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VSS IDENTIFICATION SCHEMES FOR TIME-VARYING PARAMETERS

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Abstract: In this paper, two new identification schemes based on VSS theory and sliding mode are developed to address the *time-varying* parameter identification problems. The proposed direct identification scheme can successfully estimate fast *time-varying* parameters when the process is linear in the parametric space. The proposed indirect identification scheme further exploits the possibility of estimating *time-varying* parameters for some classes of nonlinear processes not linear in parametric space, but linearizable in the parametric space when the sliding mode exists. Owing to the robustness of variable structure control, processes with fast *time-varying* parameter uncertainties and nonlinearities are now controllable, and in the sequel identifiable.

Keywords: *Time-varying* Parameter Identification, Variable Structure Control, Sliding Mode, Multi-Input Multi-Output Systems, Nonlinearity

1. INTRODUCTION

Over the past few decades, system identification has received increasing attentions. In most conventional identification methods, such as the Least Squares related methods and the Lyapunov method based adaptive methods, an identification model is required and the model structure must be identical to the real plant in order to get the accurate parameters (Zhu and Leung, 2000) (Bontayeb2k, 2000). To apply those identification schemes four conditions are indispensable.
(1) The system must be linear in the parametric space.
(2) Unknown parameters to be identified must be either constant or slow *time-varying* in nature.
(3) Process should be stable in general.
(4) Persistent Excitation condition must be satisfied.
While the fourth condition is required in general for all kinds of identification methods, the first three conditions impose great limitation to solving real problems. In (Ljung and Forssell, 1999) and (Forssell and Ljung, 2000), closed-loop identification schemes for linear system and constant parameters are well developed. However, it cannot deal with the nonlinear system with time-varying parameters.

In (Xu and Hashimoto, 1993) and (Xu and Hashimoto, 1996), two VSS-theory based parameter identification schemes have been proposed, which partially addressed the limitation arising from the first and the third conditions. Even the original process dynamics is not linear in the parametric space, it may become linear in the parametric space when in the sliding mode. This can be readily seen from the indirect identification scheme. Moreover, since the process will be put under the closed-loop, the outstanding robustness property of the sliding mode control can be fully used to stabilize highly nonlinear and uncertain processes (Utkin, 1992) (Zinober, 1994) (Man and Yu, 1997) (Young *et al.*, 1999) (Furuta and Pan, 2000) (Fridman, 2000) (Thein and Misawa, 2000) (Chen and Hiroyuki, 2000). As a consequence the identification mechanism can work under a stable control environment.

In this paper, we further extend our VSS-theory based identification methods to the *time-varying* cases, namely addressing the remaining second condition. However, in (Xu and Hashimoto, 1993) and (Xu and Hashimoto, 1996), the identification mechanism consists of a set of integrators, which indicates that only constant unknown parameters can be identified by virtue of internal model principle. In this paper, we first show that the direct identification scheme can successfully estimate the unknown parameters which could be fast *time-varying*. Next in the indirect identification scheme, a feedback controller based on variable structure control theory is designed to ensure the existence of sliding mode. When the system is in the sliding mode, an appropriate identification mechanism can be easily derived to work without using any explicit identification model. It should be noted that, when identifying arbitrary *time-varying* parameters, we have to use LSE based on instantaneous measurement to make direct calculation. Any integration based adaptive schemes or recursive Least Squares will not work properly because those methods require an invariant input-output mapping in the parametric space (Robert, 1999). In the new schemes, the concept of the equivalent control plays the key role, which leads to some important algebraic relationships from the sliding mode, therefore makes the *time-varying* parameters identifiable.

The paper is organized as follows. In Section 2, a class of nonlinear MIMO system with unknown *time-varying* parameters is presented. In Section 3, VSS theory-based direct identification scheme is proposed with the convergence analysis. In Section 4, VSS theory-based indirect identification scheme is proposed and analyzed first for the process linear in the parameters, and then extended to a more general case that the process is non-linear in the parameters. Two illustrative examples are given to show the validity of the two schemes.

