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SYSTEMATIC NONLINEAR CONTROLLER DESIGN FOR A POWER FACTOR CORRECTOR

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Abstract: A nonlinear controller for a power factor corrector is systematically constructed via Lyapunov-based controller design approach for the bilinear state averaged model. First, a nonlinear gain of the controller is derived to shape the source current and the output voltage be desired form respectively via nonlinear H^∞ control system design approach. Second, a source current reference generator is constructed, which consists of a feedforward loop given by steady state analysis and a feedback loop with output voltage error amplifier. This paper, finally, shows efficiencies of the approach through computer simulations.

Keywords: power supplies, switching rectifiers, bilinear systems, H-infinity control, Lyapunov function, convex programming

1. INTRODUCTION

There has been a steady growth of interest in control of power electronic circuits (e.g.,(Kassakian *et al.*, 1991; Banerjee and Verghese, 2001)). Many works (e.g., (Kassakian *et al.*, 1991)) discuss linear feedback control problems for power electronic circuits on the basis of linearized state averaged model of the circuits. The work (Banerjee and Verghese, 2001) is beginning to discuss nonlinear feedback control problems for the circuits via treating those as nonlinear systems.

In this paper, a nonlinear controller for a power factor corrector is systematically constructed on the basis of a bilinear state averaged model. The controller design approach consists of the following two steps.

First, a nonlinear gain of the controller is derived via nonlinear H^∞ control system design approach by using a convex programming technique. The nonlinear gain is designed to (1) shape a source current be sinusoidal and in phase with a source voltage and (2) keep an output voltage constant.

Second, a source current reference generator is derived, which consists of a feedforward loop and a feedback loop. The feedforward loop is given by a steady state analysis for the corrector model. The feedback loop includes an output voltage error amplifier.

This paper is organized as follows. Section 2 gives a power factor corrector model. Section 3 derives a nonlinear gain and a source current generator. Section 4, finally, demonstrates efficiencies of the approach through computer simulations. The simulation uses the same circuit parameters as in the work (Escobar *et al.*, 2001).

2. POWER FACTOR CORRECTOR MODEL

This paper constructs a power factor corrector control system as shown in Fig. 1. First, the power factor corrector model is derived. An inductor current i and a capacitor voltage v are treated as states of the system. A functions μ is regarded as a controller output to drive switches as shown in

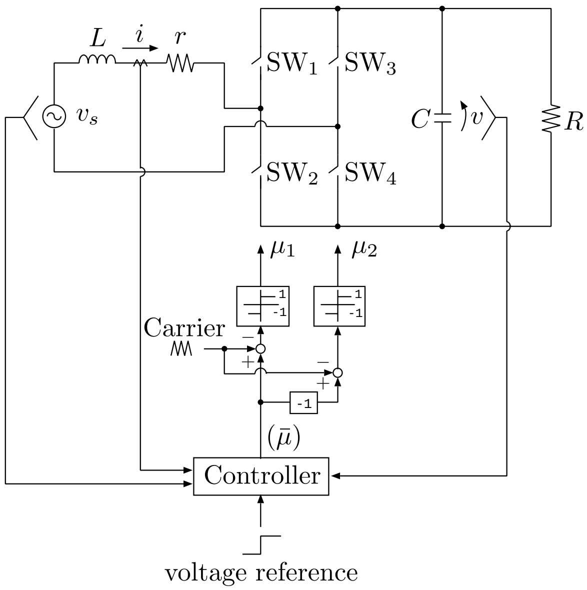

Fig. 1. Power Factor Corrector Control System

Fig. 1 and Table 1, where μ is given as $(\mu_1 - \mu_2)/2$. Then, a switched model (Σ_S) is given as the form

$$\frac{d}{dt}\begin{bmatrix} i \\ v \end{bmatrix} = \begin{bmatrix} -\frac{r}{L} & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_s + \left\{ i \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} + v \begin{bmatrix} -\frac{1}{L} \\ 0 \end{bmatrix} \right\} \mu \quad (1)$$

where v_s denotes a source voltage, L an inductance, C a capacitance, R a load resistance, and r an internal resistance.

