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HIGHER ORDER ILC VERSUS ALTERNATIVES APPLIED TO CHAIN CONVEYOR SYSTEMS

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Abstract: The norm optimal approach, both in its basic form and the extension to predictive action, where the predicted errors on a number of future trials are explicitly included in the cost function for controller design, is now a well established area in iterative learning control in terms of the underlying theory. By the fact that it includes the predicted errors on future trials in the cost function, predictive iterative learning control is clearly a higher order law. Hence it is now appropriate to ask if, in practical situations, predictive norm optimal iterative learning control can deliver significantly improved performance over its norm optimal alternative to merit the extra computational and hardware costs associated with its application. This is the area addressed in this paper using a somewhat new application area in the form of chain conveyor systems.

Keywords: iterative learning control, optimal algorithms, comaprative studies.

1. INTRODUCTION

Since the iterative learning control (denoted ILC in this paper) concept was proposed, a very large number of approaches have been considered but here we focus on so-called norm optimal ILC and its performance against alternatives. Norm optimal ILC schemes can be realized in terms of current trial mechanisms combined with feed-forward of previous trial data. The algorithm is based on splitting the two-dimensional dynamics into two separate one-dimensional problems. This is done by introducing a performance criterion as the basis of specifying the control input to be used on each trial. This norm optimal approach has a mature theoretical basis which can be extended to include predictive action where in this setting such action is included in the cost function as the predicted errors on a number of future trials. This leads naturally to an implementation law which is clearly of the higher order form.

Given the existence of well defined algorithms for this higher order ILC and its reduced version, i.e. norm optimal ILC, there is a clear need to investigate the relative performance of these algorithms and in this paper we begin this task by applying these algorithms to a chain conveyor system for which an experimental testbed is also available.

Chain conveyors are extremely simple in construction as they generally consist of parallel lengths of chain that are connected at regular intervals to form a conveying surface. The use of chain conveyors is of particular importance to the food manufacturing industry, with the twin requirement of high standards of hygiene and low overheads. As chain conveyors have no sensitive components, and most, if not all, are metal, cleaning with highpressure steam to ensure hygiene does not pose a problem. The use of standard components with no precision engineering makes them inexpensive – a very important factor in the highly competitive world of food manufacture.

Previous work has applied three term (or PID) ILC schemes to this conveyor structure both in simulation and experiment. The major conclusion was that this is a highly relevant application area for ILC. Despite this, achievable performance was limited by the PID structure in certain cases. Hence this case is ideal to compare and contrast the norm optimal/predictive ILC against themselves and both of them against simpler structure schemes. In the next section we give the necessary background results.

2. BACKGROUND

Iterative learning control (ILC) is a technique to control systems operating in a repetitive mode with the additional requirement that a specified output trajectory $r(t)$ defined over a finite interval $[0, T]$ is followed to high precision. There are numerous examples of such systems including robot manipulators that are required to repeat a given task to high precision, chemical batch processes or, more generally, the class of tracking systems. Here we will consider another application area - chain conveyor systems.

Motivated by human learning, the basic of idea of ILC is to use information from previous executions of the task in order to improve performance from trial to trial in the sense that the tracking error is sequentially reduced ((Arimoto *et al.*, 1984; Moore, 1993)). Typical ILC algorithms construct the input to the plant on a given trial from the input used on the previous trial plus an additive increment that is typically a function of past values of the observed output error, i.e. the difference between achieved and desired plant output. The objective of constructing a sequence of input functions $\{u_k(t)\}_k,\ t \in [0, T]$, such that the performance as the task is repeated is gradually improving, can be refined to a convergence condition on the input and error:

$$\lim_{k\to\infty} ||e_k|| = 0, \quad \lim_{k\to\infty} ||u_k - u_\infty|| = 0 \quad (1)$$

where $e_k(t)$ is the error on trial k, i.e. the difference between $r(t)$ and the system output $y_k(t)$, and $u_k(t)$ is the input to the system on this trial.

The above definition of convergent learning is a stability problem on an infinite-dimensional two-dimensional (2D)-product space. As such it places the analysis of ILC schemes firmly outside the scope of traditional control theory. In particular, ILC must be studied in the context of fixed-point problems or, more precisely, linear repetitive processes ((Rogers and Owens, 1992)).

As noted previously in this paper, a very large number of approaches to ILC have been considered but here we only consider so-called norm optimal ILC which can be realized in terms of current trial mechanisms combined with feedforward of previous trial data. The algorithm is based on splitting the two-dimensional dynamics into two separate one-dimensional problems. This is done by introducing a performance criterion as the basis of specifying the control input to be used on each trial.

The norm optimal approach in general has a mature theoretical basis (Amann *et al.*, 1996) and in this setting the following is the formal definition of a successful ILC algorithm.

