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A SURVEY OF APPLICATIONS OF INTERVAL ANALYSIS TO ROBUST CONTROL

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Abstract: This paper surveys some significant applications of interval analysis to robust control field. Interval analysis is specially powerful in bounding the ranges of functions while providing mathematically rigorous results. This capability is especially welcome in robust control, since a variety of analysis and design problems can be cast in the evaluation of the range of functions over intervals. To organize the paper, the different approaches are classified into three control fields, which previously are stated: parametric space, frequency methods, discrete-time systems. Moreover, some other applications of intervals are included.

Keywords: Uncertain systems, Interval analysis, Robust control

1. INTRODUCTION

The problem of robust control constitutes a relevant subject in control which has generated a number of new areas in research over the past few decades. The first project on stability analysis in the case of uncertain coefficient polynomials was presented in (Faedo, 1953). However, the first study of robust analysis using interval analysis as a tool was done by Misra (1989).

When setting the problem, it is interesting to go back to (Dorato, 1987), in which, although none of the works summarized are based on parametric methods, Kharitonov polynomials are introduced. In a later book by Dorato and Yedavalli (1990), parametric methods begin to appear, triggered by Kharitonov's results. Approaches concerning this field are set out primarily in two methods: polynomial methods and parametric methods. Barmish (1994), Ackermann (1993), Bhattacharyya *et al.* (1995) and Djaferis (1994) can be mentioned here as some of the classical references on these methods based on Kharitonov's polynomials.

Over the last fifteen years, advances on "classical" robust control for parametric systems has been run in parallel with the appearance of an increasing number of applications of interval analysis to the analysis and control of parametric systems.

Interval analysis is an unified approach to deal with parametric systems. It is generally accepted that interval methods are superior to existing techniques for parametric systems when the uncertain structure is more complex than interval or affine linear. Many authors consider intervals very useful when dealing with interval or affine uncertain structures. Moreover, the true value of interval methods is shown in their application to robust control design, when uncertainty is usually very complex.

To introduce interval analysis, let $[x]$ represent an interval of $\mathbb{R}$ (or scalar interval) that is a connected, closed and bounded set of real numbers. Interval arithmetic extends computation on real numbers to intervals in a natural and intuitive way and is the natural tool to use when dealing with interval models. The basis for this applied mathematical tool can be found in (Moore, 1966; Moore, 1979), who was followed by Alefeld and Herzberger (1983). As a result of maturing interval analysis in the mathematical field, some applications using this tool start to arise. For

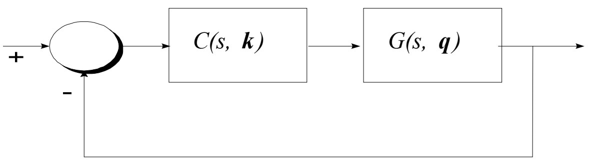


Fig. 1. Uncertain system

example, Hansen (1992), suggests the use of this powerful tool for global optimization.

Since 1990, by which time interval analysis was well established, it has been applied to different control problems. The recent book of Jaulin *et al.* (2001) as well as the special issue on application of interval analysis to systems and control Garloff and Walter (2000), appeared in the journal Reliable Computing, confirm this tendency. In this sense, applications of interval analysis to robust control have arisen. This paper attempts to show the most relevant methods concerned with the application of interval analysis to robust control developed and suggested in the literature. The approaches found are presented into four groups: parameter space approach, frequency methods, discrete-time and other applications.

Parameter space approach

The first approaches to robust stability under real parametric uncertainty, made researchers believe that the robust control problem could be approached without conservatism or overbounding. Later works offer a more realistic vision of these approaches.

To formulate the problem, giving the uncertain system of figure 1,

where $\mathbf{k}$ is the parameter vector of the controller, $\mathbf{q}$ is the parameter vector of the process,

$$X := (X', QX) \tag{1}$$

$$\mathbf{k} = [k_1 \; k_2 \; \cdots \; k_l]^T \tag{2}$$

and the uncertainty domain is defined as a box:

$$K = \{\mathbf{k} = [k_1 \; k_2 \; \cdots \; k_l]^T \mid k_i \in [\underline{k_i}, \overline{k_i}]\} \tag{3}$$

Design specifications are formulated in terms of closed-loop system stability and performances in the frequency domain. These specifications, such as bandwidth, resonance peak, control effort, etc., can be described as a set of N inequalities of the type:

$$f_i(\omega, \mathbf{q}, \mathbf{k}) > 0 \quad \omega \in \Omega, \mathbf{q} \in \mathbf{Q}, \mathbf{k} \in \mathbf{K}, i = 1, ..., N \tag{4}$$

where Ω is a subset of $\mathbb{R}^+$ (usually an interval) and $\mathbf{K}$, a nondegenerate set. Then robust design can be formulated as follows: given a controller structure, $C(s, \mathbf{k})$, the aim is to find the values of $\mathbf{k}$ which conform with the robust control specifications. Taking this formulation into account some control problems can be proposed (Vehí, 1998):