Table 1: Definition of Switching Functions

SW$_1$	SW$_2$	μ_1	SW$_3$	SW$_4$	μ_2	μ
on	off	1	off	on	−1	1
on	off	1	on	off	1	0
off	on	−1	on	off	1	−1
off	on	−1	off	on	−1	0

3. SYSTEMATIC CONTROLLER DESIGN

This section constructs a nonlinear controller to (1) shape a source current be sinusoidal and in phase with a source voltage and (2) keep an output voltage constant. First, an averaged model for the switched model (Σ_S) is derived. A steady state analysis for the averaged model gives an amplitude of source current to keep the output voltage constant, which is used for a current reference generator. Then, a model around a specified set point is derived from the averaged model. Second, a controller design specification gives a model to construct the controller, which includes controller design parameters. Third, on the basis of the design model, a nonlinear gain is concretely given by considering a domain of current and voltage and using a convex programming technique. Finally, a source current reference generator is constructed, which operates against source voltage and load resistance variations.

3.1 *Bilinear State Averaged Model and Its Steady State*

First, the switched model (Σ_S) gives a state averaged model (Σ_{SA}) of the form

$$\frac{d}{dt}\begin{bmatrix} \bar{i} \\ \bar{v} \end{bmatrix} \approx \begin{bmatrix} -\frac{r}{L} & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} \bar{i} \\ \bar{v} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_s + \left\{ \bar{i} \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} + \bar{v} \begin{bmatrix} -\frac{1}{L} \\ 0 \end{bmatrix} \right\} \bar{\mu} \quad (2)$$

where $\bar{i}$, $\bar{v}$ and $\bar{\mu}$ denote moving averages of i, v and μ, respectively (Kassakian *et al.*, 1991).

Here, a steady state of the model is analyzed. Given a source voltage $v_s = \sqrt{2}\,V_e \sin\omega t$, assume that the capacitor voltage is $\bar{v} = v_r$. Then, dc components of the system in the steady state decide a source current $\bar{i} = \sqrt{2}\,I_e \sin\omega t$, which is sinusoidal and in phase with the source voltage, given as

$$I_e = \frac{V_e}{2r} - \sqrt{\frac{V_e^2}{4r^2} - \frac{v_r^2}{rR}} =: I_e(v_r). \quad (3)$$

The decision process is discussed in the works (Escobar *et al.*, 1999; Escobar *et al.*, 2001). In the following section, the effective value $I_e(v_r)$ is used for a source current reference generator. Note that the effective value $I_e(v_r)$ depends on an effective value of source voltage V_e and a load resistance R which vary in practical circuits.

Next, for a Lyapunov-based controller design, the state averaged model (Σ_{SA}) moves to a model (Σ_A) around a specified set point $[\bar{i} \;\; \bar{v} \;\; \bar{\mu}] = [0 \;\; E \;\; 0]$, which is given as the form

$$\frac{d}{dt}\begin{bmatrix} \tilde{i} \\ \tilde{v} \end{bmatrix} \approx \begin{bmatrix} -\frac{r}{L} & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} \tilde{i} \\ \tilde{v} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} v_s \\ E \end{bmatrix} + \left\{ \begin{bmatrix} -\frac{E}{L} \\ 0 \end{bmatrix} + \tilde{i} \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} + \tilde{v} \begin{bmatrix} -\frac{1}{L} \\ 0 \end{bmatrix} \right\} \tilde{\mu} \quad (4)$$

where $[\tilde{i} \;\; \tilde{v} \;\; \tilde{\mu}] = [\bar{i} \;\; \bar{v} \;\; \bar{\mu}] - [0 \;\; E \;\; 0]$. The model ($\Sigma_A$) is rewritten as the form

$$\dot{x}_p = A_p x_p + B_{p1} w_p + B_{p2}(x_p)\tilde{\mu} \quad (5)$$

where

$$x_p := \begin{bmatrix} \tilde{i} & \tilde{v} \end{bmatrix}^T, \quad w_p := \begin{bmatrix} v_s & E \end{bmatrix}^T,$$

