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MIMO PREDICTIVE PID CONTROLS

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Abstract: This paper is concerned with the design of Multi-Inputs and Multi-Outputs (MIMO) predictive PID controllers, which have similar features to the model-based predictive controller. A PID type control structure is defined which includes prediction of the outputs and the recalculation of new set points using the future set point data. The optimal values of the MIMO PID gains are calculated using the values of gains calculated from an unconstrained generalised predictive control algorithm. Simulation studies demonstrate the performance of the proposed controller and the results are compared with generalised predictive control solutions.

Keywords: PID Control Design, Predictive PID Controller, MIMO Controller, Parallel PID structure

1. INTRODUCTION

PID controllers are used in large numbers in all industries. The controllers come in many different forms, are packaged as standard products, and are massed produced. The popularity of PID controllers is due to their functional simplicity and reliability. They provide robust and reliable performance for most systems if the PID parameters are determined or tuned to ensure a satisfactory closed-loop performance. There is a wealth of literature on PID tuning for scalar systems. Good reviews of tuning PID methods are given in Astrom *et al* (1993), Gorez and Calcev (1997) and Tan *et al*. (1999). Among these are the well known Ziegler and Nichols, (1942), Cohen and Coon (1952) and the feedback relay method (Astrom and Hagglund 1995). These algorithms are based either on time domain responses or the frequency response characteristics of the process.

Because of the wide application of the PID controllers and the problem of calculating appropriate three term gains, many researchers have attempted to use advanced control techniques such as optimal control, and MPC to restrict the structure of these controllers to PID type. Morari and Zafiriou, (1989) have shown that Internal Model Control (IMC) leads to PID controllers for virtually all models common in industrial practice. Using a first order process model, Rivera *et al*. (1986) introduced an IMC based PID controller design, and this was later extended to cover the second order process model (Chien 1988). In Wang *et al*. (2000) a frequency response approach is adopted, where a least square algorithm is used to compute the closest equivalent PID controller to an IMC design. Marques and Fliess (2000) have illustrated a simple approach for PID control of linear continuous systems based on flat output trajectory generation. Their solution is calculated off-line. In Rusnak (2000), linear quadratic regulator theory has been used to design PID controllers. A generalised PID structure was introduced and applied up to a fifth order system. In Grimble (1991), H_∞ control technique is employed to derive controllers with similar PID structure. Tan *et al.* (2000) have presented a PID control design based on the GPC approach for a second order system with time delay where the PID parameters are time-varying inside the dead time. Moradi *et al* (2001a, 2001b) have introduced the predictive PID controller based on GPC method for SISO systems. Most of these literature deal with SISO systems.

Many industrial processes are inherently multivariable and need multivariable control to provide enhanced performance. This is a strong motivation to derive a simple and effective method for developing multivariable controller design methods (Maciejowski 2001). PID control is one of most common control schemes for MIMO plants. These controllers are usually tuned using the prior knowledge of the dynamics of the system. But as MIMO processes are often also non-linear, with changing the set point the dynamics of the system are changing and the controller needs to be retuned. Therefore, to accommodate model uncertainties the robustness of MIMO controller is a very important property. Depending on the application and requirement, either a fully cross-coupled or a multi-loop controller can be adopted for MIMO processes. Multi-loop controllers (decentralised controllers) have a simpler structure and, accordingly, less tuning parameters than the fully cross-coupled controller.
This paper is an extension of the previously published method predictive PID controller (Moradi *et al.*, 2001a; 2001b) (Katebi and Moradi, 2001) to MIMO systems. The PID controller is defined by using a bank of M parallel conventional PID controllers where M is also the prediction horizon.

The paper has been organised as follows: Section 2 describes the structure of MIMO PID type predictive controller. Section 3 calculates the optimal values of the controller gain. A comparison between the proposed method and GPC technique is presented in Section 4. Finally, conclusions close the paper.

