15th Triennial World Congress, Barcelona, Spain

# DESIGN OF CONTROLLERS FOR PARAMETRIC UNCERTAIN SYSTEMS. A TWO-STEP APPROACH USING GENETIC ALGORITHMS

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Abstract: Most industrial processes are modeled as linear time invariant systems with parametric uncertainties. The design of a robust controller for these plants is formulated as a multiobjective min-max problem where certain performance objectives are minimized with respect to the controller and maximized with respect to the uncertainties. The solution of such a problem is extremely difficult. An approximated two-step approach is proposed in this paper. In the first step, an auxiliary multiobjective minimization problem is solved. The solution to this problem is the set of Pareto optimal controllers. In the second step, these controllers are checked for the worst case of parametric uncertainty by solving a multiobjective maximization problem. The MRCD genetic algorithm is used to solve both multiobjective optimization problems.

Keywords: Robust Control. Parametric Uncertainty. Multiobjective Genetic Algorithms. PID controllers.

## 1. INTRODUCTION

Most industrial processes are modeled as linear time invariant systems of first or second order with a delay. The parameters of these models are not exactly known and are subject to large uncertainties motivated by different operating conditions, neglected nonlinearities, or errors in the identification process. In addition, high order unmodeled dynamics should be taken into account in the controller design procedure, because they can affect the behavior of the closed-loop system.

Consequently, the design of controllers for industrial processes is not a trivial issue. The recent methods of robust control use a family of plants instead of a single nominal plant. The plant family involves both parametric and dynamic uncertainty and the design problem consists of computing a controller that satisfies a set of specifications for all the plants in the family.

The robust control design problem is formulated as a multiobjective min-max optimization problem where several performance objectives are minimized with respect to the controller in the worst uncertainty case. The optimization objectives are norms of certain closed-loop transfer functions, see (Zhou *et al.*, 1996; Barmish, 1994; Ackermann, 1993). This problem is extremely difficult in the general case and can only be approximated by numerical methods.

Here, a two-step approximation to the multiobjective min-max is proposed. In the first step, an auxiliary multiobjective minimization problem of several performance measures for the nominal closed-loop system is proposed. The solution to this multiobjective problem is the set of all Pareto optimal controllers. In the second step, these controllers are checked with respect to the worst uncertainty by solving a multiobjective maximization problem.

The multiobjective optimization problems of both steps are difficult to solve by using conventional methods. A multiobjective genetic algorithm, MRCD (Herreros *et al.*, 1999) has been used as a tool for solving both multiobjective problems.

Genetic algorithms (GAs) are stochastic algorithms based on the Darwinian evolution and natural selection rules. A set of candidate solutions is evolved in several steps to bring it close to the optimal solution, see (Goldberg, 1989; Mitchell, 1999). The multiobjective GA has been designed to obtain the solution of a mutiobjective problem, that is, the Pareto optimal set, see (Van Veldhuizen, 1999; Horn, 1997; Deb, 1999; Coello, 1999). The MRCD algorithm is a multiobjective GA with specific operators for robust control design problems, see (Herreros, 2000; Herreros *et al.*, 1999), and will be used to solve the two multiobjective optimization problems of the design procedure explained here.

The rest of the paper is organized as follows. In Section 2, the two-step multiobjective control design procedure is explained. In Section 3, several performance measures for the design of SISO controllers are studied. They will be used as objectives for the multiobjective optimization problems. In section 4, the MRCD genetic algorithm is briefly described. In Section 5, an example based on a real-world industrial control application is solved. Finally, in Section 6 some conclusions are given.

## 2. THE TWO-STEP ROBUST CONTROL DESIGN PROCEDURE

The block diagram of Figure 1 represents the feedback interconnection of an uncertain plant and a controller, where $G_n(s)$ is the nominal plant, $\Delta_G$ is the parametric uncertainty and $K$ is the controller to be designed.