2. PROBLEM FORMULATION

Consider the nth order nonlinear system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{p}, t) + B(\mathbf{x}, \mathbf{p}, t)\mathbf{u} \tag{1}$$

where $\mathbf{x} \in \mathcal{R}^n$ is the measurable state vector, $\mathbf{u} \in \mathcal{R}^m$ is the measurable input vector, $\mathbf{p} \in \mathcal{R}^{n_p}$ is an unknown *time-varying* parameter vector in the parameter space P, where $n_p \leq n$, $\mathbf{f} \in \mathcal{R}^n$ and $B \in \mathcal{R}^{n \times m}$ are nonlinear functions. For $\mathbf{f}$,

$$\mathbf{f}(\mathbf{x}, \mathbf{p}, t) = \mathbf{f}_0(\mathbf{x}, t) + \Phi(\mathbf{x}, t)\mathbf{p} \tag{2}$$

where $\mathbf{f}_0 \in \mathcal{R}^n$ is a known vector, $\Phi \in \mathcal{R}^{n \times n_p}$ is a known matrix and $\mathbf{p}^T = [p_1(t) \cdots p_{n_p}(t)]$ is the unknown *time-varying* parameter vector.

Assumption 1: *The matrix B can be described as*

$$B(\mathbf{x}, \mathbf{p}, t) = B_0(\mathbf{x}, t) + B_0(\mathbf{x}, t)E(\mathbf{x}, \mathbf{p}, t) \tag{3}$$

where B_0 is known and of full column rank. For matrix E, each element of E is linear in P, i.e. $e_{ij}(\mathbf{x}, \boldsymbol{p}, t) = \mathbf{e}_{ij}\boldsymbol{p}$, and

$$\begin{aligned}\forall t \in \mathcal{R}^+, \forall \mathbf{x} \in D^* \subset \mathcal{R}^n, \forall \mathbf{p} \in P,\\ \lambda_{min}(\frac{1}{2}E + \frac{1}{2}E^T) = r > -1,\end{aligned} \tag{4}$$

where λ_{min} indicates the minimum eigenvalue of the matrix E. D^* is an open subset of $\mathcal{R}^n$ in which the solution of (1) is unique with respect to the desired trajectory $\mathbf{x}_d(t)$, which is continuously differentiable at least up to (n+1)th order.

Assumption 2:

$$\forall \mathbf{p} \in P,\ p_{i,min} \leq\ p_i \leq\ p_{i,max},\ i = 1, \cdots, n_p, \tag{5}$$

where $p_{i,max}$ and $p_{i,min}$ are known constants.

From *Assumption* 1, $B\mathbf{u}$ can be expressed as

$$\begin{aligned}B(\mathbf{x}, \mathbf{p}, t)\mathbf{u} &= B_0(\mathbf{x}, t)\mathbf{u} + B_0(\mathbf{x}, t)E(\mathbf{x}, \mathbf{p}, t)\mathbf{u}\\ &= B_0(\mathbf{x}, t)\mathbf{u} + \Psi(\mathbf{x}, \mathbf{u}, t)\mathbf{p}\end{aligned} \tag{6}$$

where $\Psi \in \mathcal{R}^{n \times n_p}$ is a known matrix. Then the system in (1) can be rewritten as

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}_0(\mathbf{x}, t) + \Phi(\mathbf{x}, t)\mathbf{p} + B_0(\mathbf{x}, t)\left[I + E(\mathbf{x}, \mathbf{p}, t)\right]\mathbf{u}\\ &= \mathbf{f}_0 + B_0\mathbf{u} + A\mathbf{p},\end{aligned} \tag{7}$$

where $A = A(\mathbf{x}, \mathbf{u}, t) = \Phi(\mathbf{x}, t) + \Psi(\mathbf{x}, \mathbf{u}, t)$.

3. VSS DIRECT IDENTIFICATION SCHEME

A. Identification Algorithm

In this scheme, the identification model is selected in the form

$$\dot{\hat{\mathbf{x}}} = \mathbf{f}_0 + B_0\mathbf{u} + \mathbf{v} \tag{8}$$

where $\mathbf{v}$ is an input for identification

$$\mathbf{v} = k(\mathbf{x}, t)\frac{\boldsymbol{\sigma}}{\|\boldsymbol{\sigma}\|}, \tag{9}$$