2. MIMO SYSTEM DESCRIBTION

The block diagram of digital controller for a MIMO process under unity feedback was shown in Fig.1. The system is assumed to be square of dimension (L). The MIMO PID controller in discrete form can be represented by:

$$U(k) = K_P e(k) + K_I \sum_{j=1}^{k} e(j) + K_D[e(k) - e(k-1)] \quad (1)$$

where:
K_P, K_I and K_D are the proportional, integral and derivates gains, respectively.

$$K_P = \begin{bmatrix} k_{11}^P & k_{12}^P & \cdots & k_{1L}^P \\ k_{21}^P & k_{22}^P & \cdots & k_{11}^P \\ \vdots & \vdots & \ddots & \vdots \\ k_{L1}^P & k_{L2}^P & \cdots & k_{LL}^P \end{bmatrix} \quad \begin{matrix} e(k) = \left[e_1(k) \quad e_2(k) \quad \cdots \quad e_L(k)\right]^T \\ e(k) = r(k) - y(k) \\ r(k) = \left[r_1(k) \quad r_2(k) \quad \cdots \quad r_L(k)\right]^T \\ y(k) = \left[y_1(k) \quad y_2(k) \quad \cdots \quad y_L(k)\right]^T \end{matrix}$$

$$K_I = \begin{bmatrix} k_{11}^I & k_{12}^I & \cdots & k_{1L}^I \\ k_{21}^I & k_{22}^I & \cdots & k_{11}^I \\ \vdots & \vdots & \ddots & \vdots \\ k_{L1}^I & k_{L2}^I & \cdots & k_{LL}^I \end{bmatrix} \quad K_D = \begin{bmatrix} k_{11}^D & k_{12}^D & \cdots & k_{1L}^D \\ k_{21}^D & k_{22}^D & \cdots & k_{11}^D \\ \vdots & \vdots & \ddots & \vdots \\ k_{L1}^D & k_{L2}^D & \cdots & k_{LL}^D \end{bmatrix}$$

And the incremental representation of the controller is:

$$\begin{aligned} \Delta U(k) = U(k) - U(k-1) &= (K_P + K_I + K_D)e(k) \\ &+ (-K_P - 2K_D)e(k-1) + K_D e(k-2) \end{aligned} \quad (2)$$

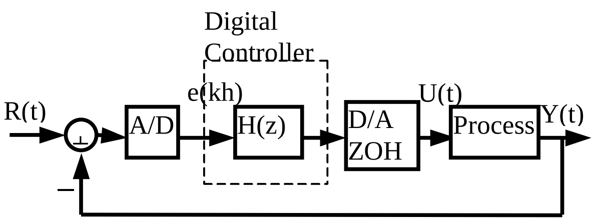


Fig.1: The closed loop block diagram of digital control of the process.

In compact matrix form, equation (2) can be written as:

$$\Delta U(k) = KE(k) = K[R(k) - Y(k)] \quad (3)$$

where:

$$K = \left[K_D \quad -2K_D - K_P \quad K_D + K_P + K_D\right]$$

$$\Delta U(k) = \left[\Delta u_1(k) \quad \Delta u_2(k) \quad \cdots \quad \Delta u_L(k)\right]^T$$

$$E(k) = \left[e^T(k-2) \quad e^T(k-1) \quad e^T(k)\right]^T \quad e^T(k) = r^T(k) - y^T(k)$$

$$R(k) = \left[r^T(k-2) \quad r^T(k-1) \quad r^T(k)\right]^T$$

$$Y(k) = \left[y^T(k-2) \quad y^T(k-1) \quad y^T(k)\right]^T$$

2.1 *Predictive form of MIMO PID Controllers*

The predictive PID controller is defined as follows:

$$\Delta u(k) = K\sum_{i=0}^{M} E(k+i) = K\sum_{i=0}^{M} R(k+i) - K\sum_{i=0}^{M} Y(k+i) \quad (4)$$

The controller consists of M parallel PID controllers. For M=0, the controller is identical to the conventional PID in equation (2). For M>0 the proposed controller has predictive capability similar to MBPC where M is prediction horizon of PID controller. The horizon, M will be selected to find the best approximation to GPC solution. The controller signal in equation (4) can be decomposed as:

$$\begin{aligned} \Delta u(k) &= K\{E(k) + E(k+1) + \cdots + E(k+M)\} \\ &= \Delta U(k) + \Delta U(k+1) + \cdots + \Delta U(k+M) \end{aligned} \quad (5)$$

It is clear from (3) and (4) that the controller consists of M PID controllers where the input of i th PID at time *k* depends on the error signal at time *(k+i)*. This implies that the current control signal value is a linear combination of the future predicted outputs. Then, the control signal can be written as:

$$\Delta u(k) = K\left\{\sum_{m=0}^{M} R(k+m) - \sum_{m=0}^{M} Y(k+m)\right\} \quad (6)$$

To calculate the control increment at time k, $\Delta u(k)$, the output for M step ahead needs to be predicted. Using a model of the system, the definition of the predictor is given in the next section.