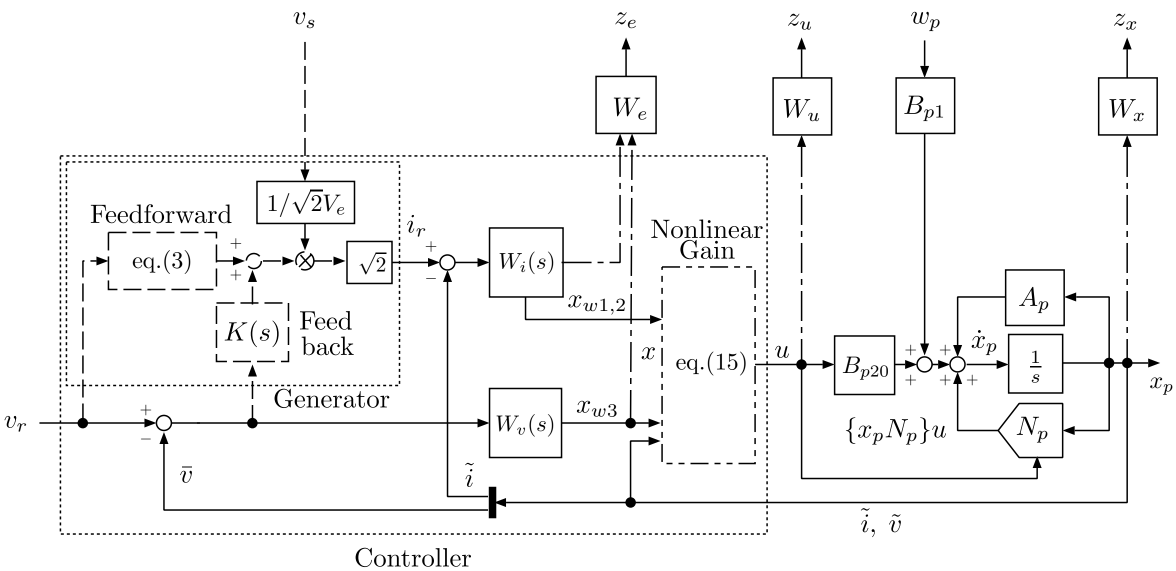

Fig. 2. Block Diagram for Power Factor Corrector Control System

$$A_p := \begin{bmatrix} -\dfrac{r}{L} & 0 \\ 0 & -\dfrac{1}{RC} \end{bmatrix}, \quad B_{p1} := \begin{bmatrix} \dfrac{1}{L} & 0 \\ 0 & -\dfrac{1}{RC} \end{bmatrix},$$

$$\begin{aligned} B_{p2}(x_p) &:= B_{p20} + \{x_p N_p\} \\ &:= B_{p20} + x_{p1} B_{p21} + x_{p2} B_{p22} \\ &:= \begin{bmatrix} -\dfrac{E}{L} \\ 0 \end{bmatrix} + \tilde{i} \begin{bmatrix} 0 \\ \dfrac{1}{C} \end{bmatrix} + \tilde{v} \begin{bmatrix} -\dfrac{1}{L} \\ 0 \end{bmatrix}. \end{aligned}$$

The model (Σ_A) is called a bilinear system in a class of nonlinear systems (Mohler, 1991).

3.2 *Controller Design Specification*

For the bilinear state averaged model (Σ_A), a nonlinear controller is constructed to meet the following design specification;

(S1) The source current $\tilde{i}$ is sinusoidal and in phase with the source voltage v_s $(i_r \to z_e)$,

(S2) The output voltage $\tilde{v}$ should track a voltage reference v_r $(v_r \to z_e)$.

The specification gives a design model (Σ_G), as shown in Fig. 2, of the form

$$\dot{x} = Ax + B_1 w + B_2(x)u, \tag{6}$$

$$z = C_1 x + D_{12} u, \tag{7}$$

where

$$x = \begin{bmatrix} x_w^T & x_p^T \end{bmatrix}^T, \qquad w = \begin{bmatrix} i_r & v_r & w_p^T \end{bmatrix}^T,$$

$$u = \tilde{\mu}, \qquad z = \begin{bmatrix} z_e^T & z_x^T & z_u \end{bmatrix}^T,$$