where $\boldsymbol{\sigma} = \mathbf{x} - \hat{\mathbf{x}}$, $k(\mathbf{x}, t) = |A|_1|\mathbf{p}|_m + \varepsilon$, ε is a positive constant, $|\mathbf{p}|_m = [|p_1|_m \cdots |p_{n_p}|_m]^T$, where $|p_i|_m = max(|p_{imax}|, |p_{imin}|)$, $i = 1, \cdots, n_p$. The column vector norm $|\mathbf{w}|_1$ and matrix norm $|W|_1$ are defined as

$$|\mathbf{w}|_1 = \begin{bmatrix} |w_1| \\ \vdots \\ |w_n| \end{bmatrix}, \quad |W|_1 = \begin{bmatrix} |w_{11}| & \cdots & |w_{1n}| \\ \vdots & \ddots & \vdots \\ |w_{n1}| & \cdots & |w_{nn}| \end{bmatrix}.$$

Theorem 1: *Sliding mode will exist under the identification model* (8) *and the control law* (9). *If the matrix A^TA is nonsingular, $n_p \leq\ n$, then the time-varying parameters can be estimated by the following LSE formula*

$$\hat{\mathbf{p}} = (A^TA)^{-1}A^T\mathbf{v}_{eq}, \qquad (10)$$

where $\mathbf{v}_{eq}$ is the equivalent control input while the system is in the sliding mode.

Proof: The Lyapunov function is $V = \frac{1}{2}\boldsymbol{\sigma}^T\boldsymbol{\sigma}$. Using (7), (8) and (9),

$$\begin{aligned}\dot{V} &= \boldsymbol{\sigma}^T\dot{\boldsymbol{\sigma}} = \boldsymbol{\sigma}^T(\dot{\mathbf{x}} - \dot{\hat{\mathbf{x}}}) \\ &= \boldsymbol{\sigma}^T\left[\mathbf{f}_0 + B_0\mathbf{u} + A\mathbf{p} - \mathbf{f}_0 - B_0\mathbf{u} - \mathbf{v}\right] \\ &= \boldsymbol{\sigma}^T\left[A\mathbf{p} - k(\mathbf{x},t)\frac{\boldsymbol{\sigma}}{\|\boldsymbol{\sigma}\|}\right] \\ &\leq\ |A|_1|\mathbf{p}|_m\|\boldsymbol{\sigma}\| - k(\mathbf{x},t)\|\boldsymbol{\sigma}\| \leq\ -\varepsilon\|\boldsymbol{\sigma}\|.\end{aligned}$$

Thus the sliding mode exists. When the system is in the sliding mode and if the matrix A^TA is nonsingular, we have

$$\begin{aligned}\dot{\boldsymbol{\sigma}}(\mathbf{v} = \mathbf{v}_{eq}) &= A\mathbf{p} - \mathbf{v}_{eq} = 0, \\ \hat{\mathbf{p}} &= (A^TA)^{-1}A^T\mathbf{v}_{eq}.\end{aligned}$$

■

Remark 1: *From* (10), *it is noted that the non-singularity of the matrix A^TA is directly related to the persistent excitation condition. Thus the dimension of the vector $\mathbf{p}$ should be lower than the dimension of the system states, i.e. $n_p \leq\ n$.*

4. VSS INDIRECT IDENTIFICATION SCHEME

In this scheme, the control law is designed as

$$\mathbf{u} = \mathbf{u}_c + \mathbf{u}_v, \qquad (11)$$

where $\mathbf{u}_c$ is the compensation part and $\mathbf{u}_v$ is the switching part. The switching surface is

$$\begin{aligned}\boldsymbol{\sigma} &= C(\mathbf{x}_d - \mathbf{x}) = \mathbf{y}_d - C\mathbf{x}, \\ \boldsymbol{\sigma}, \mathbf{y}_d &\in \mathcal{R}^n \qquad C \in \mathcal{R}^{m\times n},\end{aligned} \qquad (12)$$

where $\mathbf{x}_d$ are desired trajectories, $\mathbf{y}_d \triangleq C\mathbf{x}_d$. The matrix C is chosen such that $rank(C) = m$, and CB_0 is full rank. Define $\hat{\mathbf{p}}$ as the adjustable parameter, then $\bar{\mathbf{p}}$ is calculated in terms of the adjustable parameter $\hat{\mathbf{p}}$ and their bounds $\mathbf{p}_{max}$ and $\mathbf{p}_{min}$

$$\bar{p}_i(t) = \begin{cases} p_{imin} & if\ \ \hat{p}_i < p_{imin}, \\ \hat{p}_i(t) & if\ \ \hat{p}_i \in [p_{imin}, p_{imax}] \\ p_{imax} & if\ \ \hat{p}_i > p_{imax}, \end{cases} \qquad (13)$$