2.2 *Future Output Predictor for Predictive PID*

A CARIMA model for L-inputs, L-outputs process can be expressed as:

$$A(z^{-1})y(k) = B(z^{-1})u(k-1) \qquad (7)$$

where:

$A(z^{-1})$ and $B(z^{-1})$ are $L \times L$ polynomial matrices.
$A(z^{-1}) = I + A_1 z^{-1} + \cdots + A_{n_a} z^{-n_a}$

$B(z^{-1}) = B_0 + B_1 z^{-1} + \cdots + B_{n_b} z^{-n_b}$

The M step ahead prediction of output can be obtained from the following equation (Camacho and Bordons 1999):

$$\begin{aligned} y(k+m) &= G^m(z^{-1})\Delta u(k+m-1) + G'^m(z^{-1})\Delta u(k-1) \\ &\quad + F^m(z^{-1})y(k) \end{aligned} \qquad (8)$$

where:

$$\Delta u(k+m) = \begin{bmatrix} \Delta u_1(k+m) & \Delta u_2(k+m) & \cdots & \Delta u_L(k+m) \end{bmatrix}^T$$

$$y(k-i) = \begin{bmatrix} y_1(k-i) & y_2(k-i) & \cdots & y_L(k-i) \end{bmatrix}^T$$

$$\Delta u(k-i) = \begin{bmatrix} \Delta u_1(k-i) & \Delta u_2(k-i) & \cdots & \Delta u_L(k-i) \end{bmatrix}^T$$

$$G^m(z^{-1}) = \begin{bmatrix} G_{11}^m(z^{-1}) & \cdots & G_{1L}^m(z^{-1}) \\ \vdots & \ddots & \vdots \\ G_{L1}^m(z^{-1}) & \cdots & G_{LL}^m(z^{-1}) \end{bmatrix}$$

$$G'^m(z^{-1}) = \begin{bmatrix} G_{11}'^m(z^{-1}) & \cdots & G_{1L}'^m(z^{-1}) \\ \vdots & \ddots & \vdots \\ G_{L1}'^m(z^{-1}) & \cdots & G_{LL}'^m(z^{-1}) \end{bmatrix}$$

$$F^m(z^{-1}) = \begin{bmatrix} F_{11}^m(z^{-1}) & 0 & \cdots & 0 \\ 0 & F_{22}^m(z^{-1}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F_{LL}^m(z^{-1}) \end{bmatrix}$$

$$G_{ij}^m(z^{-1}) = g_{ij}^0 + g_{ij}^1 z^{-1} + \cdots + g_{ij}^{m-1} z^{-(m-1)} \qquad i = (1, \cdots, L)$$

$$G_{ij}'^m(z^{-1}) = g_{ij}^m + g_{ij}^{m+1} z^{-1} + \cdots + g_{ij}^{m+n_{bij}-1} z^{-(n_{bij}-1)} \qquad j = (1, \cdots, L)$$

$$F_{jj}^m(z^{-1}) = f_{jj}^m(0) + f_{jj}^m(1) z^{-1} + \cdots + f_{jj}^m(n_{aj}) z^{-(n_{aj})} \qquad (9)$$

$$E_m(z^{-1})B(z^{-1}) = G^m(z^{-1}) + z^{-i} G'^m(z^{-1}) \qquad (10)$$

n_{bij} : The order of ij th element in matrix $B(z^{-1})$

n_{aj} : The order of jj th element in matrix $A(z^{-1})$

$E_m(z^{-1})$ and $F_m(z^{-1})$ are a solution of the above Diophantine equation. In equation (10), it was assumed that the matrix $A(z^{-1})$ is diagonal, hence, matrices $E_m(z^{-1})$ and $F_m(z^{-1})$ are also diagonal matrices and the problem was reduced to the recursion of L scalar Diophantine equations, which are much simpler to program and require less computation.