The input/output relation between the exogenous input $w$ and the output $z$ is given by

$$z = \mathcal{F}(G_n, \Delta_G, K)w \tag{1}$$

Let $\mathcal{A}(G_n)$ denote the set of controllers that internally stabilize the nominal plant $G_n$. A control design problem can be formulated as follows: Given an uncertain plant $(G_n, \Delta_G)$ where $\Delta_G \in \mathbf{\Delta}$, compute a controller $K \in \mathcal{A}(G_n)$ such that the feedback interconnection $\mathcal{F}(G_n, \Delta_G, K)$ satisfy a set of specifications

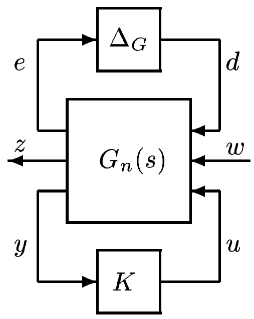


Figure 1. Control Design Framework

$$J_i(\mathcal{F}(G_n, \Delta_G, K) < \alpha_i, \text{for any } \Delta_G \in \mathbf{\Delta}. \tag{2}$$

The performance objectives $J_i(\cdot)$ are usually norms of certain weighted closed-loop operators. This design problem can be transformed in the following min-max multiobjective problem:

$$\min_{K \in \mathcal{A}(G_n)} \max_{\Delta_G \in \mathbf{\Delta}} \begin{bmatrix} J_1(\mathcal{F}(G_n, \Delta_G, K)) \\ \vdots \\ J_\ell(\mathcal{F}(G_n, \Delta_G, K)) \end{bmatrix}, \tag{3}$$

This is an infinite-dimensional multiobjective nonconvex min-max problem that is extremely difficult to solve. In the industry practice the controllers have a fixed structure, and they can be parametrized as $K = K(\theta)$. Now, the problem (3) is a parametric min-max problem, however its solution is still very complicated and only some approximations for very simple cases are found in the literature (Herreros *et al.*, 1999; Hirata *et al.*, 1999). See also (Barmish, 1994; Ackermann, 1993) for a revision of classic methods for analysis and design of control systems for parametric uncertain systems.

An alternative to the min-max problem (3) is proposed in this paper. The approach consists of two steps. In the first step the controller is parametrized $K = K(\theta)$ and an auxiliary multiobjective optimization problem is solved

$$\min_{K(\theta)} \begin{bmatrix} m_1(\mathcal{F}(G_n, 0, K)) \\ \vdots \\ m_n(\mathcal{F}(G_n, 0, K)) \end{bmatrix} \tag{4}$$

where $m_i(\mathcal{F}(G_n, K, 0))$ is a performance or robustness measure for the nominal closed-loop system. In this auxiliary problem, parametric uncertainty is not directly addressed, however, certain robustness properties can still be taken into account by a proper election of the objectives $m_i(\cdot)$. For example, robust stability against bounded unmodeled dynamics is obtained by using the $\mathcal{H}_\infty$ norm of certain weighted transfer function of the closed-loop system.

The solution of the multiobjective problem (4) is a set of controllers, *i.e.* the Pareto optimal set.

This set contains a large number of candidate controllers. If the auxiliary objectives $m_i$ have been correctly chosen, the set of candidate solutions could contain controllers that solve the control problem (2). However, the selection of one of these controllers from the Pareto set is not a trivial task.

The second step of the design procedure is a search over the Pareto optimal set in order to obtain a controller that satisfies the design specifications (2).

Let $\mathcal{P}^K$ denote the Pareto optimal set obtained after solving the multiobjective problem (4) of the first step. Choose $K_j \in \mathcal{P}^K$ and solve

$$\max_{\Delta_G \in \boldsymbol{\Delta}} \left[ J_{1j} \; \ldots \; J_{\ell j} \right]^T, \tag{5}$$

where $J_{ij} = J_i(\mathcal{F}(G_n, \Delta_G, K_j))$. Let $\mathcal{P}_j^{\Delta_G}$ denote the Pareto optimal set of the multiobjective problem (5). If any $\Delta_G \in \mathcal{P}_j^{\Delta_G}$ is such that $J_{ij} < \alpha_i$ for any $i \in \{1, \ldots, \ell\}$ then $K_j$ is a valid controller. Otherwise, $K_j$ is eliminated from $\mathcal{P}^K$ and a new controller has to be checked.