The compensation part is designed as

$$\mathbf{u}_c = [CB_0 + CB_0\bar{E}]^{-1}[\dot{\mathbf{y}}_d - \mathbf{f}_0 - C\Phi\bar{\mathbf{p}}], \qquad (14)$$

where $\bar{E} = E(\mathbf{x}, \bar{\mathbf{p}}, t)$ and $\bar{\mathbf{p}}(t)$ is the *time-varying* parameter vector defined in (13).

The switching control input is designed as

$$\mathbf{u}_v = \begin{cases} \rho(CB_0)^T\Gamma\boldsymbol{\sigma} & if\ \ \boldsymbol{\sigma} \neq 0, \\ 0 & if\ \ \boldsymbol{\sigma} = 0 \end{cases} \qquad (15)$$

$$\begin{aligned}&\boldsymbol{\alpha} = (CB_0)^T\Gamma\boldsymbol{\sigma},\ \boldsymbol{\sigma} = \sigma_1\cdots\sigma_m]^T, \\ &\mathbf{d} = [(p_{1max} - p_{1min})\cdots(p_{n_pmax} - p_{n_pmin})]^T, \\ &\rho = \frac{|\boldsymbol{\sigma}^T\Gamma A_c|_1(\mathbf{d} + \boldsymbol{\epsilon})}{(1+r)\boldsymbol{\alpha}^T\boldsymbol{\alpha}},\ A_c = C\left[\Phi + \Psi(\mathbf{x}, \mathbf{u}_c, t)\right],\end{aligned} \qquad (16)$$

where $\Gamma = \Gamma^T > 0$ is constant matrix and $\boldsymbol{\epsilon} = [\epsilon\cdots\epsilon]^T$ is a vector with n_p elements.

Theorem 2: *Sliding mode can be reached in finite time under the control law* (11) *with* (14) *and* (15). *If the matrix F^TF, where $F = C\left[\Phi(\mathbf{x},t) + \Psi(\mathbf{x}, \mathbf{u}_{eq}, t)\right]$, is nonsingular, then the time-varying parameter $\hat{\Psi}$ can be estimated by the following LSE formula*

$$\hat{\mathbf{p}} = (F^TF)^{-1}F^T(\dot{\mathbf{y}}_d - C\mathbf{f}_0 - CB_0\mathbf{u}_{eq}), \qquad (17)$$

where $\mathbf{u}_{eq}$ is the equivalent control input forcing the system in the sliding mode.

Proof: The Lyapunov function is $V = \frac{1}{2}\boldsymbol{\sigma}^T\Gamma\boldsymbol{\sigma} > 0$. Using (7), (11) and (14), then the derivative of V is

$$\begin{aligned}\dot{V} &= \boldsymbol{\sigma}^T\Gamma\dot{\boldsymbol{\sigma}} = \boldsymbol{\sigma}^T\Gamma(\dot{\mathbf{y}}_d - C\dot{\mathbf{x}}) \\ &= \boldsymbol{\sigma}^T\Gamma\left[\dot{\mathbf{y}}_d - C(\mathbf{f}_0 + \Phi\mathbf{p}) - CB_0(I+E)\mathbf{u}\right] \\ &= \boldsymbol{\sigma}^T\Gamma\left[\dot{\mathbf{y}}_d - C(\mathbf{f}_0 + \Phi\mathbf{p}) - CB_0(I+E)\mathbf{u}_c\right. \\ &\quad \left.-CB_0(I+E)\mathbf{u}_v\right] \\ &= \boldsymbol{\sigma}^T\Gamma\left[\dot{\mathbf{y}}_d - C(\mathbf{f}_0 + \Phi\mathbf{p}) - CB_0(I+\bar{E})\mathbf{u}_c\right. \\ &\quad \left.+CB_0(\bar{E} - E)\mathbf{u}_c - CB_0(I+E)\mathbf{u}_v\right] \\ &= \boldsymbol{\sigma}^T\Gamma\left[C\Phi(\bar{\mathbf{p}} - \mathbf{p}) + CB_0(\bar{E} - E)\mathbf{u}_c\right. \\ &\quad \left.-CB_0(I+E)\mathbf{u}_v\right] \\ &= \boldsymbol{\sigma}^T\Gamma\left[C(\Phi + \Psi(\mathbf{x}, \mathbf{u}_c, t))(\bar{\mathbf{p}} - \mathbf{p})\right. \\ &\quad \left.-CB_0(I+E)\mathbf{u}_v\right] \\ &= \boldsymbol{\sigma}^T\Gamma A_c(\bar{\mathbf{p}} - \mathbf{p}) - \boldsymbol{\sigma}^T\Gamma CB_0(I+E)\mathbf{u}_v \\ &= \boldsymbol{\sigma}^T\Gamma A_c(\bar{\mathbf{p}} - \mathbf{p}) - |\boldsymbol{\sigma}^T\Gamma A_c|_1(\mathbf{d} + \boldsymbol{\epsilon})\rho'.\end{aligned}$$

