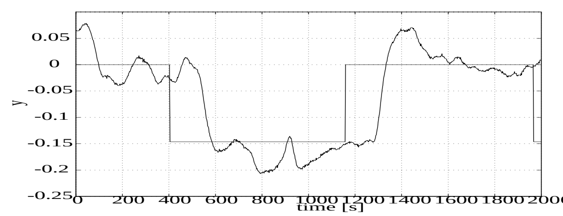


(a) plant output

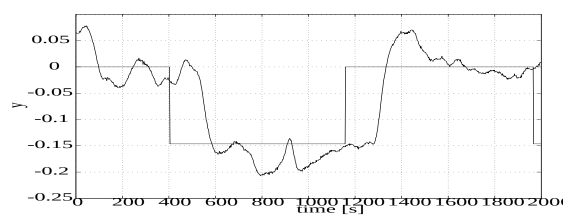


(b) manipulated value

Fig. 10. one-degree of freedom GPC



(a) plant output



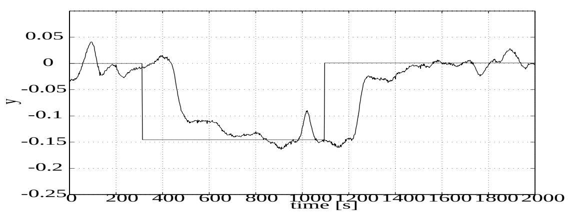
(b) manipulated value

Fig. 11. GPC with a feedforward control

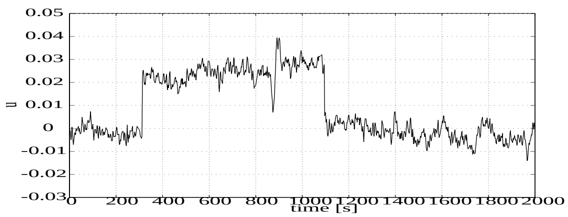
Figure 13 shows a plant start up operation with a conventional PI plus a feedforward control. Figure 14 shows the test results by the proposed control. In the both figures, the control system started around 1200 seconds. Figure 14 shows not only smaller fluctuation but also a wide usefulness from the start to the rated condition by the proposed control.

6. CONCLUSION

In this paper we described a two-degree of freedom configuration of a generalized predictive control. We also describe that the presented T-DOF GPC is similar to the state feedback control with the minimal order observer. Finally we show experimental results accomplished on a commercial power plant.

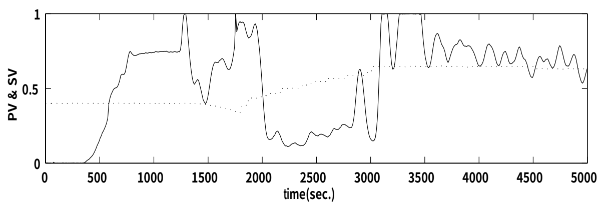


(a) plant output

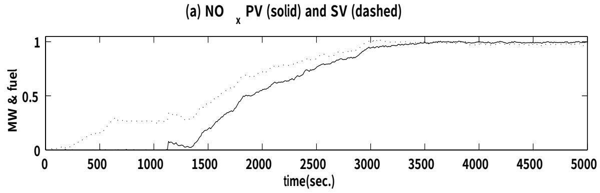


(b) manipulated value

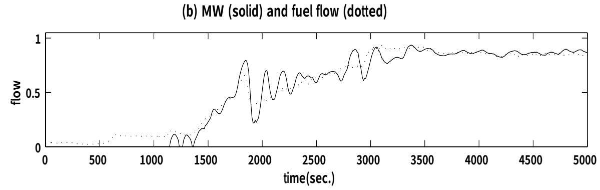
Fig. 12. GPC desgin with a feedforward model



(a) NO_x PV (solid) and SV (dashed)

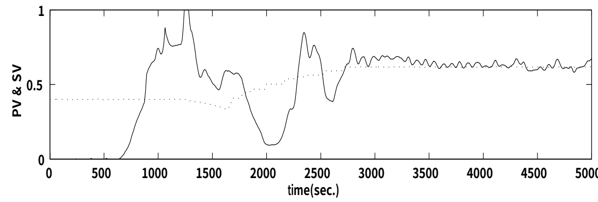


(b) MW (solid) and fuel flow (dotted)

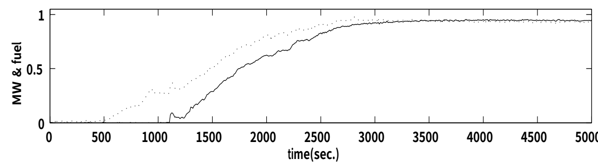


(c) GT NO_x flow (solid) and NH_3 flow (dotted)

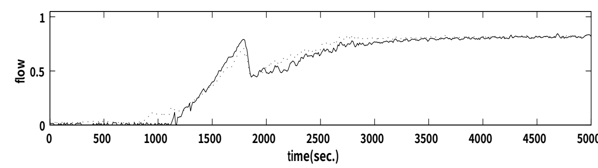
Fig. 13. Start up - conventional control



(a) NO_x PV (solid) and SV (dashed)



(b) MW (solid) and fuel flow (dotted)



(c) GT NO_x flow (solid) and NH_3 flow (dotted)

Fig. 14. Start up - GPC

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