1. *Performance checking.* Given the uncertain system $G(s, \mathbf{q})$ and the uncertain domain $\boldsymbol{P}$, the problem is to see if the designed controller $C(s, \mathbf{k}^0)$, achieve the robustness specifications obtained from 4:

$$f_i(\omega, \mathbf{q}, \mathbf{k}^0) > 0 \tag{5}$$

2. *Performance margin computation.* Taking set Π as a function of its radius ρ:

$$\Pi(\rho) = \left\{ \mathbf{q} : \left\| \mathbf{q} - \mathbf{q}^0 \right\|_\infty^w \leq \rho \right\} \tag{6}$$

find the maximal set $\Pi(\rho^*)$ so that the designed controller $C(s, \mathbf{k}^0)$ achieves robust performances for all $\boldsymbol{\rho}$ belonging to $\Pi(\rho^*)$.

3. *Robust controller design.* Taking a particular structure for the controller and specifying the uncertain domain where the parameters of the system $\mathbf{Q}$ can vary, find a fixed structure controller $C(s, \mathbf{k}^0)$ so the controlled closed loop system achieves the robustness specifications:

$$f_i(\omega, \mathbf{q}, \mathbf{k}^0) > 0 \tag{7}$$

4. *Obtaining the set* $\mathbf{K}$ *for the robust controller problem.* Given an uncertain plant $G(s, \mathbf{q})$, the variation domain of the system parameters, $\mathbf{Q}$, and the controller structure, find the robust set $\mathbf{K}$ which allows $C(s, \mathbf{k})$ to achieve the robustness specifications:

$$f_i(\omega, \mathbf{q}, \mathbf{k}) > 0 \tag{8}$$

5. *Estimating the stability region problem , for a given* $\mathbf{k}^0$. This problem consists of constructing a set of all $\boldsymbol{\rho}$'s giving closed loop stability when given $\mathbf{k}^0$.

Frequency methods

Two methods are presented next. Both are based on the same problem: how to generate sets.

- *Value sets.* Some gridding approaches have the length of time it takes to compute the set of frequency plots as a drawback. An alternative and faster technique, suggested by Ackermann (1993), is to compute the frequency plot $p(jw, \mathbf{k})$, $w \geq 0$, for each $\mathbf{k}$ on a grid of K. It is advisable to compute the *value set* for each w on a grid of frequencies from 0 to $+\infty$.

$$P(jw, K) = \{p(jw, \mathbf{k}) \in \mathbb{C} \mid \mathbf{k} \in K\} \tag{9}$$

The *value set* problem is usually considered as a graphical control tool for analysis using frequency plots.

Theorem 1. (zero-exclusion theorem). Given a polynomial family $P(s, K) = \{p(s, k) \mid k \in K\}$. This set is robustly stable if and only if

1) A stable polynomial $p(s, \mathbf{k}) \in P(s, K)$ exists and

2) $0 \notin P(jw, K)$ for all $w \geq 0$.

-QFT (Quantitative feedback theory*).* While value sets are mainly concerned with analysis, when the aim is finding a compensator to satisfy design specifications, QFT is the most important approach. It can be considered as a natural extension of classical frequency-domain design approaches. One of the main objectives is to design a simple low-order controller where the bandwidth of the feedback controller being as small as possible. At a fixed frequency, the plant's frequency response set is called a template. In the bound generation step of QFT design procedure, the plant template is used to translate the given robustness specifications in domains in the Nichols chart where the controller gain-phase values are allowed to lie.

Discrete-time

Some problems concerned with discrete-time are:

-*Model conversion*. Conversion is needed in both senses, from a continuous-time interval state-space model to a discrete-time interval model and also from a discrete-time uncertain system to a continuous-time uncertain model.

-*Schur stability test*. The problem of checking the stability of a discrete-time system is reduced to the determination of whether or not the roots of the characteristic polynomial of the system lie strictly within the unit disc, that is whether or not the characteristic polynomial is a Schur polynomial. Given a polynomial

$$P(z) = p_n z^n + p_{n-1} z^{n-1} + ... + p_1 z + p, \tag{10}$$

where the z_i are the n roots of $P(z)$. Then, if $P(z)$ is Schur, all these roots are located inside the unit circle, $|z| < 1$, so that when z varies along the unit circle, $z = e^{j\theta}$, the argument of $P(e^{j\theta})$ increases monotonically. For a Schur polynomial of degree n, $P(e^{j\theta})$ has a net increase of argument of $2n\pi$, and thus the plot of $P(e^{j\theta})$ encircles the origin n times. This can be used as a frequency domain test for Schur stability (Bhattacharyya *et al.*, 1995; Garloff and Graft, 1999).