The output prediction for the i th PID in each loop can be written as:

$$Y(k+m) = \begin{bmatrix} y(k+m-2) \\ y(k+m-1) \\ y(k+m) \end{bmatrix} \qquad (11)$$

$$= \begin{bmatrix} G^{m-2}(z^{-1})\Delta u(k+m-1) + G'^{m-2}(z^{-1})\Delta u(k-1) + F^{m-2}(z^{-1})y(k) \\ G^{m-1}(z^{-1})\Delta u(k+m-1) + G'^{m-1}(z^{-1})\Delta u(k-1) + F^{m-1}(z^{-1})y(k) \\ G^{m}(z^{-1})\Delta u(k+m-1) + G'^{m}(z^{-1})\Delta u(k-1) + F^{m}(z^{-1})y(k) \end{bmatrix}$$

Equation (11) can be simplified to:

$$\begin{bmatrix} y(k+m-2) \\ y(k+m-1) \\ y(k+m) \end{bmatrix} = \begin{bmatrix} G_{m-3} & \cdots & G_0 & 0 & 0 \\ G_{m-2} & G_{m-3} & \cdots & G_0 & 0 \\ G_{m-1} & G_{m-2} & G_{m-3} & \cdots & G_0 \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_u-1) \end{bmatrix}$$

$$+ \begin{bmatrix} G'_{(m-2)1} & G'_{(m-2)2} & \cdots & G'_{(m-2)n_b} \\ G'_{(m-1)1} & G'_{(m-1)2} & \cdots & G'_{(m-1)n_b} \\ G'_{m1} & G'_{m2} & \cdots & G'_{mn_b} \end{bmatrix} \begin{bmatrix} \Delta u(k-1) \\ \Delta u(k-2) \\ \vdots \\ \Delta u(k-n_b) \end{bmatrix}$$

$$+ \begin{bmatrix} F_0^{m-2} & F_1^{m-2} & \cdots & F_{n_a}^{m-2} \\ F_0^{m-1} & F_1^{m-1} & \cdots & F_{n_a}^{m-1} \\ F_0^{m} & F_1^{m} & \cdots & F_{n_a}^{m} \end{bmatrix} \begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-n_a) \end{bmatrix} \qquad (12)$$

where:

$$G_i = \begin{bmatrix} g_{11}{}^i & g_{12}{}^i & \cdots & g_{1L}{}^i \\ g_{21}{}^i & g_{22}{}^i & \cdots & g_{2L}{}^i \\ \vdots & \vdots & \ddots & \vdots \\ g_{L1}{}^i & g_{L2}{}^i & \cdots & g_{LL}{}^i \end{bmatrix}_{L\times L} \qquad G'_{i(j+1)} = \begin{bmatrix} g_{11}^{i+j} & g_{12}^{j+i} & \cdots & g_{1L}^{j+i} \\ g_{21}^{j+i} & g_{22}^{i+j} & \cdots & g_{2L}^{i+j} \\ \vdots & \vdots & \ddots & \vdots \\ g_{L1}^{j+i} & g_{L2}^{j+i} & \cdots & g_{LL}^{i+j} \end{bmatrix}_{L\times L}$$

$$F_j^i = \begin{bmatrix} f_{11}^i(j) & 0 & \cdots & 0 \\ 0 & f_{22}^i(j) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & f_{LL}^i(j) \end{bmatrix}_{L\times L}$$

In equation (12), future control inputs $\{\Delta u(k+i) \quad i = 1{:}(N_u-1)\}$ are needed to calculate the current control signal. Rewriting the output prediction gives straightforwardly:

$$Y(k+m) = G^m \Delta\hat{u}(k) + F^m y_0(k) + G'^m \Delta u_0(k) \qquad (13)$$

where:

$$G^m = \begin{bmatrix} G_{m-3} & \cdots & G_0 & 0 & 0 \\ G_{m-2} & G_{m-3} & \cdots & G_0 & 0 \\ G_{m-1} & G_{m-2} & G_{m-3} & \cdots & G_0 \end{bmatrix}_{(3\times L)\times(L\times N_u)}$$

$$F^m = \begin{bmatrix} F_0^{m-2} & F_1^{m-2} & \cdots & F_{n_a}^{m-2} \\ F_0^{m-1} & F_1^{m-1} & \cdots & F_{n_a}^{m-1} \\ F_0^{m} & F_1^{m} & \cdots & F_{n_a}^{m} \end{bmatrix}_{(3\times L)\times(L\times(n_a+1))}$$

$$G'^m = \begin{bmatrix} G'_{(m-2)1} & G'_{(m-2)2} & \cdots & G'_{(m-2)n_b} \\ G'_{(m-1)1} & G'_{(m-1)2} & \cdots & G'_{(m-1)n_b} \\ G'_{m1} & G'_{m2} & \cdots & G'_{mn_b} \end{bmatrix}_{(3\times L)\times(L\times n_b)}$$

