15th Triennial World Congress, Barcelona, Spain

SEMIGLOBAL STABILIZATION OF CONTINUOUS SYSTEMS WITH BOUNDED DELAYED INPUTS [1]

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Abstract: The semiglobal stabilization of null-controllable linear systems with delayed inputs which are bounded is achieved and the robustness of the closed loop scheme with respect to uncertainty in the parameters, in the delay and input bounded disturbances is proved. A delayed oscillator is stabilized with a bounded control law.

Keywords: time-delay, bounded inputs, predictor, periodic control, robustness.

1. INTRODUCTION

Many processes present delay in their input or output variables due to transport phenomena, time consuming information processing and sensors design among others. In many cases we can model their behavior with a linear multivariable system with delay h^* described by

$$\dot{x}(t) = A^* x(t) + B^* u(t - h^*) \qquad (1)$$

where the parameters are $A^* \in \mathbb{R}^{n\times n}, B^* \in \mathbb{R}^{n\times m}$, and the initial condition is $x(t) = f(t), t \in [-h^*, 0]$.

Many approaches for the control of systems with delay in the input include in an explicit or implicit manner a predictor of the state at time $t+h$. Some of the more widely used are the Smith predictor (Smith, 1959; Palmor, 1996), Process-Model Control schemes (Watanabe and Ito, 1981), and finite spectrum assignment techniques (Manitius and Olbrot, 1979; Artstein, 1982). A common drawback, linked to the internal instability of the prediction, is that they fail to stabilize unstable systems. As shown in Mondié *et al.* (2001a), it is possible to overcome this problem by introducing a periodic resetting of the predictor.

Another long lasting concern for control engineers is the use of bounded control laws. It is well known that for systems with poles in the open right half plane, only locally stabilyzing control laws can be obtained and that global or semiglobal stabilizability can be achieved only for systems with poles in the closed left half plane, named null-controllable (Sussmann *et al.*, 1994). The works of Teel (1992), Sussmann *et al.* (1994), Mazenc and Praly (1996), gave answers to this query in the framework of systems with no delay. The approach of Teel (1992), based on saturated control laws, was successfully generalized to input delay chains of integrators (Mazenc *et al.*, 2001). In the present paper, we focus our attention on the semiglobal stabilization for null-controllable linear systems based on a simple periodic linear controller proposed in Lozano *et al.* (1999). This approach is particularly well suited because a refreshing of the control law occurs naturally due to the periodicity of the control law.

The paper is organized as follows. The control scheme is presented in Section 2, and the closed loop system when there is no uncertainty is obtained in Section 3. The robustness of the closed loop system with respect to delay and parameter uncertainty is analyzed in Section 4. Conditions focusing on the uncertainty in the delay are given in Section 5. A delayed oscillator is stabilized in Section 6 and some concluding remarks end the paper. Previous results concerning the approach introduced in Lozano *et al.* (1999) are summarized in Appendix 7.

[1] Supported by Conacyt, Mexico 31951-A and by GDR-Automatique, France.
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2. CONTROL SCHEME

The systems considered here are those described by (1) such that the pair (A^*, B^*) is null-controllable (Sussmann *et al.*, 1994), and with no loss of generality, A^* is assumed to be in the Jordan real canonical form. The system response is

$$x(t) = e^{A^*(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A^*(t-\sigma)}B^*u(\sigma - h^*)d\sigma. \quad (2)$$

In a robust analysis framework, the delay and the parameters used in the design, h, A and B can be different from h^*, A^* and B^*. A prediction $x_p(t)$ for the variable $x(t+h)$, is then given by

$$x_p(t) = e^{Ah}x(t) + \int_{t}^{t+h} e^{A(t+h-\sigma)}Bu(\sigma - h)d\sigma \quad (3)$$

Defining $\tau := \sigma - h$, $x_p(t)$ can be rewritten as

$$x_p(t) = e^{Ah}x(t) + \int_{t-h}^{t} e^{A(t-\tau)}Bu(\tau)d\tau. \quad (4)$$

The above depends only on past and present values of $x(t)$ and $u(t)$. Thus, $x_p(t)$ is available at time t, and we introduce a predictor that is periodically reinitialized at time kT.