Definition 1. Consider a dynamic system with input u and output y. Let $\mathcal{Y}$ and $\mathcal{U}$ be the output and input function spaces respectively and let $r \in \mathcal{Y}$ be a desired reference trajectory from the system. Then an ILC algorithm is successful if, and only if, it constructs a sequence of control inputs $\{u_k\}_{k\geq 0}$ which, when applied to the system or plant (under identical experimental conditions), produces an output sequence $\{y_k\}_{k\geq 0}$ with the following properties of convergent learning:

$$\lim_{k\to\infty} y_k = r, \quad \lim_{k\to\infty} u_k = u_\infty \quad (2)$$

Here convergence is interpreted in terms of the topologies assumed in $\mathcal{Y}$ and $\mathcal{U}$ respectively.

Note: This general description includes linear and nonlinear dynamics, continuous or discrete plants, and time-invariant or time-varying systems.

Now let the space of output signals $\mathcal{Y}$ be a real Hilbert space and $\mathcal{U}$ also be a real (and possibly distinct) Hilbert space of input functions. Then the respective inner products (denoted by $\langle\cdot,\cdot\rangle$) and norms $||\cdot||^2 = \langle\cdot,\cdot\rangle$ are indexed in a way that reflects the space if it is appropriate to the discussion.

The dynamics of the plant considered here are approximated by a linear model which in operator form can be written as

$$y = Gu + z_0 \quad (3)$$

where no loss of generality arises from setting $z_0 = 0$. Also it is clear that the ILC procedure, if convergent, solves the problem $r = Gu_\infty$ for u_∞ and, if G is invertible, the formal solution is just $u_\infty = G^{-1}r$. A basic premise of the ILC approach is that the direct inversion of G is regarded as an impractical solution because it requires exact knowledge of G and involves derivatives of r. This high-frequency gain characteristic would make the approach sensitive to noise and other disturbances. Also it can be argued that inversion of the whole plant G is unnecessary as the solution only requires finding the pre-image of r under G.

The above problem is easily shown to be equivalent to finding the minimizing input u_∞ for the optimization problem

$$min_u\{||e||^2 \, : \, e = r - y, y = Gu\} \qquad (4)$$

The optimal error $||r - Gu_\infty||^2$ is a measure of how well the ILC algorithm has solved the inversion problem. It also represents the best that the system can do in tracking the signal r. The case of interest here is when the optimal error is zero, i.e. u_∞ is a solution of $r = Gu_\infty$. Also (4) is clearly a singular optimal control problem which by its very nature requires an iterative solution.

There are an infinity of potential iterative procedures for solving (4) and of these the gradient approach has the simplest form and has been extensively investigated in the ILC literature. A gradient based ILC algorithm has the form

$$u_{k+1} = u_k + \epsilon_{k+1} G^* e_k \qquad (5)$$

where $G^* : \mathcal{Y} \rightarrow \mathcal{U}$ is the adjoint operator to G, and ϵ_{k+1} is a step length to be chosen at each iteration. This general approach suffers from the need to choose this step length on each trial and the feedforward structure of the iteration takes no account of current trial effects - including disturbances and plant modeling errors.

Norm optimal ILC has the following two crucial properties relative to the gradient based algorithms discussed above.
1. Automatic choice of step size.
2. Potential for improved robustness through the use of causal feedback of current trial data and feedforward of data from previous trials.
In particular, the class of ILC algorithms considered here compute, at the completion of trial k, the input on trial $k+1$ as the solution of the minimum norm optimization problem

$$u_{k+1} = \arg\, min_{u_{k+1}}\{J_{k+1}(u_{k+1})\} \qquad (6)$$

subject to

$$e_{k+1} = r - y_{k+1},\; y_{k+1} = Gu_{k+1} \qquad (7)$$

where the performance index (or optimality criterion) used is

$$J_{k+1}(u_{k+1}) = ||e_{k+1}||^2_{\mathcal{Y}} + ||u_{k+1} - u_k||^2_{\mathcal{U}} \quad (8)$$

The initial control $u_0 \in \mathcal{U}$ can be arbitrary but, in practice, will be a good first guess at the solution of the problem. Also the relative weighting of reducing the current trial error against minimizing the deviation in the control input signals used on successive passes can be absorbed into the definitions of the norms in $\mathcal{Y}$ and $\mathcal{U}$.

The benefits of this approach are immediate from the simple interlacing result

$$||e_{k+1}||^2 \leq J_{k+1}(u_{k+1}) \leq ||e_k||^2,\; \forall\, k \geq 0 \quad (9)$$

which follows from optimality and the fact that the (non-optimal) choice of $u_{k+1} = u_k$ would lead to the relation $J_{k+1}(u_k) = ||e_k||^2$. This result states that the algorithm is a descent algorithm as the norm of the error is monotonically non-increasing in k and also equality holds if, and only if, $u_{k+1} = u_k$, i.e. when the algorithm has converged and no more input-updating takes place.