$$\Delta\hat{u}(k) = \begin{bmatrix} \Delta u^T(k) & \Delta u^T(k+1) & \cdots & \Delta u^T(k+N_u-1) \end{bmatrix}^T$$

$$\Delta u(j) = \begin{bmatrix} \Delta u_1(j) & \Delta u_2(j) & \cdots & \Delta u_L(j) \end{bmatrix}^T$$

$$Y(k+m) = \begin{bmatrix} y^T(k+m-2) & y^T(k+m-1) & y^T(k+m) \end{bmatrix}^T$$

$$y_0(k) = \begin{bmatrix} y^{\circ T}(k) & y^{\circ T}(k-1) & \cdots & y^{\circ T}(k-n_a) \end{bmatrix}^T$$

$$y^\circ(j) = \begin{bmatrix} y_1(j) & y_2(j) & \cdots & y_L(j) \end{bmatrix}^T$$

$$\Delta u_0(k) = \begin{bmatrix} \Delta u^{\circ T}(k-1) & \Delta u^{\circ T}(k-2) & \cdots & \Delta u^{\circ T}(k-n_b) \end{bmatrix}^T$$

$$\Delta u^\circ(j) = \begin{bmatrix} \Delta u_1(j) & \Delta u_2(j) & \cdots & \Delta u_L(j) \end{bmatrix}^T$$

Put equation (13) in the (6):

$$\Delta u(k) = K\left\{R(k+m) - \sum_{m=0}^{M} Y(k+m)\right\}$$
$$= K\left\{\begin{array}{l}\sum_{m=0}^{M} R(k+m) - (\sum_{m=0}^{M} G^m)\Delta\hat{u}(k) \\ +(\sum_{m=0}^{M} F^m) y_0(k) + (\sum_{m=0}^{M} G^{'m})\Delta u_0(k)\end{array}\right\} \quad (14)$$

Equation (14) can be simplified to:

$$\Delta u(k) = K\left\{R_t(k) - \alpha\Delta\hat{u}(k) + F_f y_0(k) + G_g \Delta u_0(k)\right\} \quad (15)$$

where:

$$R_t(k) = \sum_{m=0}^{M} R(k+m) \quad \alpha = \sum_{m=0}^{M} G^m \quad F_f = \sum_{m=0}^{M} F^m \quad G_g = \sum_{m=0}^{M} G^{'m}$$

3. OPTIMAL VALUES of PID-TYPE PREDICTIVE GAINS

To obtain the optimal values of the gains, the Generalised Predictive Control (GPC) algorithm is used. For process control problems default setting of output cost horizon $\{N_1 : N_2\} = \{1 : N\}$, and the control cost horizon $N_u = 1$ can be used in GPC to give reasonable performance (Clarke *et al.*,1987). GPC consists of applying a control sequence that minimises the following cost function:

$$J(1,N) = [Gu(k) + Fy_0(k) + G'\Delta u_0(k) - W(k)]^T$$
$$[Gu(k) + Fy_0(k) + G'\Delta u_0(k) - W(k)] + \lambda\Delta u(k)^T \Delta u(k) \quad (16)$$

The minimum of J, assuming there are no constraints on the control signals, is obtained using the usual gradient analysis, which leads to [Camacho and Bordons, 1999]:

$$\Delta u(k) = K_{GPC}[W(k) - Fy_0(k) - G'\Delta u_0(k)] \quad (17)$$

which can be summarised as:

$$\Delta u(k) = K_{GPC} W(k) - K_{GPC}[F \quad G'] \begin{bmatrix} y_0(k) \\ \Delta u_0(k) \end{bmatrix}$$
$$= K_{GPC} W(k) - K_0 \begin{bmatrix} y_0(k) \\ \Delta u_0(k) \end{bmatrix} \quad (18)$$

where:

$$K_0 = K_{GPC}[F \quad G']$$
$$K_{GPC} = (G^T G + \lambda I)^{-1} G^T$$
$$W(k) = \left[w^T(k) \quad w^T(k+1) \quad \cdots \quad w^T(k+N)\right]^T$$
$$w(k) = \left[w_1(k) \quad w_2(k) \quad \cdots \quad w_L(k)\right]^T$$
$$y_0(k) = \left[y^T(k) \quad y^T(k-1) \quad \cdots \quad y^T(k-n_a)\right]^T$$
$$\Delta u_0(k) = \left[\Delta u^T(k-1) \quad \Delta u^T(k-2) \quad \cdots \quad \Delta u^T(k-n_b)\right]^T$$