-*Non-linear discrete-time control of uncertain systems*. This problem can be formulated as:

Find one c

$$c \in S_c = \{c \in C \quad | \quad \forall p \in P, \quad f(c, p) > 0\} \tag{11}$$

where f is a vector function that can be evaluated using algorithms based on interval analysis.

-*Robust analysis*. Given a plant assumed to be described by the following uncertain discrete-time transfer function (Vehí *et al.*, 2000*a*):

$$G(z^{-1}, q) = \frac{b_1(q)\, z^{-1} + \cdots + b_m(q)\, z^{-m}}{1 - a_1(q)\, z^{-1} - \cdots - a_n(q)\, z^{-n}} \tag{12}$$

which depends on a structured perturbation characterized by (2) and (3).

Here, modal interval analysis (Gardeñes *et al.*, 1985) can be introduced showing that its use in the analysis of the robustness of predictive controllers allows us to convert the robust stability problem into a problem of checking the positivity of a rational function.

-*Modeling uncertainties through interval values.* Consider the plant to be controlled described in (Bravo *et al.*, 2000) is described by the following non-linear time-varying state-space model:

$$\begin{aligned} x(k) &= f(x(k-1), u(k)p(k)) \\ y(k) &= g(x(k)) \end{aligned} \tag{13}$$

where $u(k)$ is a vector of inputs or manipulated variables, $x(k)$ is a vector of state variables, $p(k)$ is a vector of uncertain and $y(k)$ is a vector of controlled variables or outputs.

The problem to be solved at each sampling time may be stated as follows:

$$\min_u \; J(u(k), y(k), w(k), \theta(k)) \tag{14}$$

subject to

$$C_1(u(k)), C_2(y(k)), C_3(\theta(k)) \tag{15}$$

J defines an objective function over a finite control horizon, $w(k)$ defines the set point sequence and $C_i(k)$ are sets of non linear constrains.

Other applications of intervals

Problems of estimating the unknown parameters of a model from bounded-error data, including identification methods and problems based on the use of gain scheduling, are also been included in order to see application fields in this area.

2. PARAMETER SPACE APPROACH

Studies found in this field have been classified into four major categories, namely robustness analysis, robust design, state space and an H_2/H_∞ approach.

2.1 *Robustness analysis*

The parametric approach to robust stability analysis has received a great deal of attention in the past few years. In (Piazzi and Marro, 1996) has been already conducted a survey based primarily on robust stability. Several other approaches illustrate these problems. In (Walter and Jaulin, 1994), is suggested that characterization of the stability domain can be approached as a problem of set inversion which can be solved with interval analysis tools. In (Garloff *et al.*, 1998), is presented an approach with a new algorithm which relies on the expansion of a multivariate polynomial into Bernstein polynomials and that is based on the inspection of the value set of the family of polynomials on an imaginary axis. In (Jaulin and Burger, 1999), is used interval analysis as a tool to develop a new algorithm able to prove that the feasible set is included in the stability domain. In (Malan *et al.*, 1992), is given an approach to finding global minima of multimodal optimization problems. There, they propose a Bernstein, Branch and Bound algorithm (B^3), which offers an efficient and easy way to check if the polynomial reaches its minimum on one of the vertices of the domain.

To conclude this section, an approach proposed by Didrit *et al.* (1997) will be presented and their work will be used to illustrate the approaches presented above. The idea of this example is to show the performance and the limitations of branch-and-bound algorithms, including modifications based on modal intervals to improve their effectiveness in non-monotonic regions (Vehí *et al.*, 1997).

Example 1. Given the third-degree uncertain polynomial:

$$p(s, \boldsymbol{q}) = 2 + r^2 + 6q_1 + 6q_2 + 2q_1q_2 +$$
$$+(2 + q_1 + q_2)s + (2 + q_1 + q_2)s^2 + s^3$$

classify the parameter space in unstable regions, regions with a stability degree between 0 and 0.1 and regions with a stability degree greater than or equal to 0.1.