$$x_p(kT) = e^{Ah}x(kT) + \int_{kT-h}^{kT} e^{A(kT-\tau)}Bu(\tau)d\tau,$$

$$x_p(kT+\sigma) = e^{A\sigma}x_p(kT) + \int_{kT}^{kT+\sigma} e^{A(kT+\sigma-\tau)}Bu(\tau)d\tau,$$

$$\sigma \in [0, T), \quad k = 1, 2, ...$$

Equivalently, $x_p(t)$ is given by

$$x_p(kT+\sigma) = e^{A(h+\sigma)}x(kT) + \int_{kT-h}^{kT+\sigma} e^{A(kT+\sigma-\tau)}Bu(\tau)d\tau, \quad \sigma \in [0, T), \quad (5)$$

Combining the ideas on the use of a resetted predictor for delayed systems and those on semiglobal stabilization, let us propose the control law

$$u(\sigma + kT) = B^\top e^{-A^\top \sigma}W^{-1}(\Gamma - I)x_p(kT),$$
$$\sigma \in [0, T), \quad k = 1, 2, ... \quad (6)$$

where W is the controllability grammian

$$W := W(T) = \int_0^T e^{-A\tau}BB^\top e^{-A^\top \tau}d\tau, \quad (7)$$

which is invertible for all $T > 0$ because the system is controllable, and where Γ is defined according to the the design of the control law for null-controllable systems with no delay proposed in Lozano *et al.* (1999), recalled in Appendix 7.

3. STABILIZING CONTROL SCHEME: IDEAL CASE

In the ideal case, when the parameters and the delay size are known with certainty ($A = A^*$, $B = B^*$, $h = h^*$), and the integrals are performed with exactitude, one can write the response of the system at time $kT + T + h$ as

$$x(kT+T+h) = e^{AT}x(kT+h) + \int_{kT+h}^{kT+T+h} e^{A(kT+T+h-\tau)}Bu(\tau - h),$$

Defining the variable $\sigma = \tau - kT - h$ we obtain

$$x(kT+T+h) = e^{AT}\{x(kT+h) + \int_0^T e^{-A\sigma}Bu(kT+\sigma)d\sigma\},$$

Substituting the control law (6) in this expression, it follows that for $k = 1, 2, ...$

$$x(kT+T+h) = e^{AT}\{x(kT+h) + (\Gamma - I)x_p(kT)\}. \quad (8)$$

By comparing the expressions (2) for the system response and (3) for the predictor, one can see that under ideal conditions, namely known parameters and exact computation of integrals, we have that

$$x_p(t) = x(t+h), \quad t \geq 0.$$

Hence, substituting the above into (8) leads to

$$x_p(kT+T) = e^{AT}\Gamma x_p(kT).$$
$$x(kT+T+h) = e^{AT}\Gamma x(kT+h).$$

Now, for a given bounded prespecified set Ω and for any given bound Δ, according to the results in Lozano *et al.* (1999), it is possible to design Γ so that, for all initial condition $x_p(0) = e^{Ah}x(0) \in \Omega$, the closed loop system is such that the state $x(kT)$ converges to zero exponentially as $k \to \infty$, and the control law is bounded by Δ. Furthermore, $\|x(t)\|$ and the control law are bounded by exponentially decreasing functions.

4. ROBUSTNESS ANALYSIS

Now, we turn our attention to more realistic situations. First, discrepancies in the values of the real and design parameters are very common in practice. In particular, it is very difficult to measure accurately the delay value.

As we shall see below, the exact cancellations of the ideal case do not occur anymore. Here we show that as long as the parameters and delay mismatch is within a certain range, the stability of the closed loop is preserved. The result is proved in two steps. First, we show that the control stabilizes the predictor. Then, we show that the state of the system is stabilized as well.

4.1 *Stabilization of the predictor*

Proposition 1. At time instants kT, $k = 1, 2, ...$ the resetted predictor (5) in closed loop with the control law (6) is governed by the difference equation

$$x_p(kT+T) - \alpha_1 x_p(kT) - \alpha_2 x_p(kT-T) = 0 \quad (9)$$

where the matrices α_1, α_2 are given by

$$\alpha_1 = e^{Ah}e^{A^*T}e^{-Ah} + e^{AT}(\Gamma - I)$$
$$+\{e^{Ah}e^{A^*(T-h^*)}\int_0^{T-h^*} e^{A^*\sigma}B^*B^\top e^{-A^\top\sigma}d\sigma$$
$$-e^{AT}\int_0^{T-h} e^{-A\sigma}BB^\top e^{-A^\top\sigma}d\sigma\}W^{-1}(\Gamma - I), \quad (10)$$