Using *Assumption* 2 and the definition (16),

$$\bar{p}_i - p_i \leq d_i,$$
$$\rho' = \frac{\boldsymbol{\alpha}^T(I + \frac{1}{2}E + \frac{1}{2}E^T)\boldsymbol{\alpha}}{(1+r)\boldsymbol{\alpha}^T\boldsymbol{\alpha}} > 1. \quad (18)$$

Thus $\dot{V} \leq -|\boldsymbol{\sigma}^T \Gamma A_c|_1 \boldsymbol{\epsilon}$, which means the sliding mode can be reached in finite time. When the system is in sliding mode, we have

$$\begin{aligned}\dot{\boldsymbol{\sigma}}(\mathbf{u} = \mathbf{u}_{eq}) &= \dot{\mathbf{y}}_d - C\left[\Phi(\mathbf{x},t) + \Psi(\mathbf{x},\mathbf{u}_{eq},t)\right]\mathbf{p} \\ &\quad - C\mathbf{f}_0(\mathbf{x},t) - CB_0(\mathbf{x},t)\mathbf{u}_{eq} \\ &= \dot{\mathbf{y}}_d - F\mathbf{p} - C\mathbf{f}_0 - CB_0\mathbf{u}_{eq} = 0.\end{aligned}$$

If the matrix F^TF is nonsingular, then

$$\hat{\mathbf{p}} = (F^TF)^{-1}F^T(\dot{\mathbf{y}}_d - C\mathbf{f}_0 - CB_0\mathbf{u}_{eq}).$$

■

Now let us extend the second identification scheme to a class of systems nonlinear in parametric space P.

Theorem 3: *Consider the nonlinear system*

$$\dot{\mathbf{x}} = G^{-1}(\mathbf{x},\boldsymbol{p},t)\left[\boldsymbol{f}(\mathbf{x},\boldsymbol{p},t) + B(\mathbf{x},\boldsymbol{p},t)\boldsymbol{u}\right] \quad (19)$$

where the definitions of the nonlinear vector $\boldsymbol{f}(\mathbf{x},\boldsymbol{p},t)$ *and the matrix* $B(\mathbf{x},\boldsymbol{p},t)$ *are the same as those defined in* (1) *except that* $B \in \mathcal{R}^{n\times n}$. $G\dot{\mathbf{x}}$ *can be expressed in a matrix form*

$$G(\mathbf{x},\boldsymbol{p},t)\dot{\mathbf{x}} = \boldsymbol{g}_0(\dot{\mathbf{x}},\mathbf{x},t) + M(\dot{\mathbf{x}},\mathbf{x},t)\boldsymbol{p}, \quad (20)$$

where $\boldsymbol{g}_0$ *is a known vector and* M *is a known matrix. Define* $M_d = M(\dot{\mathbf{x}}_d,\mathbf{x}_d,t)$ *and* $N = A - M_d$. *If the matrix* N^TN *is nonsingular, and the sliding mode exists, then the time-varying parameter* $\hat{\Psi}(t)$ *can be estimated by the following LSE formula*

$$\hat{\boldsymbol{p}} = (N^TN)^{-1}N^T(\boldsymbol{g}_0 - B_0\boldsymbol{u}_{eq} - \boldsymbol{f}_0), \quad (21)$$

where $\boldsymbol{u}_{eq}$ *is the equivalent control input when the system is in the sliding mode.*