The condition of a stability degree greater than a is obtained by substituting s with $s = -a + j\alpha, \quad \alpha \geq 0$ in the characteristic polynomial and applying classic Hurwitz test:

$$\begin{aligned} F(\boldsymbol{q}) = & -14q_2a - 14q_1a - 32a^2 - r^2 + 8a^3 + 2q_2^2a \\ & +4q_1aq_2 - 8q_1 + 24a + 32 + q_1^2 + q_2^2 \\ & +2q_1^2a + 8q_1a^2 + 8q_2a^2 - 8q_2 > 0 \end{aligned} \quad (16)$$

For $a = 0$ and $a = 0.1$, the stability conditions are, respectively:

$$32 - r^2 - 8q_1 - 8q_2 + q_1^2 + q_2^2 > 0 \quad (17)$$

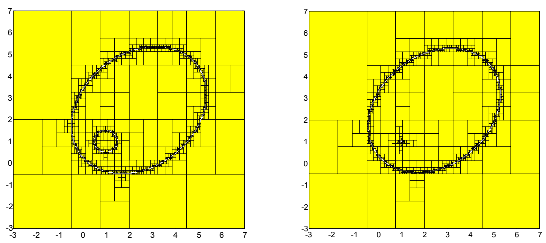
and

$$\begin{aligned} & 1.2\left(q_1^2+q_2^2\right)+34.08-9.32\left(q_1+q_2\right)-r^2 \\ & +0.4q_1q_2>0 \end{aligned} \quad (18)$$

To calculate the absolute stability region (*degree of stability 0*) and the region which reaches *stability degree 0.1*, a branch-and-bound algorithm based on modal intervals is used. Using it, two functions are analyzed and four linked lists are generated:

(1) Unstable regions
(2) Regions of stability degree between 0 and 0.1
(3) Regions of stability degree greater than 0.1
(4) Residual rectangles.

Figure 2 shows the results obtained using the modal intervals based algorithm. In Figure 2-a, the outer region has a stability degree greater than 0.1, the inner region corresponds with unstable regions, while regions into intermediate zone have a stability degree between 0 and 0.1. A particular case, such as when $r = 0$, is represented in figure 2-b. Here, the unstable region of the parameter space is a point, thus there is one unique unstable polynomial.



a b

Fig. 2. Stability regions for a: $r = 0.5$ b: $r = 0$

Note that, contrary to other methods, modal interval algorithms do not leave the unstable point out as there is an undetermined region around it.

2.2 *Robustness design*

To illustrate this problem, an example suggested by Fiorio *et al.* (1993) and later studied by Malan *et al.* (1997) with the aim of tuning a PI controller for an interval plant is presented. In (Vehí, 1998), modal intervals are applied to this example. Fiorio suggests a method based on Bernstein's polynomial expansion for the problem of designing robust controllers of fixed structure dependent on some free design parameters.

Example 2. The plant is described by the transfer function

$$G(s, \mathbf{q}) = \frac{q_1}{1 - \frac{s}{q_2}} \tag{19}$$

where q_1 and q_2 are the uncertain parameters remaining inside an interval $q_i = [0.8,\ 1.25]$.

The PI controller is expressed as:

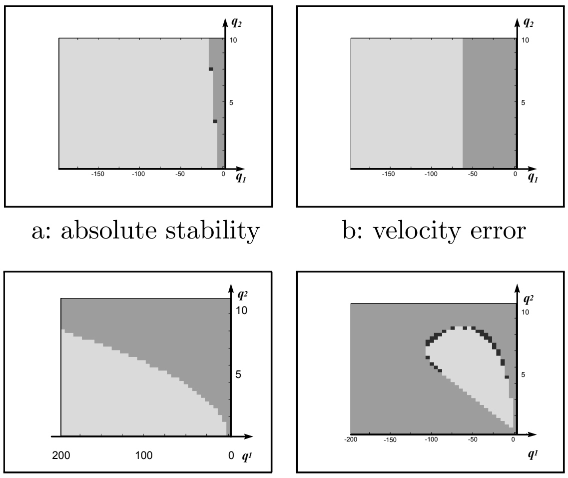
$$C(s, \mathbf{k}) = \frac{k_1 \left(1 + \frac{s}{k_2}\right)}{s} \tag{20}$$

where $\mathbf{k} = [k_1\ k_2]^T$ is the design parameter vector.

The design aim is to find the parameter set K of the controller that completes the following performance specifications:

(1) Closed-loop stability.
(2) Velocity error less than 2%.
(3) Control signal less than 20.
(4) Resonance peak of the closed-loop transfer function less than 3 dB.

Given the initial range $\mathbf{K}_{init}$ as a starting point, the algorithm calculates the set of controllers $\mathbf{K}$ which fulfill the performance specifications. In this example, as illustrated in Figure 3, the regions of the feasible controllers for each one of the specifications are computed in a separate form.



a: absolute stability b: velocity error

c: control effort d: resonance peak

Fig. 3. Feasible regions

The parameter space region of the feasible controllers is the intersection of the four regions determined by application of the four different specifications.