The controller on trial $k+1$ is given by

$$u_{k+1} = u_k + G^* e_{k+1},\; \forall k \geq 0 \qquad (10)$$

This relationship, together with the error update relation

$$e_{k+1} = (I + GG^*)^{-1} e_k,\; \forall\, k \geq 0 \qquad (11)$$

and the recursive input update relation

$$u_{k+1} = (I + G^*G)^{-1}(u_k + G^* r),\; \forall\, k \geq 0 \quad (12)$$

can be used to undertake a detailed analysis of the (theoretical) properties of this class of ILC laws (Amann *et al.*, 1996).

In this paper, we will only consider the special case of $J_{k+1}(u_{k+1})$ defined as follows applied to a linear time invariant differential plant model with state space matrices (A, B, C) (state, input and output respectively)

$$\begin{aligned} J_{k+1}(u_{k+1}) = & \frac{1}{2}\int_0^T \{e^T_{k+1}(t) Q e_{k+1}(t) \\ & + H^T R H\} dt \\ & + \frac{1}{2} e^T_{k+1}(T) F e_{k+1}(T) \end{aligned} \qquad (13)$$

where

$$H = u_{k+1}(t) - u_k(t) \qquad (14)$$

and the symmetric matrices $Q, R,$ and F satisfy the normal linear quadratic optimal control assumptions. Standard optimal control theory now gives the solution as

$$\begin{aligned} \dot{\psi}_{k+1}(t) &= -A^T \psi_{k+1}(t) - C^T Q e_{k+1}(t) \\ u_{k+1}(t) &= u_k(t) + R^{-1} B^T \psi_{k+1}(t) \end{aligned}$$

$$\psi_{k+1}(T) = C^T F e_{k+1}(T),\ t \in [0,T] \quad (15)$$

This representation is non-causal (in the standard sense) but it can be transformed into a causal implementation as detailed next for the case of a relaxation factor α.

Transform the costate vector $\psi_{k+1}(t)$ using

$$\psi_{k+1}(t) = -K(t)\left[x_{k+1}(t) - \alpha x_k(t)\right] + \zeta_{k+1}(t) \quad (16)$$

where the feedback gain matrix $K(t)$ satisfies the well known Riccati (matrix) differential equation

$$\begin{aligned} \dot{K}(t) &= -A^T K(t) - K(t)A \\ &\quad + K(t)B^T R^{-1} B^T K(t) - C^T Q C \\ K(T) &= C^T F C \end{aligned} \quad (17)$$

Note that this last equation is independent of the inputs, states and outputs of the system and hence only needs to be computed once before the sequence of trials begin.

The predictive or 'feedforward' term $\zeta_{k+1}(t)$ must be computed on each trial using

$$\begin{aligned} \dot{\zeta}_{k+1}(t) &= -(A - BR^{-1}B^T K)^T \zeta_{k+1}(t) \\ &\quad - \alpha C^T Q e_k(t) \\ &\quad + (1-\alpha) K B u_k(t) \\ &\quad - (1-\alpha) C^T Q r(t) \end{aligned} \quad (18)$$

with terminal boundary condition

$$\zeta_{k+1}(T) = C^T F\left[\alpha e_k(T) + (1-\alpha) r(T)\right] \quad (19)$$

The algorithm is now causal since (18) and (19) can be solved off-line by reverse time simulation using available previous trial data.

Predictive optimal ILC (Amann *et al.*, 1998) extends the cost function to the form

$$J_{k+1,N}(u_{k+1}) = \sum_{i=1}^{N} \lambda^{i-1}(||e_{k+i}||^2_{\mathcal{Y}} + ||u_{k+i} - u_{k+i-1}||^2_{\mathcal{U}}) \quad (20)$$

This criterion includes the error of the next N trials as well as the corresponding changes in the control input signals The weighting parameter $\lambda > 0$ determines the importance of more distant (future) errors and incremental inputs compared with the current ones. By including more future signals into the performance criterion, the algorithm becomes less 'short sighted'.

The theory given above extends in a natural manner to this case and an obvious question to ask is: when does the extra (computational) cost become worthwhile?