Apparently, this second step seems to be very time consuming because a multiobjective maximization problem has to be solved for any candidate controller in the Pareto set $\mathcal{P}^K$. However, the use of multiobjective genetic algorithms permits a very efficient implementation of this step. The controllers in $\mathcal{P}^K$ are ordered by some performance measure $J_i$ and are checked successively starting from the controller with smaller $J_i$ value. Two successive controllers $K_j$ and $K_{j+1}$ are very similar and so are their Pareto sets $\mathcal{P}_j^{\Delta_G}$ and $\mathcal{P}_{j+1}^{\Delta_G}$. Therefore, the computation of $\mathcal{P}_{j+1}^{\Delta_G}$ is very fast if $\mathcal{P}_j^{\Delta_G}$ has been previously computed.

## 3. PERFORMANCE OBJECTIVES

Consider the block diagram of Figure 2. The process to be controlled is modeled as a linear SISO system with transfer function $P_n$. The process input and output are denoted by $u(t)$ and $y(t)$, respectively, and the signal $r(t)$ is the setpoint. The control system is subject to disturbances. Two types of disturbance are usually considered on a control loop: load disturbance $l(t)$, entering at the input of the process, and measurement noise $d(t)$, entering at the output of the process. Load disturbances will be modeled as step functions and measurement noise as a stochastic process (e.g. filtered gaussian white noise). The filtered measured output signal is denoted by $\tilde{y}(t)$. The controller $K(s)$ has two degrees of freedom.

Any sensible formulation for a $K(s)$ controller design problem should at least consider the following

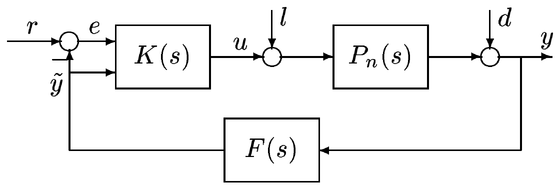


Figure 2. Block diagram of a feedback system with a controller

specifications, see (Herreros *et al.*, 2000; Åström *et al.*, 1998):

*Setpoint Tracking.* Some suitable integral performance indexes for setpoint following are:

$$J_{er} = \left( \int_0^\infty |te(t)|^p dt \right)^{\frac{1}{p}} = \left\| \frac{d}{ds} \left( T_{er} \frac{1}{s} \right) \right\|_p \tag{6}$$

where $e(t)$ is the output to a unit step in the input $r(t)$.

*Robustness to Modeling Uncertainties* This can be expressed in terms of the closed loop sensitivity function $S(s) = \frac{1}{1+L(s)}$, where $L(s)$ is the loop transfer function. In order to obtain robustness against unmodeled uncertainties, the value of $J_{yd} = \|S(s)\|_\infty$ must be small.

*Load Disturbance Rejection* This will be expressed in terms of an integral performance measurement of the output to a unit step load disturbance input

$$J_{yl} = \int_0^\infty |y(t)|^p dt = \left( \left\| T_{yl} \frac{1}{s} \right\|_p \right)^{\frac{1}{p}}, \; p \in \mathbb{N}. \tag{7}$$

There are different possibilities depending on the value of $p$. Normally $p$ is chosen as 1 or 2.

*Measurement Noise Reduction* The measurement noise is modeled as a stochastic process with a given spectrum power density $\Phi_n(\omega)$. The objective is to attenuate the effect of this disturbance at the output $y(t)$. This objective can be formulated using a weighted $\mathcal{H}_2$ norm

$$\|T_{yd}(s)W(s)\|_2 \tag{8}$$

where the filter $W(s)$ weights the band of frequencies where the noise should be attenuated.