$$A = \begin{bmatrix} A_w & -B_w \\ 0 & A_p \end{bmatrix}, \qquad B_1 = \begin{bmatrix} B_w & 0 \\ 0 & B_{p1} \end{bmatrix},$$

$$B_2(x) = \begin{bmatrix} 0 \\ B_{p20} \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ B_{p21} \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ B_{p22} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} W_e C_w & 0 \\ 0 & W_x \\ 0 & 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 \\ W_u \end{bmatrix}.$$

x_w, A_w, B_w, C_w are state and coefficient matrices of state space description of the form

$$\dot{x}_w = A_w x_w + B_w\left(\begin{bmatrix} i_r \\ v_r \end{bmatrix} - x_p\right), \quad x_w = 0, \tag{8}$$

$$y_w = C_w x_w, \tag{9}$$

where

$$A_w = \begin{bmatrix} 0 & \omega_n & 0 \\ -\omega_n & 0 & 0 \\ 0 & 0 & -\varepsilon \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 & 0 \\ k_v & 0 \\ 0 & k_i \end{bmatrix}, \tag{10}$$

$$C_w = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

for weighting functions

$$W_i(s) = \frac{k_i \omega_n}{s^2 + \omega_n^2}, \qquad W_v(s) = \frac{k_v}{s + \varepsilon} \tag{12}$$

with respect to the specification. The weighting functions $W_i(s)$ and $W_v(s)$ are used for the current and voltage to track the references i_r and v_r, respectively. Parameters k_i and k_v adjust gains of the weighting functions for the tracking to be achieved. ω_n is a principal value of angular frequency in the voltage source. Matrices W_e, W_u and W_x are weighting parameters to be used only for the controller design.

3.3 *Nonlinear Gain*

On the basis of the design model (Σ_G), this section gives concretely a nonlinear gain, that is a block from x to u in Fig. 2, via nonlinear H^∞ control system design approach. The work (Sasaki and Uchida, 1998) says that for a given γ in a specified domain of x, a matrix Y satisfying inequalities of the form

$$(1)\quad Y > 0, \tag{13}$$

$$(2)\quad \begin{bmatrix} AY + YA^T + \gamma^{-2}B_1B_1^T & YC_1^T \\ -B_2(x)(D_{12}^T D_{12})^{-1}B_2(x)^T & \\ C_1 Y & -I \end{bmatrix} < 0 \tag{14}$$

gives the nonlinear gain of the form

$$u = -(D_{12}^T D_{12})^{-1}B_2(x)^T Y^{-1} x. \tag{15}$$

For any state x, which is current and voltage in a specified domain, the matrix Y satisfying the inequalities is concretely given by using a convex programming technique (Sasaki and Uchida, 1998). The gain guarantees a closed-loop system stability and a tracking performance for output voltage and source current references. Note that the nonlinear gain (15) does not depend on a structure to generate the source current reference i_r.

3.4 *Source Current Reference Generator*

This section gives a mechanism to generate a source current reference i_r. For a given output voltage reference v_r, in a steady state of the system, a source current amplitude is given by the equation (3). The source current amplitude, however, depends on the source voltage v_s and the load resistance R which vary in practical circuits. The source and load variations need change an amplitude of current reference i_r so that the power balance between ac and dc ports are not violated. For this purpose, a gain block $K(s)$ is used, which is

$$K(s) := k_P + \frac{k_I}{s} + k_D s \tag{16}$$

where k_P, k_I, and k_D are constant parameters decided by a system designer. Then, the source current reference i_r is given as the form

$$i_r = \sqrt{2}\,[I_e(v_r) + K(s)\{v_r - \bar{v}\}]\,\frac{1}{\sqrt{2}V_e} v_s. \tag{17}$$

Note that the $v_s/\sqrt{2}V_e$ means that for a voltage source the generator (17) requires only an angular frequency of the source.

As shown in Fig. 2, a feedback loop with the block $K(s)$ adjusts the amplitude of current reference such that the output voltage keeps constant even if the source or load changes, thus operating as output voltage error amplifier. The feedback loop is the same as a conventional loop used in many works (e.g., (Redl, 1994)).