To compute the optimal values of predictive control PID gains with $N_u = 1$ [$\hat{u}(k) = u(k)$], the PID control signals should then be made the same as GPC controller. This means using equation (15) and (17) the following optimal problem should be solved:

$$Min_{K \in K^S_{PID}, M} J(K, K_0) \quad (19)$$

$K^S_{PID} =$ Set of stability gain for PID

where:

$$J(K,K_0) = \left\| \begin{array}{l} (I+K\alpha)^{-1} K[W - \left[F_f \quad G_g\right] Z(K)] \\ -\left\{K_{GPC} w - K_0[Z(K_0)\right\} \end{array} \right\|_2$$

$Z = \begin{pmatrix} y_0(k) \\ \Delta u_0(k) \end{pmatrix}$ is dependent on the controls gains used

Write $Z(K) = Z(K_0) + \Delta Z$. Then the optimisation cost function can be stated as:

$$J(K,K_0) = \left\| \begin{array}{l} (1+K\alpha)^{-1} K[R_t(k) - \left[F_f \quad G_g\right] Z(K)] \\ -\left\{K_{GPC} W(k) - K_0[Z(K_0)\right\} \end{array} \right\|_2$$
$$\le \left\| (-(I+K\alpha)^{-1} K\left[F_f \quad G_g\right] + K_0) Z(K_0) \right\|_2$$
$$+ \left\| -(I+K\alpha)^{-1} K\left[F_f \quad G_g\right] \Delta Z \right\|_2$$
$$+ \left\| (1+K\alpha)^{-1} K R_t(k) - K_{GPC} W(k) \right\|_2$$
$$\le \left\| (-(I+K\alpha)^{-1} K\left[F_f \quad G_g\right] + K_0) \right\|_2 \left\| Z(K_0) \right\|_2$$
$$+ \left\| -(I+K\alpha)^{-1} K\left[F_f \quad G_g\right] \right\|_2 \left\| \Delta Z \right\|_2$$
$$+ \left\| (I+K\alpha)^{-1} K R_t(k) - K_{GPC} W(k) \right\|_2$$

Thus:

(i) A minimum norm solution is sought from: $\left\| -(I+K\alpha)^{-1} K\left[F_f \quad G_g\right] + K_0 \right\|_2$
This is found as, $(I+K\alpha)^{-1} K\left[F_f \quad G_g\right] = K_0$

(ii) It is assumed that it is possible to find suitable gain K close to K_0 so that $\left\|\Delta Z\right\|_2$ is suitably small.

(iii) It is assumed that $R_t(k)$ (rebuilt future set point) will be calculated so that: $\left\| (I+K\alpha)^{-1} K R_t(k) - K_{GPC} W(k) \right\|_2 = 0$

The above reasoning leads to:

$$J(K,K_0) = \left\| -(I+K\alpha)^{-1} K\left[F_f \quad G_g\right] \Delta Z \right\|_2$$
$$\le \left\| -(I+K\alpha)^{-1} K\left[F_f \quad G_g\right] \right\|_2 \left\|\Delta Z\right\|_2$$

if:

$$\begin{cases} (I+K\alpha)^{-1} K R_t(k) = K_{GPC} W(k) \\ (I+K\alpha)^{-1} K\left[F_f \quad G_g\right] = K_0 \end{cases} \quad (20)$$

To calculate the control signals, steady state values of the control inputs and outputs are used. These values are assumed to be almost constant at steady state plant operation, hence, the assumption (ii) will be satisfied, $\left\|\Delta Z\right\|_2$ is suitably small, and the cost function will be minimised.

The solution for K can be found in terms of K_0 from second equation in (20), and then W (rebuilt future set point) will be calculated from the first equation. After some straightforward algebra:

$$K_0 = (I+K\alpha)^{-1} K\left[F_f \quad G_g\right] \to K_0 = (I+K\alpha)^{-1} K S_0$$
$$K_0(I+K\alpha) = K S_0 \to K(S_0 - \alpha K_0) = K_0 \quad (21)$$

where:

$$S_0 = \left[F_f \quad G_g\right]$$

A unique solution to equation (21) always exists and takes the form (Levine, 1996):

$$K = K_0(S_0 - \alpha K_0)^T [(S_0 - \alpha K_0)(S_0 - \alpha K_0)^T]^{-1} \quad (22)$$

From first equation in (20) the rebuilt future set point will be calculated as:

$$R_t(k) = K^{-1}(I+K\alpha) K_{GPC} W(k) = K_S W(k) \quad (23)$$

where: $K_S = K^{-1}(I+K\alpha) K_{GPC}$

Predictive Control Algorithm

The predictive PID controller can be implemented using the following procedure.