*Actuator limits* Every physical control signal always has a limited range. The following condition can be introduced to avoid saturation for unit setpoint changes:

$$J_{ur} = \left\| T_{ur} \frac{1}{s} \right\|_{\mathcal{L}_\infty} < \alpha \tag{9}$$

where $\alpha$ is the saturation level.

*Unit Step Time Response* Specifications on setpoint following may also include requirements on rise time, settling time, decay ratio, overshoot and steady-state offset for step changes in setpoint or in the load disturbance.

In the industrial practice, the specifications are usually established by using the unit step time response and quantities as rise time, settling time or overshoot. However, other performace measures are more appropriate for setting the auxiliary multiobjective problem (4) of the first step. Note that this optimization problem is crucial in the two-step procedure, because it is important that the Pareto optimal set should contain a great variety of controllers to be checked later in the second step. If the filter $F(s)$ has a fixed value and the measured noise is not relevant, the multiobjective optimization problem (4) can be formulated as follows: "Find a set of internally stabilizing controllers $K(s)$ that minimize the following vector valued performance index:

$$\begin{bmatrix} J_{er} \\ J_{yd} \\ J_{yl} \end{bmatrix} = \begin{bmatrix} \left\| \frac{d}{ds}\left( T_{er}(s)\frac{1}{s} \right) \right\|_q \\ \| T_{yd}(s) \|_r \\ \left\| T_{yl}(s)\frac{1}{s} \right\|_p \end{bmatrix} \tag{10}$$

where $p, q, r \in \{1, 2, \infty\}$, and which also satisfy the non saturation condition $\|T_{ur}\frac{1}{s}\|_{\mathcal{L}_\infty} < \alpha$. Normally $p = 1$, $q = 2$, iand $r = \infty$, but other selections are possible depending on each particular problem. This multiobjective optimization problem is very versatile, as has been shown in (Herreros *et al.*, 2000). Therefore, it is a good choice for the first step, independently of the specific performance objectives $J_i$ of the control design problem (2).

## 4. MULTIOBJECTIVE OPTIMIZATION AND GENETIC ALGORITHMS

Genetic algorithms (GAs) are random heuristic search methods where an initial set of possible solutions (the so-called population) is modified in successive steps to converge towards the optimal solution, see (Goldberg, 1989; Mitchell, 1999).

The main GA operators are selection, crossover and mutation. Selection is used to choose the best individuals in a population, crossover produces new individuals by mixing couples of selected individuals and mutation induces random changes in the individuals.

A genetic algorithm for multiobjective optimization must address two important issues: diversity and Pareto optimality. Diversity is accomplished by maintaining a set of candidate solutions in order to cover the entire extension of the Pareto front. In addition, the algorithm must incorporate strategies for directing the population towards the Pareto-optimal solutions.

The multiobjective GA used in this article is the MRCD algorithm (Herreros *et al.*, 1999). It is an algorithm specially developed for multicriteria design control problems and some of its main features are:

- Adaptive Search Space. The initial search space is modified as a function of the individual values.
- Selection method of (Horn *et al.*, 1994) with ranking function (Fonseca and Fleming, 1995) and fitness sharing.
- Coarse Grained Parallel GA, see (Mitchell, 1999).
- External Migration. Random individuals are added to the population in every generation.
- Crossover, Mutation and Mate Restriction.
- Constraints treated as Objectives.
- Elite filter. The best individuals of a generation are filtered by the phenotype or genotype criterium and are added to the rest.

A more detailed explanation of the algorithm and its features is given in (Herreros *et al.*, 1999).