The $I_e(v_r)$ (i.e., the equation (3)) in the equation (17) can be directly changed if the source and load changes are exactly measured. The works (Escobar *et al.*, 1999; Escobar *et al.*, 2001) directly adjust the $I_e(v_r)$ only to load variations via an adaptive control technique.

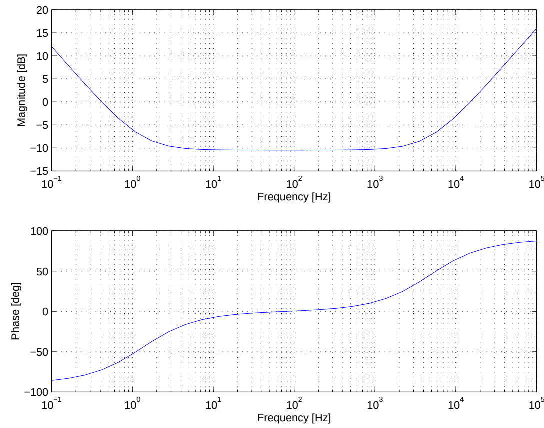

Fig. 3. Bode plots of output voltage error amplifier $K(s)$

4. COMPUTER SIMULATIONS

This section shows efficiencies of the approach through computer simulations. A software package; MATLAB, Simulink, LMI Control Toolbox is used for the simulations. Circuit parameters are used as

$$L = 1\ [\text{mH}],\quad C = 2200\ [\mu\text{F}],\quad R = 240\ [\Omega],$$
$$r = 2.2\ [\Omega]$$

which are the same as in the work (Escobar *et al.*, 2001). The load resistance $R = 240$ denotes the nominal value.

For the circuit, a nonlinear controller is derived. First, a nonlinear gain (15) is designed. Design parameters for the gain are chosen as

$$W_e = \text{diag}\left[10^2\ \ 10^{-4}\right],\quad W_u = 10,$$
$$W_x = \text{diag}\left[10^{-10}\ \ 10^{-10}\right],\quad \omega_n = 100\pi,$$
$$\varepsilon = 10^{-4},\quad k_i = 50,\quad k_v = 0.003,\quad \gamma = 0.98$$

where ω_n focuses on a voltage source with a frequency 50 hertz and W_e is set such that the tracking performance for source current has more weight than that for output voltage. Now, consider that a set point is [0 ampere, 180 volts] and the current and voltage are varying as ± 6 amperes, ± 50 volts (i.e., $-6 \le i \le 6,\ 130 \le v \le 230$). Then the convex programming technique in the section 3.3 gives the solution Y, as shown in (18), to obtain the nonlinear gain (15). Next, a voltage error amplifier $K(s)$ is designed. The gain is chosen in order to be low around the input line frequency (i.e., 50 hertz) as shown in Fig. 3, whose parameters are given as

$$k_P = 0.3,\quad k_I = 2.5,\quad k_D = 10^{-5}.$$

For an output voltage tracking, the smaller the parameter k_P is, the longer the transient response time is and the smaller the overshoot is.

Now, the nonlinear controller gives computer simulation results. Figs. 4 to 7 show responses for the switched model (Σ_S) in the following cases,

$$Y = \begin{bmatrix} 4.80 \times 10^{-1} & -5.11 \times 10^{0} & -6.12 \times 10^{-11} & -2.60 \times 10^{2} & -3.86 \times 10^{-8} \\ -5.11 \times 10^{0} & 7.18 \times 10^{1} & 1.10 \times 10^{-9} & 4.11 \times 10^{3} & 6.93 \times 10^{-7} \\ -6.12 \times 10^{-11} & 1.10 \times 10^{-9} & 1.31 \times 10^{4} & 7.30 \times 10^{-8} & 1.56 \times 10^{6} \\ -2.60 \times 10^{2} & 4.11 \times 10^{3} & 7.30 \times 10^{-8} & 2.52 \times 10^{5} & 4.61 \times 10^{-5} \\ -3.86 \times 10^{-8} & 6.93 \times 10^{-7} & 1.56 \times 10^{6} & 4.61 \times 10^{-5} & 9.84 \times 10^{8} \end{bmatrix} \quad (18)$$

where the initial state is $[\, i \;\; v \,] = [\, 0 \;\; 150 \,]$ and the switching frequency is 13 kilohertz;

(C1) An output voltage reference v_r changes from 160 volts to 200 volts at 0.3 seconds for a source voltage $v_s = 150 \sin 100\pi t$ volts and a load resistance $R = 240$ ohms ;

(C2) A load resistance R changes from 240 ohms to 80 ohms at 0.3 seconds for a source voltage $v_s = 150 \sin 100\pi t$ volts ;

(C3) An amplitude of source voltage v_s changes from 150 volts to 120 volts at 0.3 seconds for a load resistance $R = 240$ ohms.