$$\alpha_2 = e^{Ah}\{-e^{A^*T}e^{A(T-h)}\int_{T-h}^{T} e^{-A\sigma}BB^\top e^{-A^\top\sigma}d\sigma$$
$$+e^{A^*(2T-h^*)}\int_{T-h^*}^{T} e^{-A^*\sigma}B^*B^\top e^{-A^\top\sigma}d\sigma\}W^{-1}(\Gamma - I)(11)$$

Proof. According to (5) the predictor at time kT and $kT+T$ is respectively

$$x_p(kT) = e^{Ah}x(kT) + \int_{kT-h}^{kT} e^{A(kT-\tau)}Bu(\tau)d\tau, \quad (12)$$

and

$$x_p(kT+T) = e^{Ah}x(kT+T)$$
$$+\int_{kT+T-h}^{kT+T} e^{A(kT+T-\tau)}Bu(\tau)d\tau. \quad (13)$$

From the system response we have that

$$x(kT+T) = e^{A^*T}x(kT)$$
$$+\int_{kT}^{kT+T} e^{A^*(kT+T-\sigma)}B^*u(\sigma - h^*)d\sigma, \quad (14)$$

or, if we let $\tau = \sigma - h^*$,

$$x(kT+T) = e^{A^*T}x(kT)$$
$$+\int_{kT-h^*}^{kT+T-h^*} e^{A^*(kT+T-h^*-\tau)}B^*u(\tau)d\tau. \quad (15)$$

Substituting (15) into (13) we obtain

$$x_p(kT+T) = e^{Ah}e^{A^*T}x(kT)$$
$$+e^{Ah}\int_{kT-h^*}^{kT+T-h^*} e^{A^*(kT+T-h^*-\tau)}B^*u(\tau)d\tau$$
$$+\int_{kT+T-h}^{kT+T} e^{A(kT+T-\tau)}Bu(\tau)d\tau. \quad (16)$$

Next, because of the invertibility of e^{Ah}, it follows from (12) that

$$x(kT) = e^{-Ah}[x_p(kT) - \int_{kT-h}^{kT} e^{A(kT-\tau)}Bu(\tau)d\tau]$$

Substituting $x(kT)$ into (16) leads to

$$x_p(kT+T) = e^{Ah}e^{A^*T}e^{-Ah}x_p(kT)$$
$$-e^{Ah}e^{A^*T}e^{-Ah}\int_{kT-h}^{kT} e^{A(kT-\tau)}Bu(\tau)d\tau$$
$$+e^{Ah}\int_{kT-h^*}^{kT+T-h^*} e^{A^*(kT+T-h^*-\tau)}B^*u(\tau)d\tau$$
$$+\int_{kT+T-h}^{kT+T} e^{A(kT+T-\tau)}Bu(\tau)d\tau,$$

or equivalently,

$$x_p(kT+T) = e^{Ah}e^{A^*T}e^{-Ah}x_p(kT)$$
$$-e^{Ah}e^{A^*T}e^{-Ah}\int_{kT-h}^{kT} e^{A(kT-\tau)}Bu(\tau)d\tau$$
$$+e^{Ah}\int_{kT-h^*}^{kT} e^{A^*(kT+T-h^*-\tau)}B^*u(\tau)d\tau$$
$$+e^{Ah}\int_{kT}^{kT+T-h^*} e^{A^*(kT+T-h^*-\tau)}B^*u(\tau)d\tau$$
$$+\int_{kT+T-h}^{kT+T} e^{A(kT+T-\tau)}Bu(\tau)d\tau.$$

Defining the variables $\sigma = \tau - (kT - T)$ and $\sigma = \tau - kT$ in the second and third rows respectively,

$$x_p(kT+T) = e^{Ah}e^{A^*T}e^{-Ah}x_p(kT)$$
$$-e^{Ah}e^{A^*T}e^{-Ah}\int_{T-h}^{T} e^{A(T-\sigma)}Bu(\sigma + kT - T)d\sigma$$
$$+e^{Ah}\int_{T-h^*}^{T} e^{A^*(2T-h^*-\sigma)}B^*u(\sigma + kT - T)d\sigma$$
$$+e^{Ah}\int_0^{T-h^*} e^{A^*(T-h^*-\sigma)}B^*u(\sigma + kT)d\sigma$$
$$+\int_{T-h}^{T} e^{A(T-\sigma)}Bu(\sigma + kT)d\sigma.$$