## 5. AN EXAMPLE

In this section an example of control design using the two-step procedure is presented. The plant to be controlled is a pelleting machine at a factory of animal food. The input of the pelleting machine is the feeding rate of raw material that is manipulated by the voltage of the feeding motor $u(t)$. The output is the current intensity $i(t)$ of the main motor. The objective is to drive the main motor to its nominal current because the production process is energetically optimized at this operating condition.

The process has been modeled as a first order system with a delay and its parameters have been obtained by system identification under different operation conditions

$$\frac{I(s)}{U(s)} = G(s) = \frac{Ke^{-Ls}}{Ts+1}$$
$$\begin{cases} \overline{K} = 51, \;\; 29 < K < 90 \\ \overline{T} = 4, \;\; 3 < T < 6 \\ \overline{L} = 18, \;\; 13 < L < 21. \end{cases} \tag{11}$$

The nominal plant is $G_n(s) = \frac{51e^{-18s}}{4s+1}$ and the parametric uncertainty $\Delta_G$ is parametrized in terms of $\Delta_K$, $\Delta_T$ and $\Delta_L$.

The controller to be designed is a PID digital controller with the following structure

$$\frac{U(z)}{E(z)} = K_c \left[1 + \frac{h}{T_I(1-z^{-1})} + \frac{T_D(1-z^{-1})}{h}\right]. \quad (12)$$

where $E(z)$ is the z-transform of the output error signal $e(t) = r(t) - i(t)$, $r(t)$ is the setpoint and $h$ is the sampling time that has been chosen to 1 second.

The control problem is formulated as follows: Design an internally stabilizing PID digital controller (12) such that the overshoot is less than 1.3 and the rise time less than 160 seconds for any operating condition. In addition, for the nominal plant and a unit step setpoint change, the absolute value of the control input cannot be greater than 0.08 volts. and the control input rate of change must be less than 0.01 volts.

The multiobjective problem for the first step is

$$\min_{K_c, T_I, T_D} \begin{bmatrix} J_{er} \\ J_{yd} \\ J_{yl} \end{bmatrix}$$

subject to $\sup_t |u(t)| < 0.08$ and $\sup_t |u(t+1) - u(t)| < 0.01$ for a setpoint change of 1 volt. The auxiliary performace objectives $J_{er}$, $J_{yd}$ and $J_{yl}$ are the discrete version of the objectives given in (10), *i.e.* $J_{er} = \sum_{t=0}^{\infty} |te(t)|$, $J_{yd} = \|T_{yd}\|_2$ and $J_{yl} = \left\|T_{yl}\frac{z}{z-1}\right\|_2$.

The parameter values of all the controllers in the Pareto optimal set found by the MRCD algorithm are shown in Figure 3. and the Pareto Front for the objectives $[J_{re}, J_{yd}, J_{yl}]$ is depicted in Figure 4. The Pareto optimal set contains 490 controllers that will be checked for the overshoot and rise time specifications in the second step.

The second step multiobjective problem is

$$\max_{\Delta_G} \begin{bmatrix} \mathcal{D}_{os}(G_n, K, \Delta_G) \\ \mathcal{D}_{t_r}(G_n, K, \Delta_G) \end{bmatrix}$$

where $\mathcal{D}_{os}(G_n, K, \Delta_G)$ and $\mathcal{D}_{t_r}(G_n, K, \Delta_G)$ are the overshoot and the rise time for the uncertain plant family.

A controller that satisfies the design specifications has been computed by solving the second step multiobjective problem. Their parameters are:

$$K_c = 0.00163, T_I = 4.203, T_D = 0.3147. \quad (13)$$

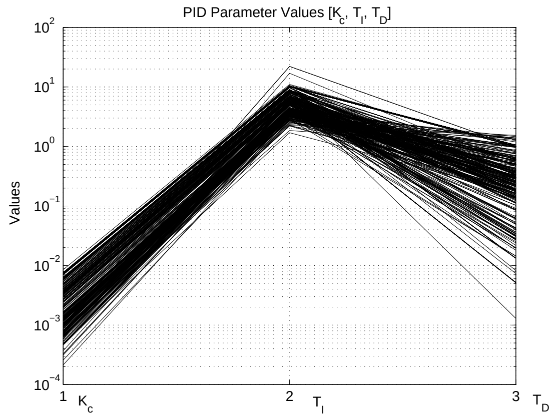


Figure 3. Values of the Pareto Optimal Controller Set

The Pareto optimal uncertainty set for this controller is shown in Figure 5 and the attained Pareto Front is depicted in Figure 6. The worst case parametric uncertainties are located at the extreme sides of the Pareto front. They have been represented by solid lines in Figure 5. Finally, Figure 7 shows the time response $y(t)$ for a unit step input $r(t)$ in the nominal and worst uncertainty cases.

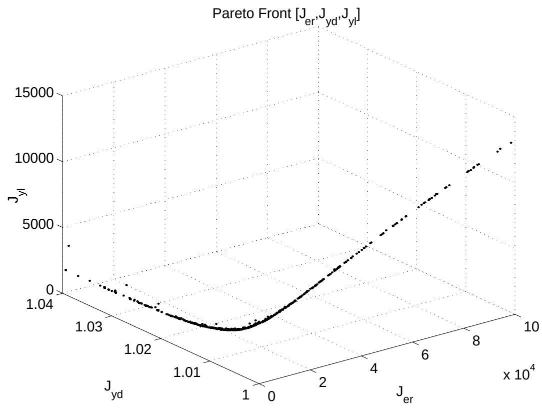


Figure 4. Pareto Front $[J_{er}, J_{yd}, J_{yl}]$.