Figs. 4, 6 and 7 show responses by the nonlinear gain. Fig. 5 shows responses by a linear gain which is given for linearized model of (Σ_G) (i.e., a solution Y satisfying inequalities (13) and (14) given as $B_2(x) = [\, 0 \;\; B_{p20}^T \,]^T$).

In all cases, the source current i is almost sinusoidal and in phase with the source voltage v_s. Fig. 4 shows that the capacitor voltage v tracks the output voltage reference v_r. Figs. 6 and 7 show that the capacitor voltage v keeps constant with short transient response time for changes of source voltage or load resistance. Comparisons between Fig. 4 and Fig. 5 show that the nonlinear gain gives a slightly better performance than the linear gain because a source current in Fig. 4 includes smaller high frequency components than in Fig. 5 and for an output voltage tracking an overshoot in Fig. 4 is smaller than in Fig. 5.

Than in the work (Escobar *et al.*, 2001), in all cases, the overshoots are greater and so the transient response times are much shorter. Moreover, the work (Escobar *et al.*, 2001) does not have any adaptation for source voltage variations.

The above computer simulation results show that the nonlinear controller works very well. Efficiencies of the systematic nonlinear controller design approach was shown.

5. CONCLUSION

A power factor corrector control system was clearly constructed via systematic nonlinear controller design approach. A nonlinear gain was derived via a nonlinear H^∞ control system design approach by using a convex programming technique. The gain guarantees a closed-loop system stability and a tracking performance for output voltage and source current references. A source current reference generator was composed of a feedforward loop derived by steady state analysis for the averaged model and a feedback loop with output voltage error amplifier. Computer simulations demonstrate efficiencies of the approach.

The systematic approach had been also applied to design the other power converter systems in the works (Sasaki *et al.*, 1999; Sasaki and Inoue, 2000).

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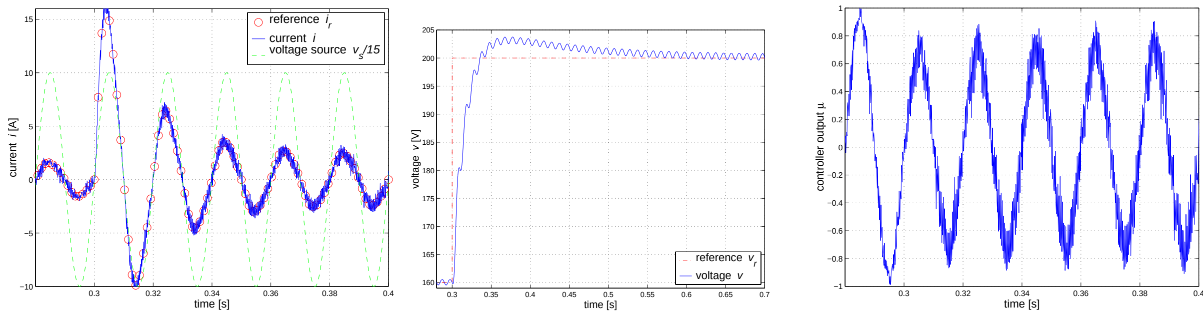
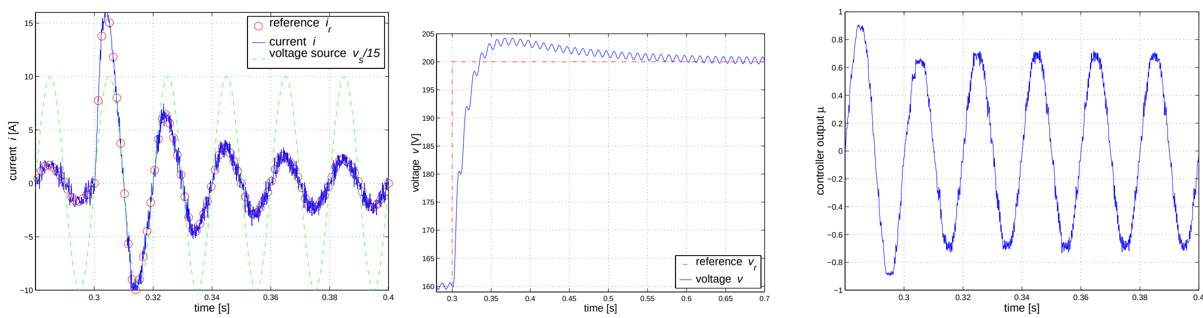