Now, assuming that $h < T$, $h^* < T$, substituting

$$u(\sigma + kT) = B^\top e^{-A^\top \sigma} W^{-1}(\Gamma - I)x_p(kT),$$
$$0 \le \sigma \le T, k = 1, 2, ...$$

and performing simple algebraic manipulations lead to the result. ■

Remark 1. When there is no uncertainty, $B = B^*$, $A = A^*$ and $h = h^*$, one can see that $\alpha_1 = -e^{AT}\Gamma, \alpha_2 = 0$, and (9) reduces to the ideal case.

Lemma 2. The predictor (4) in closed loop with the control law (6), is exponentially stable if the roots of

$$\det(z^2 I - \alpha_1 z - \alpha_2) = 0, \qquad (17)$$

for α_1 and α_2 respectively defined in (10) and (11), belong strictly inside the unit disk. Moreover the control law is exponentially stable and bounded.

Proof. If the roots of (17) are strictly inside the unit disk, the difference equation (9) is exponentially stable, and the predictor at time kT tends exponentially to zero. The expression (6) implies that the control law is exponentially stable. This implies that the intersampling behavior of the predictor described by (5) is also exponentially stable. The boundedness of the control law is due to the design of Γ as in Lozano *et al.* (1999). The result is semiglobal because the design procedure for Γ depends on initial conditions. ■

4.2 *Stabilization of the system*

We are now ready to establish the stability of the state of the system.

Theorem 3. The state $x(t)$ of system (1) in closed loop with control law (6), (5) is bounded if the system delay $h^*$and parameters A^*, B^*, and design parameters h, A, B, l, G are such that the roots of (17), for α_1 and α_2 respectively defined in (10) and (11), are strictly inside the unit disk.

Proof. If the roots of (17) are strictly inside the unit disk, it follows from Lemma 2 that the predictor is exponentially stable. By substituting (6) into (12), we obtain that

$$x_p(kT) = e^{Ah}x(kT)$$
$$+\{\int_{kT-h}^{kT} e^{A(kT-\tau)}BB^\top e^{-A^\top \tau}d\tau\}W^{-1}(\Gamma - I)x_p(kT - T),$$

hence the state $x(kT)$ is exponentially stable as well. Finally, the intersampling behavior is described by (2), and the stability of the state for all $t \ge 0$ follows from the exponential stability of the state at time kT and from the exponential stability of the control law. ■

Remark 2. If the system delay $h^*$and parameters A^*, B^* are known, one can elaluate α_1 and α_2 in (10) and (11) and verify if the roots of (17) are strictly inside the unit disk. However, a more usefull result would be to determine a bound on the deviation from the nominal parameters for which (17) remain stable. A bound on the delay deviation, when the parameters of the system are known is given in the next section. Future research include the obtention of sufficient conditions depending on $\|A^* - A\|$ and $\|B^* - B\|$.

Remark 3. Even when the exact values of the parameters and of the delay are known, inaccuracies in the computation of the integral of the predictor are always present. When dealing with unstable null-controllable systems (chains of integrators, oscillators of order greater or equal to two), this is critical because the prediction error explodes and the system is internally unstable. The periodical resetting of the predictor introduced in Mondié *et al.* (2001a) permits to overcome this problem. One can consider roughly that the inaccuracies in the integration process can be modeled as parametric uncertainty, hence we can infer that the above results guarantee robustness with respect to computational errors.

Remark 4. When a bounded disturbance is present at the input of system (1), we have that

$$\dot{x}(t) = A^*x(t) + B^*u(t - h^*) + d(t)$$

and the term $\int_{kT}^{kT+T} e^{A^*(kT+T-\tau)}d(\tau)d\tau$ is added to the response described by (15). It is present in all the steps of the proof of Proposition 1 and the behavior of the predictor is now decribed by

$$x_p(kT + T) - \alpha_1 x_p(kT) - \alpha_2 x_p(kT - T)$$
$$= \int_{kT}^{kT+T} e^{A^*(kT+T-\tau)}d(\tau)d\tau, \quad k = 1, 2, ...$$

where α_1 and α_2 are as in Proposition 1. If the conditions of Lemma 2 hold, the stability of the predictor in closed loop is guaranteed, but it is not exponential. As in Theorem 3, we can conclude that the closed loop system is stable.