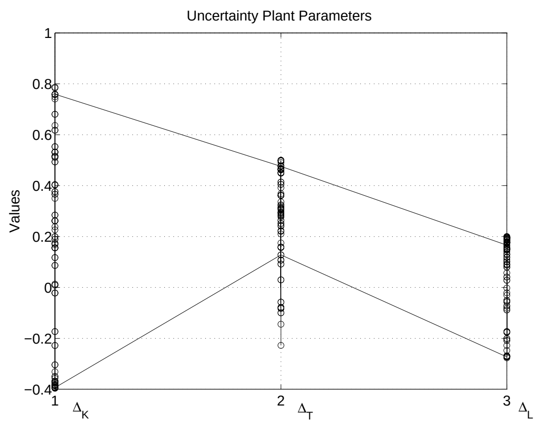


Figure 5. Values of the Pareto Optimal Uncertainty Set

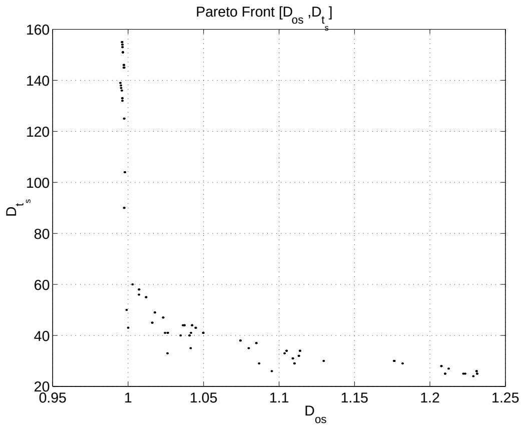


Figure 6. Pareto Front $[\mathcal{D}_{os}, \mathcal{D}_{t_r}]$

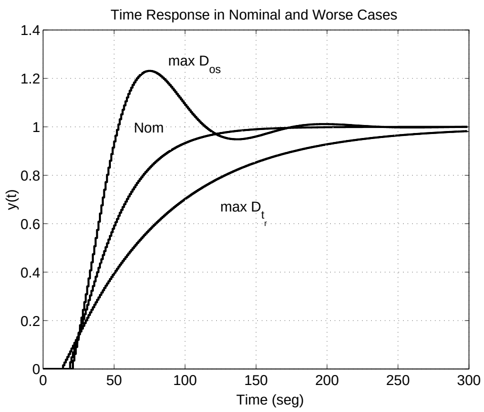


Figure 7. Time Response in Nominal and Worst Cases

## 6. CONCLUSION

The design of robust controllers for systems with parametric uncertainty is a very difficult problem. In this paper a two-step procedure based on multiobjective optimization problems has been proposed. The first step computes a set of Pareto optimal controllers for an auxiliary multiobjective minimization problem without uncertainty. In the second step the Pareto set of controllers is checked for the parametric uncertainty. The two-step procedure has turned out to be very efficient in the design of fixed structure industrial controllers.

## 7. REFERENCES


Ackermann, J. (1993). *Robust Control, System with Uncertain Physical Parameters.* Springer-Verlag.

Barmish, J. Ross (1994). *New Tools for Robustness of Linear Systems.* first ed.. Macmillan.

Coello, C.A. (1999). A Comprehensive Survey of Evolutionary-Based Multiobjective Optimization Techniques. *Knowledge and Information Systems. An International Journal.* (Accepted for publication).

Deb, K. (1999). Evolutionary Algorithms for Multi-Criterion Optimization in Engineering Design. In: *Proceedings of Evolutionary Algorithms in Engineering and Computer Science (EUROGEN'99).*

Fonseca, C.M. and P.J. Fleming (1995). Multiobjective Genetic Algorithms Made Easy: Selection, Sharing and Mate Restriction. In: *Proceedings of the First International Conference on Genetic Algorithms in Engineering Systems, Innovations and Applications, GALESIA.* IEE. University of Sheffield, UK. pp. 45–52.

Goldberg, D.E. (1989). *Genetic Algorithms in Search, Optimization, and Machine Learning.* Addison-Wesley.

Herreros, A. (2000). Design of Robust Controllers using Genetic Algorithms (in Spanish). PhD thesis. University of Valladolid (Spain).

Herreros, A., E. Baeyens and J.R. Perán (1999). Design of Multiobjective Robust Controllers Using Genetic Algorithms. In: *Proceedings of the 1999 Genetic and Evolutionary Computation Conference. Workshop Program* (Annie S. Wu, Ed.). Orlando, Florida. pp. 131–132.

Herreros, A., E. Baeyens and J.R. Perán (2000). Design of pid Controllers Using Multiobjective Genetic Algorithms. In: *Proceedings of the 2000 IFAC Workshop on Digital Control. Past, present and Future of PID Control..* Terrassa, Spain. pp. 309–315.

Hirata, K., Y. Yanase, T. Katayama and T. Kawabe (1999). A min-max design of robust i-pd controller for time-delay systems with parametric uncertainty. In: *Proceeding of the 14th World Congress of IFAC.* pp. 259–264.

Horn, J. (1997). Multicriterion Decision making. In: *Handbook of Evolutionary Computation* (Thomas Bäck, David Fogel and Zbigniew Michalewicz, Eds.). Vol. 1. IOP Publishing Ltd. and Oxford University Press. pp. F1.9:1 – F1.9:15.

Horn, J., N. Nafplio tis and D.E. Goldberg (1994). A Niche Pareto Algorithm for Multiobjective Optimization. In: *First IEEE Conference on Evolutionary Computation.* IEEE World Congress on Computational Intelligence.

Mitchell, M. (1999). *An Introduction to Genetic Algorithm.* fitht ed.. MIT press.

Åström, K.J., H. Panagopoulos and Hägglund (1998). Design of pi controllers based on nonconvex optimization. *Automatica.*

Van Veldhuizen, D. (1999). Multiobjetive Evolutionary Algorithms: Classification, Analyses, and New Innovations. PhD thesis. Air University. AFIT/DS/ENG/99-01.

Zhou, K., J. C. Doyle and K. Glover (1996). *Robust and Optimal Control* . Prentice Hall. New Jersey.