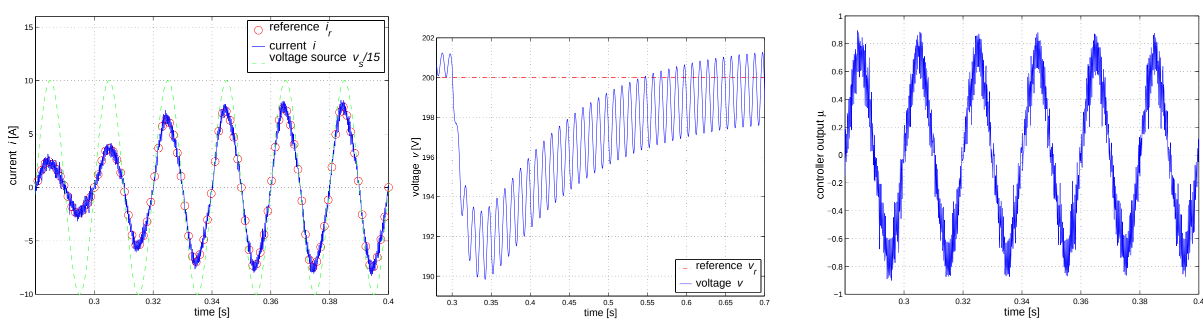
(a) inductor current i (b) capacitor voltage v (c) controller output $(\bar{\mu})$

Fig. 4. (C1) Output voltage reference v_r changes from 160 volts to 200 volts



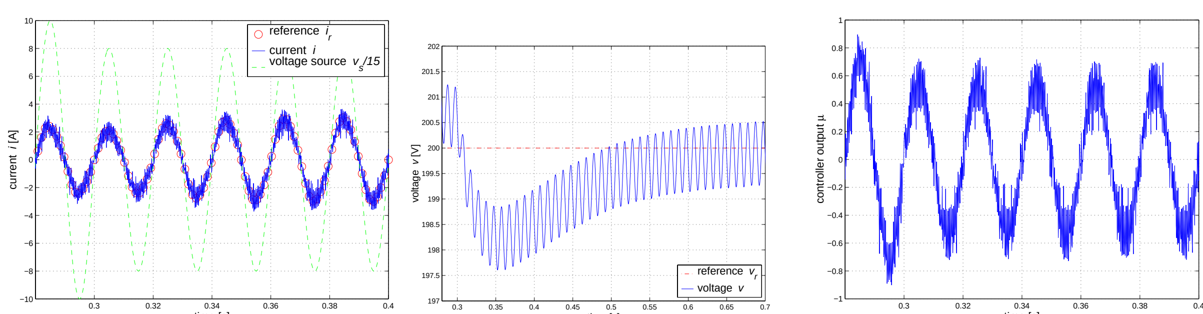
(a) inductor current i (b) capacitor voltage v (c) controller output $(\bar{\mu})$

Fig. 5. (C1) Under linear gain, output voltage reference v_r changes from 160 volts to 200 volts



(a) inductor current i (b) capacitor voltage v (c) controller output $(\bar{\mu})$

Fig. 6. (C2) Load resistance R changes from 240 ohms to 80 ohms



(a) inductor current i (b) capacitor voltage v (c) controller output $(\bar{\mu})$

Fig. 7. (C3) Source voltage changes from 150 volts to 120 volts