The application of coercion theorems from modal interval theory reduces the computation time to less than half in the worst cases.

Following this line of research, Malan *et al.* (1997) make an overview of some of the main interval mathematical algorithms to test their efficiency on several problems such as robustness design. Fiorio and Malan present their approaches taking into direct account the presence of parametric perturbations, which has not been treated by many authors. Piazzi and Visioli (1998*b*) propose a new feedforward/feedback synthesis design with the aim of minimizing the worst-case settling-time relative to the transition. Rocco *et al.* (2001) propose an approach based on the combination of two tools: Interval Analysis and CES (Cellular Evolutionary Strategies). The result is the achieving of only one interval evaluation and of the exact range of the constraint functions inside the generated box.

An approach suggested by Shashikhin (2001) considers the problem of design of the robust controller for uncertain time delay systems. The proposal consists on finding solutions of two algebraic Riccati equations with real coefficients that corresponds to boundary values of interval coefficients. In this case the techniques of interval analysis are used to solve an interval matrix Ricatti type equation, which result gives the parameters of the robust controller. This design method guarantees properties of robust stability of the closed system concerning to the structured parameter perturbations and values of finite time delays.

2.3 *State space*

Misra (1989), who is considered the first to have used interval arithmetic in an explicit form to study stability analysis, proposes the problem of finding the state feedback vector which achieves stability for an interval coefficient polynomial.

The solution suggested by Misra uses Routh's table for the feedback interval polynomial and calculates the elements of the first column via the evaluation of its natural extension. As is known, the result of computing the elements of the table using interval arithmetic overestimates the exact range, so sufficient conditions for stability are obtained.

Then Misra suggests a problem with many restrictions which only consider the case of polynomials with interval coefficients and only has a partial solution. The problem can be solved more easily with the Kharitonov theorem.

Another interesting approach, more general than Misra's one, is related by Kwon and Cain (1995), Kwon and Cain (1996). They consider the problem of finding a state feedback vector for an uncertain system with two matrices $\mathbf{A}$ and $\mathbf{B}$, which depend on an uncertain parameter vector.

They consider a linear system with parametric uncertainty and a static state feedback control, given by

$$\dot{\mathbf{x}}(t) = \mathbf{A}(p)\mathbf{x}(t) + \mathbf{B}(p)\mathbf{u}(t) \quad (21)$$
$$\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector, $\mathbf{u}(t) \in \mathbb{R}^m$ the input vector, p an unknown parameter vector and $\mathbf{K} \in \mathbb{R}^{m\times n}$ the constant matrix. The purpose of control is to find a constant matrix which can stabilize the perturbed system against all parametric perturbations over a variation domain Q. By putting $\mathbf{k} = vec(\mathbf{K})$ and calling $g(\mathbf{k}, \mathbf{q})$ the vector formed by the coefficients of the characteristic polynomial, Kwon and Cain show that the problem of robust pole-location is equivalent to finding a vector k^* such that $g(\mathbf{k},\mathbf{q}) > 0$, $\forall q \in Q$. Then the problem can be formulated and solved as an NP-hard optimization problem. The algorithm used to implement this optimization is based on interval analysis and the theory of semi-infinite programming.

Another approach in this field is presented in (Smagina, 1997) and (Smagina and Brewer, 2000) where is proved the direct correlation between interval dynamical system controllability and existence of robust regulators. The main advantage, is the result of comparing it with the analytic synthesis of regulators, achieving a smaller number of operations with polynomial matrices.

2.4 H_2/H_∞

Optimal H_2/H_∞ controller design aims to find a k^* so that a feedback controller $C(s; k^*)$ internally stabilizes the plant while minimizing a nominal H_2 cost, subject to the robust closed-loop stability constraint (H_∞).

With this aim of determining a global minimizer k^* , an effective option is a global optimization algorithm with hybrid features. This algorithm is based on a genetic algorithm at the upper level and on an interval procedure at the lower level to handle the semi-infinite constrains. Another possibility is to use a technique based on the partially elitistic genetic algorithm. Applying this method, the computation of some terms makes a special interval procedure necessary, i.e. a deterministic algorithm which uses concepts of interval analysis. An algorithm of this kind is presented by Guarino Lo Bianco and Piazzi (1996).

Taking into account the problem of calculating compensators, Weinhofer and Haas (1997) present a new H_∞-approach which builds the compensator using interval arithmetic in order to avoid numerical problems, thus obtaining a precision bound on the results. This approach uses frequency domain methods for the design process which offer a minimal degree for the compensators. As a result, a numerically stable design tool is presented in which the obtained interval can be adjusted dynamically, making it possible to calculate results with a predefined precision, (Haas, 1998).