Algorithm 1: Predictive PID controller for MIMO process.

Step 1: Initialisation

1. Find a system model and calculate the discrete polynomials matrices, A and B
2. Choose the value of prediction horizon, M, and formulate the future set point vectors w

Step 2: Off line Calculation

1. Calculate the matrices α, F_f, G_g in equation (15) using equation (13)
2 Calculate the GPC gain, K_{GPC}, using equation (18)
3 Calculate the optimal value of predictive PID gains using equation (22)
4 Iterate over the value of M to minimize the cost function.

Step 3: On line Calculation

1 Calculate the following signals
(a) $\begin{bmatrix} F_f & G_g \end{bmatrix} Z(K)$
(b) $R_t(k)$ using equation (23)
2 Calculate the control increment
$u(k) = u(k-1) + (I + K\alpha)^{-1} K$
$[R_t(k) - \begin{bmatrix} F_f & G_g \end{bmatrix} Z(K)]$

Step 4: Assessment

1 Apply the control signal.
2 Check closed loop performance.

4. SIMULATION RESULTS

In this section, the performance comparison of proposed method and GPC for two industrial systems will be discussed, the systems are:

1- A small signal model for stirred tank reactor was described by the following transfer matrix (Camacho and Bordons, 1999):

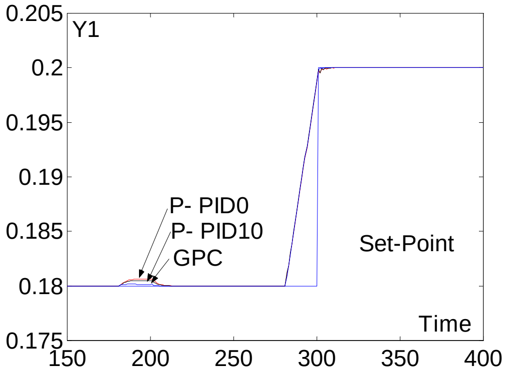
$$G_1(s) = \begin{bmatrix} \dfrac{1}{1+0.7s} & \dfrac{5}{1+0.3s} \\ \dfrac{1}{1+0.5s} & \dfrac{2}{1+0.4s} \end{bmatrix} \qquad (24)$$

where the manipulated variable $u_1(s)$ and $u_2(s)$ are the feed flow rate and the flow of coolant in the jacket respectively. The control variables $Y_1(s)$ and $Y_2(s)$ are the effluent concentration and the reactor temperature respectively.

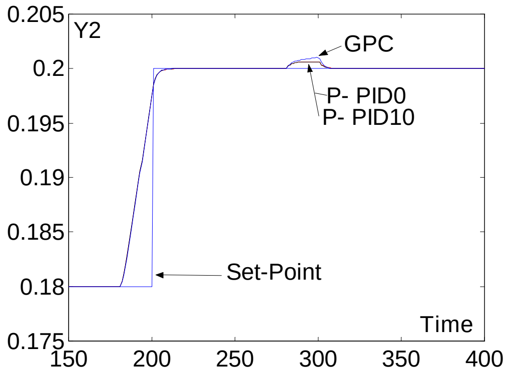
2- The boiler model has been considered. (Katebi *et al*, 2000)

$$G_2(s) = \begin{bmatrix} \dfrac{-.007s + .00078}{s^2 + .0178s + .00055} & \dfrac{-.005s + 7\times10^{-5}}{s^2 + .017s + .0004} \\ \dfrac{-.0067s + .00062}{s^2 + .017s + .00053} & \dfrac{s^2 - .043s + .00065}{s^2 + .067s + .0006} \end{bmatrix} (25)$$

where the manipulated variable $u_1(s)$ and $u_2(s)$ are the feed/air demand and the control valve position respectively. The control variables $Y_1(s)$ and $Y_2(s)$ are throttle pressure and the steam flow respectively.