Remark 5. The above robustness analysis is based on the assumption that the period of the controller is greater than the delay. For large delays, this may be a problem because it implies that the system is in open loop during a long time. In this case a period substancially smaller than the delay is required, and as many predictors as the number of periods in the delay must run simultaneously. A simpler approach in this case is to use a discrete predictor because the value of the state at time

kT is all the state information required for the computation of the control law in the time interval $(kT, kT+T)$. This approach is studied in detail in the framework of the Continuous and Discrete Ressetted Smith Predictor (Mondié *et al.*, 2001b) and can be extended with no difficulty to the control strategy presented here.

5. UNCERTAINTY IN THE DELAY

In this section we focus our attention on the case where there is only uncertainty in the delay, namely, $A = A^*$, $B = B^*$ and $h^* = h + \delta$.

Lemma 4. The linear system with delay in the input described by (1) in closed loop with the control law (6), (5), when the system parameters are known ($A = A^*$, $B = B^*$) and the uncertain delay is such that $h^* = h + \delta$, is exponentially stable if the roots of

$$z^2 - \alpha_1^\delta z - \alpha_2^\delta = 0, \qquad (18)$$

where α_1^δ and α_2^δ are given by

$$\begin{aligned}\alpha_1^\delta &= \left\|e^{AT}\Gamma\right\| \\ &+\{\delta \left\|e^{-A\delta}\right\| \left\|e^{AT}\right\| \sup_{T-h-\delta\leq\sigma\leq T-h} \left\|e^{-A\sigma}BB^\top e^{-A^\top\sigma}\right\| \\ &+ \left\|e^{-A\delta} - I\right\| \left\|e^{AT}\right\| \|W\|\} \left\|W^{-1}\right\| \|(\Gamma - I)\|\end{aligned}$$

$$\begin{aligned}\alpha_2^\delta &= \{\delta \left\|e^{-A\delta}\right\| \sup_{T-h-\delta\leq\sigma\leq T-h} \left\|e^{-A\sigma}BB^\top e^{-A^\top\sigma}\right\| \\ &+ \left\|e^{-A\delta} - I\right\| \left\|e^{A(2T)}\right\| \|W\|\} \left\|W^{-1}\right\| \|(\Gamma - I)\|,\end{aligned}$$

are located inside the unit disk.

Proof. When there is no uncertainty in the parameters, ($A = A^*$, $B = B^*$), the difference equation (9) that describes the predictor dynamics, after some simple manipulations, reduces to

$$\begin{aligned}x_p(kT+T) &= e^{AT}\Gamma x_p(kT) \\ &+\{-e^{-A\delta}e^{AT}\int_{T-h-\delta}^{T-h} e^{-A\sigma}BB^\top e^{-A^\top\sigma}d\sigma \\ &+(e^{-A\delta} - I)e^{AT}\int_0^{T-h} e^{-A\sigma}BB^\top e^{-A^\top\sigma}d\sigma\}W^{-1}(\Gamma - I)x_p(\\ &+\{(e^{-A\delta} - I)e^{2AT}\int_{T-h}^{T} e^{-A\sigma}BB^\top e^{-A^\top\sigma}d\sigma \\ &+e^{-A\delta}e^{2AT}\int_{T-h-\delta}^{T-h} e^{-A\sigma}BB^\top e^{-A^\top\sigma}d\sigma\}W^{-1}(\Gamma - I)x_p(k\end{aligned}$$

Now observe that for $0 \leq a \leq \sigma \leq b \leq T$,

$$\left\|\int_a^b e^{-A\sigma}BB^\top e^{-A^\top\sigma}d\sigma\right\|.$$

is bounded by $(b-a)\sup_{a\leq\sigma\leq b}\left\|e^{-A\sigma}BB^\top e^{-A^\top\sigma}\right\|$ and by $\|W\|$. Substituting at our convenience leads to

$$\|x_p(kT+T)\| \leq \alpha_1^\delta \|x_p(kT)\| + \alpha_2^\delta \|x_p(kT-T)\|$$

Now, if we consider the difference equation associated to this inequality, we can proceed as in the general case, and the result follows. ■

Remark 6. The above lemma provides indeed a conservative bound for the maximal allowed delay deviation δ^*. When δ is zero, the roots of (18) are all located inside the unit circle. By a continuity argument, the system reaches an instability region, when for the first time, as δ increases, a root of (18) is on the unit circle. Therefore, one can determine δ^* by substituting $z = e^{j\omega}$ into (18), and by solving numerically the two nonlinear equations in two unknowns δ and ω obtained by setting to zero the real and imaginary part.