Haas and Weinhofer (1996) have made an approach in the case of design methods for H_2-compensators (two parameter compensators) for linear multivariable plants. The method presented offers two main advantages over most published H_2-design methods: first the decoupling of the design of reference tracking and disturbance rejection and second the optimization of a cost function with respect to non-square integrable deterministic signals. To get a bound on the precision of the results, interval arithmetic is introduced.

Once approaches H_2 and H_∞ have been presented, a mixed study between these two techniques is presented by Guarino Lo Bianco and Piazzi (1999*b*). They propose a solution to a H_2/H_∞ fixed-structure controller design problem via a global optimization approach. To implement this approach a necessary preliminary step is to convert the H_∞ constraint into semi-infinite inequality over a real bound interval. Thus, the optimization problem is reduced to a bound-constrained problem. In order to solve this new problem, the genetic/interval algorithm in Guarino Lo Bianco and Piazzi (1996), is presented as the best option. In a previous study by these authors, (Guarino Lo Bianco and Piazzi, 1997), interval analysis was also applied to a global optimization problem, in this case, via an *ad hoc* interval procedure which uses concepts of interval analysis to obtain the convergence with certainty within the prespecified numerical tolerance.

The approach by Haas (1998), presents a control toolbox in which the most common algorithms used in the frequency domain are implemented.

3. FREQUENCY METHODS

In order to carry out frequency response analysis and design incorporating robustness with respect to parameter uncertainty we need to be able to determine the complex plane images of various parameterized sets. Two relevant lines of investigation in this field are remarkable: qualitative-feedback-theory (QFT) and value sets.

3.1 *Value sets*

Ohta *et al.* (1990) introduced PIA (Polygon Interval Arithmetic) as a powerful tool, which solves robust controller design (and robust stability analysis) more efficiently than other classical methods. Later in (Ohta *et al.*, 1994*a*), he presented an improvement on this tool to reduce the computing time of execution in. The same year, Ohta proposed two design methods as applications of PIA. The first, (Ohta *et al.*, 1994*c*), is based on the gain phase shaping approach, with the main advantage of guaranting the worst case performance. The second one, (Ohta *et al.*, 1994*b*), proposes a method of computing an almost exact gain margin. Another field of control in which Ohta applies PIA is solving zero-exclusion problems efficiently, (Ohta *et al.*, 1995). In these cases, PIA is used to estimate the value sets of multi-linear functions.

A new version called NPIA (Non-convex Polygon Interval Arithmetic) in (Ohta *et al.*, 1996), can be seen as the arithmetic defined in the set of all polygons in the complex plane. It is very useful to compute estimates of value sets of transfer functions including uncertain physical parameters in a reasonable computing time. In a more applied paper, (Ohta *et al.*, 1997), is proposed a method to compute a region of PID parameters which guarantees robust stability and several robust performances for systems with uncertain parameters. Again in the field of PID controllers, Ohta (1999) addresses the design of two degrees of freedom robust PID controller by solving minimization problems. Because in this last approach the regions obtained are not convex, set operations are used to compute level sets of the minimization problems. In further work, (Ohta, 2000), is described the implementation of NPIA as tool for the analysis and design of robust control systems to show that its definition and implementation allows estimating of value sets in an efficient computation cost. For nonlinear control systems with uncertain parameters and constant reference inputs (Lur'e systems), the possibility of shifting in the equilibrium state or a loss of stability increases. Wada *et al.* (1998) check conditions for parametric absolute stability in this case by computing value sets using the PIA algorithm, because its use considerably simplifies computation.

Other approaches are focused on more applied fields. For instance, Hedrich and Barke (1999) formulate a possible verification technique of linear analog circuits with parameter tolerances based on a curvature driven bound computation for value sets using interval arithmetic.

3.2 *QFT*

In several applications of quantitative feedback theory (QFT), templates of non-rational transfer functions have to be numerically generated. Sardar and Nataraj (2000) propose an algorithm for generating templates of uncertain non-rational transfer functions which completes its task without the requirement of rational transfer function approximation. Nataraj and Sardar (1999) introduce an algorithm based on the Moore-Skelboe global optimization technique of interval mathematics to generate Bode plot envelopes for uncertain transfer functions. In a further approach, Nataraj and Sardar (2000*a*) present QC (quadratic constrains) algorithms for computing QFT bounds to achieve robust sensitivity reduction and gain-phase margin specifications. As improvement on other existing algorithms where discrete controller phase values are used, these new algorithms can generate bounds over intervals of controller phase values solving the difficulty that at the non-selected phase the bound values are not actually computed. In this sense, in (Nataraj and Sardar, 2000*b*) are presented algorithms which use interval analysis as tool to build plant templates, achieving important improvements as eliminating safety problems associated with the phase discretization process in QFT bound generation and security of computed results.