Proof: Using (19), when the system is in the sliding mode, $\mathbf{u} = \mathbf{u}_{eq}$, $\mathbf{x} = \mathbf{x}_d$ and $\dot{\mathbf{x}} = \dot{\mathbf{x}}_d$,

$$\begin{aligned}\dot{\boldsymbol{\sigma}}(\mathbf{u}_{eq}) = \dot{\mathbf{x}}(\mathbf{u} = \mathbf{u}_{eq}) - \dot{\mathbf{x}}_d &= 0 \\ \mathbf{f}_0 + A\mathbf{p} + B_0\mathbf{u}_{eq} - M_d\mathbf{p} - \mathbf{g}_0 &= 0 \\ \mathbf{f}_0 + N\mathbf{p} + B_0\mathbf{u}_{eq} - \mathbf{g}_0 &= 0.\end{aligned}$$

Thus if the matrix N^TN is nonsingular, then we can directly derive the LSE formula

$$\hat{\mathbf{p}} = (N^TN)^{-1}N^T(\mathbf{g}_0 - B_0\mathbf{u}_{eq} - \mathbf{f}_0).$$

■

Remark 2: *In the indirect identification scheme (Theorem 2 and Theorem 3), the existence of the sliding mode is ensured by properly designing the VSC based feedback controller. For time-varying parameters identification, any integration based identification schemes, which have developed mostly for constant parameters identification, will not work. In the proposed scheme, the instantaneous information of the equivalent control signal is used to assist LSE based identification.*

5. ILLUSTRATIVE EXAMPLE

Consider the following system which is a nonlinear MIMO system

$$G(\mathbf{x},\mathbf{p})\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{p}) + B(\mathbf{x},\mathbf{p})\mathbf{u} \quad (22)$$

where $\mathbf{x} = [x_1 \;\; x_2]^T$ is the measurable state vector, $x_1(0) = -2$, $x_2(0) = 2$, $\mathbf{u} = [u_1 \;\; u_2]^T$ is the measurable input vector, $\mathbf{p} = [a_1(t) \;\; a_2(t)]^T$ is the unknown *time-varying* parameter vector,

$$G(\mathbf{x},\mathbf{p}) = \begin{bmatrix} 3 & 1 \\ 0 & 1 + a_1(t)|sin(x_1)| \end{bmatrix},$$
$$\mathbf{f}(\mathbf{x},\mathbf{p}) = \begin{bmatrix} a_2(t)x_1^2 \\ -2x_1^2 - x_1x_2 \end{bmatrix},$$
$$B(\mathbf{x},\mathbf{p}) = \begin{bmatrix} 3 & \frac{1}{2}cos(x_1) \\ a_1(t) & 3 \end{bmatrix}.$$

The unknown parameters are

$$\begin{aligned}a_1(t) &= 2 + sin(5\pi t), \\ a_2(t) &= 2 - 0.5cos(5\pi t) + 0.5sin(10\pi t),\end{aligned}$$

and their bounds are given by

$$a_{1max} = a_{2max} = 4, \qquad a_{1min} = a_{2min} = 1.$$

The switching surface are

$$\sigma_1 = x_{1d} - x_1, \qquad \sigma_2 = x_{2d} - x_2,$$

where the desired trajectory are

$$\begin{aligned}x_{1d} &= 0.5cos(\pi t) + 0.5cos(2\pi t) + 0.3sin(3\pi t) - 5, \\ x_{2d} &= 0.3sin(3\pi t) + 0.2cos(5\pi t) \\ &\quad + 0.15sin(6\pi t) + 2.\end{aligned}$$