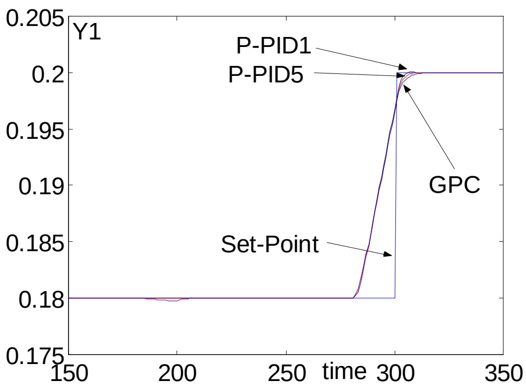


(a) Output 1

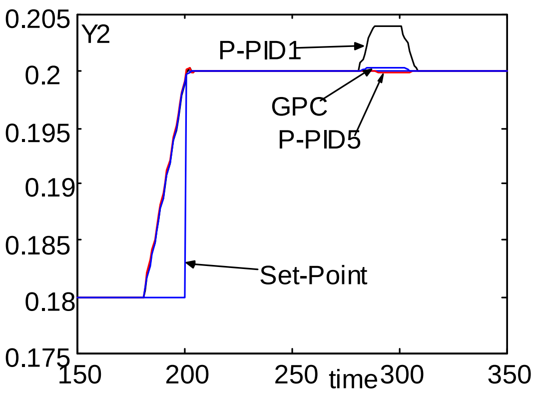


(b) Output 2

Fig.2: The comparison of GPC with Proposed PID Method for a stirred tank reactor.



(a) Output 1



(b) Output 2

Fig.3: The comparison of GPC with proposed Method for Boiler model.

Table 1: Comparison between GPC and Proposed PID control design for G_1 and G_2.

System	G_1
GPC Gains:	$\begin{bmatrix} -.072 & .285 & .019 & -.062 & -.0006 & .0031 & -.02 & .01 \\ .236 & .018 & -.064 & -.0038 & .002 & .0002 & -.008 & -.273 \end{bmatrix}$
Predictive PID Gains: M=0	$\begin{bmatrix} -.02 & -.054 & -.0006 & .055 & .23 & .003 \\ .06 & .174 & .002 & .0034 & .014 & .0002 \end{bmatrix}$
M=10	$\begin{bmatrix} -.019 & -.029 & -.005 & .0067 & .038 & .0017 \\ -.037 & -.04 & .012 & -.013 & -.031 & .0026 \end{bmatrix}$
System	G_2
GPC Gains:	$\begin{bmatrix} .087 & -.0 & .0597 & .0001 & .041 & .0001 & .011 & -.00 & .003 & .00 & .162 & .033 & .055 & .014 & .015 & .003 \\ -.003 & .026 & -.002 & .003 & -.0013 & .001 & -.0004 & -.002 & -.0001 & .0 & -.003 & .27 & -.0087 & .056 & -.0005 & -.023 \end{bmatrix}$
Predictive PID Gains: M=1	$\begin{bmatrix} -0.05 & 0.14 & 0.04 & -0.0003 & -0.00031 & -0.0002 \\ 0.03 & 0.03 & 0.01 & -0.0034 & 0.0135 & -0.0058 \end{bmatrix}$
M=5	$\begin{bmatrix} -0.004 & 0.05 & -0.013 & .0005 & .0015 & 0.0002 \\ .0005 & .0015 & .0002 & .017 & .047 & -0.061 \end{bmatrix}$

GPC and Multivariable predictive PID methods were used to design the controller for two systems G_1 and G_2 with transfer function mentioned in equations (24) and (25), respectively. For GPC, the horizon prediction of output N=20, control input horizon N_u=1 were assumed. The controller gains for the two methods are shown in Table1. It is clear from the Table 1 that for the first order 2I2O system conventional PID is enough to achieve the GPC performance (M=0). For second order 2I2O system also M=1 is sufficient to approximate the GPC performance. The step response of the closed loop system for two methods has been shown in Fig 2 and Fig 3. The results show that the proposed method can achieve GPC performance levels provided M is chosen correctly.

5. CONCLUSIONS

From the simulations performed, it can be observed that the control system has an adequate behaviour. In the case in which a reference trajectory is used, the system transitions are smoother and use less energy during the initial transient response. This behaviour is understandable since the zero frequency quadratic error (cost) has a lower magnitude than when the set point is used directly, so control actions need less energy to achieve the control objective.

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