6. BOUNDED INPUT STABILIZATION OF A SIMPLE OSCILLATOR

The control strategy developped in this paper is illustrated with the stabilization, by a bounded input, of the delayed oscillator with transfer function $e^{-\frac{\pi}{4}s}/s^2+1$. The design parameters are the delay $h = \frac{\pi}{4}$ and the frequency $\omega = 1$; The real parameters are $h^* = 1.15(\frac{\pi}{4})$ and $\omega^* = 1.15$. The parameters of the control law are chosen so that the input is bounded by 10 for all initial conditions $\|x(0)\| \leq 5$. The behavior of the state variables and input, when the real parameters are equal to those used in the design, and when there is a mismatch are shown in Fig. 1 and Fig. 2, respectively. One can verify that for the design and real parameters given above, the roots of (17) are inside the unit disk, hence the stability of the response in Fig. 2 is expected.

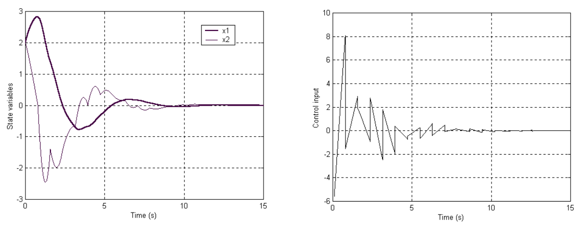

Fig1 : States and input, known parameters

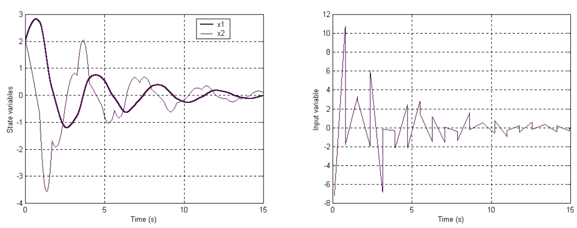

Fig2 : States and input, uncertain parameters

7. CONCLUDING REMARKS

The control scheme presented in this paper allows the semiglobal stabilization of linear systems with delayed input which are bounded. The control law is basically a combination of the predictor based techniques used frequently for systems with delayed input (Smith, 1959), (Manitius and Olbrot, 1979), (Artstein, 1982), (Watanabe and Ito, 1981), and of the periodic controllers methods developed for the stabilization of linear systems with bounded inputs (Lozano *et al.*, 1999). A basic feature is that the periodic controller introduces naturally the resetting of the predictor, which was shown to be crucial to have robust predictor based control laws (Mondié *et al.*, 2001a).

The robustness of the scheme with respect to uncertainty in the parameters, in the size of the delay and input disturbances is established.

Current research include the analysis of use of a resetting predictor in control schemes for null-controllable systems that insure global stabilization (Sussmann *et al.*, 1994).

8. APPENDIX

We recall here the result on the semiglobal stabilization of null-controllable systems obtained in Lozano *et al.*(1999), which is the starting point for the present work.

Lemma 5. Consider the null-controllable single input system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

and the control law with period T

$$u(\sigma + kT) = B^\top e^{-A^\top \sigma} W^{-1}(\Gamma - I)x(kT), \quad \sigma \in [0, T), \quad k = 1, 2, ... \tag{19}$$

where W is the controllability grammian. Let a bound Δ and a bounded prespecified set Ω be given. Then, there exists a Γ (see Lozano *et al.*(1999) for the design procedure) such that, for all initial condition $x(0) \in \Omega$, the closed loop system is such that the state $x(kT)$ converges to zero exponentially as $k \to \infty$, and the control law (19) is bounded by Δ. Furthermore, $\|x(t)\|$ and the control law are bounded by exponentially decreasing functions.

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