The identification mechanism is constructed according to (11) and (17). The equivalent control input can be obtained from $\dot{\boldsymbol{\sigma}} = 0$. However, in practice, the existence of a discontinuity and the limited sampling rate will influence the acquisition of the equivalent control signal and as a result deteriorate the identification result. One way to recover the equivalent control is to get the average value of the switching control by using a low pass filter (Utkin, 1992). The average signal will approach the ideal one when the switching frequency approaches infinity. An alternative is to use a smoothing scheme. Here a fractional interpolation is employed and the high switching component in (15) is replaced by

$$\mathbf{u}_v = \begin{cases} \dfrac{|\boldsymbol{\sigma}^T \Gamma A_c|_1 \mathbf{d}(\boldsymbol{\alpha}^T\boldsymbol{\alpha} + \delta_1)\boldsymbol{\alpha}}{(1+r)(\boldsymbol{\alpha}^T\boldsymbol{\alpha} + \delta)^2} & if \;\; \boldsymbol{\sigma} \neq 0, \\ 0 & if \;\; \boldsymbol{\sigma} = 0 \end{cases}$$

where δ is a small positive constant, $\delta_1 = 2\delta + \frac{\delta^2}{\eta}$. It ensures that the equivalent control signal can be approximated to any degree of accuracy by choosing the precision bound $\boldsymbol{\alpha}^T\boldsymbol{\alpha} = \eta$ sufficiently small. The sampling period is selected as $T_s = 0.1 \;\; ms$.

Case 1: Indirect identification without measurement noise

In this case, $\eta = 0.01$, $\delta = 0.003$. $Fig.1$ shows the identification result for the *time-varying* parameters $a_1(t)$. It is shown that the estimated parameters take some time to approach their true values. This is because during the reaching phase, the sliding mode does not exist, hence the identification mechanism cannot work properly.

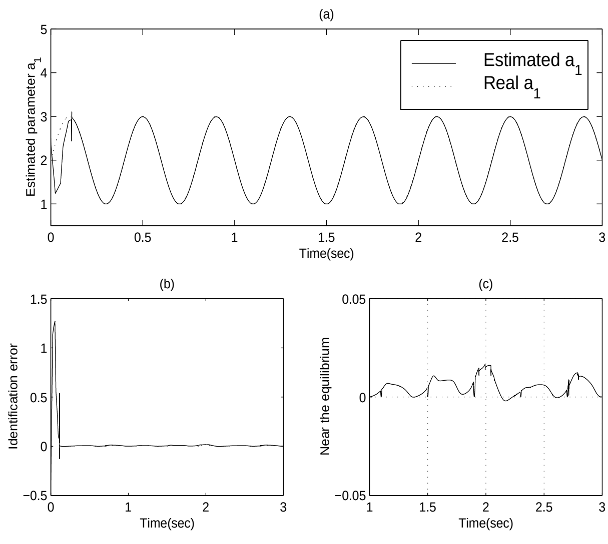

Fig. 1. Estimated *time-varying* parameter $\hat{a}_1(t)$ and identification error: (a) Evolution of $\hat{a}_1(t)$ (Solid-line: $\hat{a}_1(t)$; Dashed line: real $a_1(t)$); (b) Identification error of $a_1(t)$; (c) Identification error near the equilibrium.

Case 2: Indirect identification with measurement noise

In this case, measurement noise with the magnitude 0.01 is considered. In consideration of noise, $\eta = 0.5$ and $\delta = 0.5$ is selected. The identification results of the *time-varying* parameters $a_1(t)$ is shown in $Fig.2$. It is observed that there is chattering in the identified parameters. This is due to the effect of the noise on the acquisition of the equivalent control signal and the calculation of the LSE formula in $Theorem$ 3.

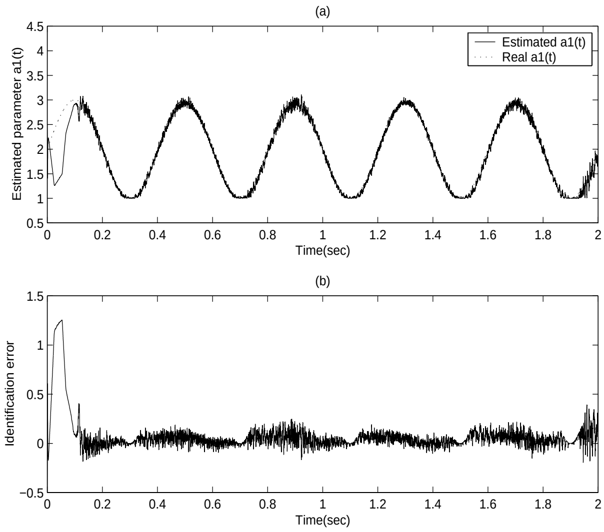

Fig. 2. Estimated *time-varying* parameter $\hat{a}_1(t)$ and identification error: (a) Evolution of $\hat{a}_1(t)$ (Solid-line: $\hat{a}_1(t)$; Dashed line: real $a_1(t)$); (b) Identification error of $a_1(t)$.