4. DISCRETE-TIME CONTROL

Shieh *et al.* (1996) present a methodology of conversion from a continuous-time interval model to an enclosing discrete-time interval model, which uses interval arithmetic.

In a further study, (Shieh *et al.*, 1999), is proposed a method to convert a discrete-time uncertain system to an equivalent continuous-time uncertain model. In this case, interval analysis is used for the construction of the proposed procedure which is based on an interval geometric-series method.

In reference to robust Schur stability problem, an approach is presented by Garloff and Graft (1999), where they consider it with coefficients depending polynomially on parameters varying in given intervals. They propose an algorithm which relies on the Bernstein expansion of the symmetric and anti-symmetric parts for the polynomial family to reach the verification of this Schur stability.

Many design problems, including control and signal processing, can be formulated within the framework of guaranteed tuning. In (Jaulin and Walter, 1996*b*) is given an approach concerned with this framework, presenting a prototype numerical algorithm. This algorithm is based on interval analysis, a very useful tool in this case because in the algorithm it is necessary to characterize sets defined by inequalities, and for this function interval analysis is the best tool.

The same authors in another approach, (Jaulin and Walter, 1997), use interval analysis to compute all the sequences of control driving a deterministic nonlinear discrete-time state-space system from a given initial state to a given desired set of terminal states.

In (Vehí *et al.*, 2000*a*), interval techniques are applied to the analysis of the robustness of predictive controllers. The basic tool used is modal interval analysis, (Gardeñes *et al.*, 1985). The authors base their approach on the affirmation that checking the robustness of a predictive controller is equivalent to verifying the positiveness of the range of a set of functions.

Model Predictive Control (MPC) is one of the most popular control strategies. A new version of the MPC strategy is presented in (Bravo *et al.*, 2000), called Interval Model Predictive Control (IMPC). This approach appeared in order to complete the MPC strategy in the case that the process or the constrains were not linear or the cost function was not quadratic. To solve these special cases, global optimization algorithms which use interval analysis, are introduced as the best tool.

5. OTHER APPLICATIONS OF INTERVALS

There are a few other problems that do not fit with any of the control subjects studied up to now, but also use interval analysis.

Estimation

Bounded-error parametric estimation approaches have arisen in the last decade, mainly for two reasons:

-These approaches can deal with deterministic structural errors not adequately described by random variables.

-They are well suited to the guaranteed characterization of parameter uncertainty.

Some authors have built their approaches around these kinds of control problems applying interval techniques with the aim to obtain improvements over classical methods.

Given a model structure, one problem is the choice of the criterion to be optimized in order to find the best model in the class to be defined. That is to say, the criterion for this selection is the optimization of a scalar cost function $j(\boldsymbol{\rho})$ with respect to the model parameters p.

An estimator is said to be robust if its performance does not deteriorate too much when the hypotheses on which it is based are not satisfied. Some approaches to obtain robust estimators have been studied in (Walter and Pronzato, 1997).

Following the same line of study, (Jaulin and Walter, 1993; Jaulin and Walter, 1996*a*; Jaulin and Walter, 1999), cast non-linear bounded-error estimation into the framework of set inversion, and present an algorithm to solve it using interval analysis. In (Jaulin and Walter, 1999) the SIVIA (Set Inverter Via Interval Analysis) is presented as an adapted algorithm from (Jaulin and Walter, 1993) which characterizes the set of all values of the parameter vector to be estimated. In (Jaulin *et al.*, 1999), is applyied interval analysis to bounded-error parametric estimation for discrete-event systems (DES), in which behaviors are governed by occurrences of different types of events rather than by ticks of a clock. Problems involving DES systems are generally nonlinear, non-convex and non-differentiable, so classic methods often fail to give reliable results.

Kieffer *et al.* (1998) present a state estimator based on interval analysis, which evaluates a set estimate guaranteed to contain all the values of the state which are consistent with the available observations, given the noise bounds and a set containing the initial value of the state.

Another algorithm for parameter estimation is presented by Feng *et al.* (1999). In this case it is a recursive algorithm for calculating axis-aligned orthotopes, or boxes, which bound the set of feasible parameters. It is shown that interval mathematics can be used as an efficient tool to calculate these orthotopic bounds at each iteration, providing very accurate estimates.

Markov and Popova (1996) consider the problems of interpolation and curve fitting in the presence of unknown but bounded errors in the output measurements, using generalized polynomials under bounded measurement uncertainties.