Case 3: Indirect identification with integrators

In this case, the integration based indirect identification scheme is implemented for *time-varying* parameters (Xu and Hashimoto, 1996). There is no measurement noise applied. Here $\eta = \delta = 0.1$. The identification results of the *time-varying* parameters $a_1(t)$ is shown in $Fig.3$. It is observed that the identified parameter $\hat{a}_1(t)$ cannot approach the real one. This is because the integration based identification scheme is not for *time-varying* parameters though it will work well for constant parameters identification.

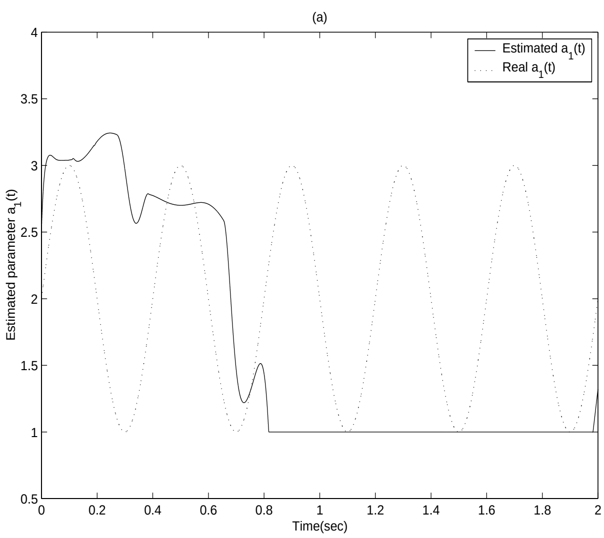

Fig. 3. Estimated *time-varying* parameter $\hat{a}_1(t)$ (Solid-line: $\hat{a}_1(t)$; Dashed line: real $a_1(t)$).

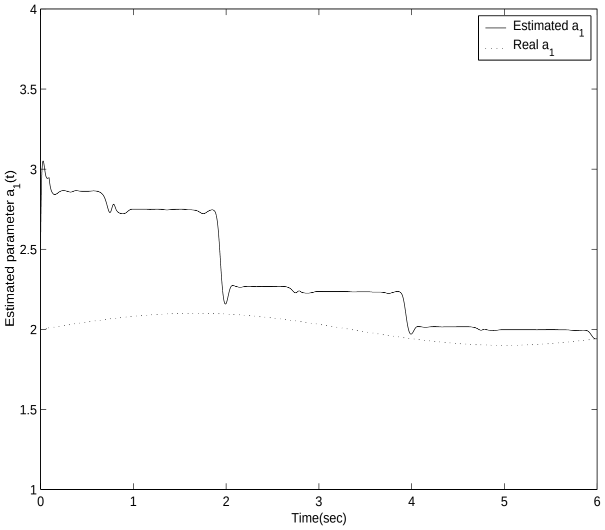

Fig. 4. Estimated time-varying parameter $\hat{a}_1(t)$ (Solid-line: $\hat{a}_1(t)$; Dashed line: real $a_1(t)$).

To explore the capability of the integral type identification scheme, a number of simulations are conducted where the magnitude and frequency of the time varying part in $a_1(t)$ are kept decreasing. It shows that the profile of $\hat{a}_1(t)$ is able to approach the real $a_1(t)$ ($Fig.4$) only when the magnitude is reduced from 1 to 0.1, and frequency from 5π to 0.3π, i.e. $a_1(t) = 2 + 0.1sin(0.3\pi t)$.

6. CONCLUSIONS

In this paper, two new identification approaches, which are based on the variable structure control theory, have been developed to identify *time-varying* parameters. In the direct identification scheme, an identification model is used and it can be applied to those systems which are linear in the parametric space. In the indirect identification scheme, a variable structure controller is first used to ensure the existence of the sliding mode. When the system is in the sliding mode, the *time-varying* parameters can be achieved by an appropriate identification mechanism which can be easily derived to work without using any explicit identification model. An immediate advantage of the indirect scheme is that it can be applied to systems that, originally not linear in parameters but becomes linear in the parameters when entering the sliding mode.

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