In (Munoz and Kearfott, 2000) is suggested an approach to estimate accurate model parameters that provide the best fit to measured data, despite small-scale noise in the data or occasional large-scale measurement errors. In contrast classical methods, the proposed techniques use interval arithmetic to compute in a reliable way the global optimum for the nonlinear parameter estimation problem. The estimators used are: nonsmooth least absolute value and minimax.

In the approach suggested in (Brahim-Belhouari *et al.*, 2000) Set Inversion Via Interval Analysis (SIVIA) is used to make the model selection for measurement purpose.

Robotics

Staying with estimation approaches but now applying them to robot localization, Kieffer *et al.* (1999) introduce a methodology which makes it possible to compute a set guaranteed to contain all values of the parameter vector, and that provides that an upper bound on the number of tests at fault is available. The estimator used is obtained from an adaptation of the SIVIA algorithm (Kieffer *et al.*, 1998).

Guarino Lo Bianco and Piazzi (1999*a*) try to compute optimal robot trajectory planning. Their approach combines two optimization techniques: stochastic optimization using a genetic algorithm and deterministic optimization using an interval algorithm, to arrive at a feasible certain estimation of the global solution. In later papers from (Piazzi and Visioli, 1998*a*; Piazzi and Visioli, 2000), the same planning problem is presented but in this case it is solved by using a global deterministic approach based on a procedure which uses interval analysis tools. In their first study (Piazzi and Visioli, 1998*a*), the aim was to compute the total traveling time required to perform the robotic task, which is stated as an optimal trajectory planning problem with minimum-time criterion. Their second, (Piazzi and Visioli, 2000), was concerned with jerk constrains due mainly to the fact that joint position errors increase when the jerk increases, and to limit excessive wear on the robot and the excitation of resonances so that the robot life-span is extended.

An example is used in (Vehí *et al.*, 2001) to show that the controller obtained using modal interval analysis tools, exhibits a much more robust performance in the convergence of the tracking error and the stabilization of the controlled process, than a nominal predictive controller based on a single value of the parameter vector.

Another basic robot control problem consists of computing the actuating torques required to make the robot follow a desired trajectory. Vehí *et al.* (2000*b*) , concentrating on this kind of problem, take a non-holonomic mobile robot as a prototype and then obtain a velocity control based on an estimated model. They present a method to design and implement an interval model based on a PI controller. The method they use is by means of Modal Interval analysis (Gardeñes *et al.*, 1985), which provides tools to solve the interval equations that appear during the design process and to compute control laws as interval functions.

Gain Scheduling

Gain-scheduling compensators are usually used in closed loop in order to achieve good performance in spite of large parameter variations. Fadali and Bebis (1998), use a linear time-invariant (LTI) system and propose a robust design synthesis approach based on the solution of a Diophantine equation. Interval analysis is used to extend the synthesis procedure proposed by Fadali et al. to systems which contain uncertain transfer function coefficients. As result, a satisfactory controller or family of controllers for bounded parameter uncertainty are obtained. Following along the same line of controller design, Fadali and McNichols (2000), propose a methodology to control non-linear systems with slowly varying dynamics using gain scheduling (GS). Again, interval analysis is used to extend this approach including slowly varying non-linear interval systems by applying GS.

McNichols and Fadali (2001) use an interval arithmetic tool to determine intervals which restrict the closed loop poles of the system to regions around the ideal transfer function coefficients. The aim of their design is to determine a minimal set of design points which connect with GS. As a result they have obtained its ideal controller coefficients which in turn place the closed loop poles at their nominal design locations.

6. CONCLUSIONS

In this paper, some key approaches concerned with robust control have been surveyed in order to point out the improvements suggested by interval analysis. Interval analysis, over all the survey is presented as a very useful tool that allows to avoid numerical problems. In addition, it can mainly be interesting for problems of set characterization involving optimization, nonlinear inequalities and quantifiers, for which interval analysis should be much helpful. This is due to its ability to produce guaranteed results even in a nonlinear context. Interval analysis has been also shown to be very powerful in bounding the ranges of functions efficiently while providing mathematically rigorous results. This capability is especially welcome in robust control since a variety of analysis and design problems can be cast in the evaluation of the range of functions over intervals. The arising relevance that this tool has been taking is shown by the high number of approaches that have been appeared in order to achieve a perfect match with the corresponding problem. In this sense, the survey shows interval analysis variants as: PIA (Polygon Interval Arithmetic), NPIA (Non-convex Polygon Arithmetic), Modal interval analysis, and so on.

As a final assessment, it can be stated that from this study of state-of-the-art works, the lack of approaches concerned with design is noted. This fact gives encourage to following this research line, because the impression is that it rests a lot of work to do.

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