Suppose now that we use a quadratic cost function which is just the natural generalization of (13) to the predictive setting. Then the following is the final form (with no relaxation factor) of the implementation algorithm for predictive optimal ILC (for complete details see (Amann *et al.*, 1998))

$$\begin{bmatrix} u_{k+1} \\ u_{k+2} \\ \vdots \\ u_{k+N} \end{bmatrix} = \begin{bmatrix} u_k \\ u_k \\ \vdots \\ u_k \end{bmatrix} - R_N^{-1} B_N^T (K(t) \times \left(\begin{bmatrix} x_{k+1} \\ x_{k+2} \\ \vdots \\ x_{k+N} \end{bmatrix} - \begin{bmatrix} x_k \\ x_k \\ \vdots \\ x_k \end{bmatrix} \right) - \xi_{k+1,N}(t)) \quad (21)$$

$$\begin{aligned} \dot{K} &= -A_N^T K - K A_N + \\ &\quad K B_N R_N^{-1} B_N^T K - C_N^T Q C_N, \\ K(T) &= C_N^T(T) F C_N(T) \end{aligned} \quad (22)$$

$$\dot{\xi}_{k+1,N}(t) = -\left(A_N - B_N R_N^{-1} B_N^T K\right)^T \xi_{k+1,N}(t) - C_N^T Q_N \begin{bmatrix} e_k(t) \\ e_k(t) \\ \vdots \\ e_k(t) \end{bmatrix} \quad (23)$$

with final condition

$$\xi_{k+1,N}(T) = C_N^T F_N \times \left[e_k^T(T) \dots\ e_k^T(T)\right]^T \quad (24)$$

where A_N is a block diagonal matrix with each diagonal entry equal to the state matrix A, and the matrices B_N and C_N are constructed in an identical manner using the matrices B and C respectively. For the precise form of the weighting matrices Q_N, R_N and F_N, see again (Amann *et al.*, 1998).

Next we detail the application area studied in this paper.

3. CHAIN CONVEYOR SYSTEMS

Previous work [6] has applied three term (or PID) ILC schemes to this conveyor structure both in simulation and experiment. The major conclusion was that this is a highly relevant application area for ILC. Despite this, achievable performance was limited by the PID structure in certain cases. Hence the decision was made to apply norm optimal based ILC schemes outlined in the previous section to chain conveyor systems. The eventual goal is to assess the performance of such schemes in 'real world' operation - both stand alone and comparatively. In the remainder of this paper, we describe the chain conveyor system to be used, its mathematical modeling and configuration for the actual implementation of control action, and the design of the candidate ILC schemes.

The System

The chain conveyor systems considered in this work have two possible operational modes - indexing and synchronization. When operating in an indexing mode the conveyor moves one item at a time under a dispenser. The dispenser remains stationary and product is dispensed when the conveyor comes to rest. This motion is then repeated for the next item. In synchronization mode the conveyor moves at constant velocity and the dispenser moves back and forth. Product is dispensed when the position and velocity of the dispenser are synchronized to that of item on the conveyor. The system is measured by its accuracy combined with rate of throughput and reliability. Each requirement introduces difficulties and accuracy will degrade with time due to component wear. Commonly this is overcome by regular manual re-calibration of the system.

High rates of throughput imply large accelerations. These produce large electrical and mechanical stresses in the system components that increase wear and reduces accuracy. Ultimately high stresses will cause the premature failure of components, reducing reliability and overall throughput. It is therefore necessary to ensure that the controller demand does not require the actuator to perform outside of the manufacturers specifications. As described in (Barton *et al.*, 2000) the system has many problems that a PID controller cannot deal with at high enough accuracy to meet typical performance requirements.

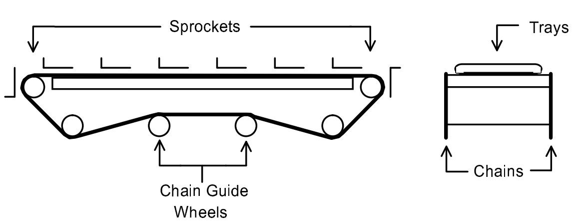

Fig. 1. Schematic of Conveyor Construction.

The conveyor, see Figure 1, is constructed from a 3m long framework of right angle steel section and consists of two parallel strings of 0.5 pitch steel roller chain. At 300 mm intervals there is an aluminum plate supported on a rod that is pinned through a bushing on each chain. A standard squirrel cage induction motor supplied by a variable voltage variable frequency (VVVF) inverter, that is delta connected to a 3 phase pulse width modulated (PWM) inverter, drives the conveyor through a timing belt drive with a 5:2 reduction ratio. The induction motor is oversized for the mechanical load to ensure that the actuator will not limit system performance. A 500 pulse per revolution optical encoder, measured on the motor shaft with differential outputs, provides position feedback. Processing using a DEVA 004 motion control card increases the resolution to the equivalent of 2000 pulses per revolution.

The dispenser, see Figure 2, consists of a trolley that moves linearly above the conveyor. The trolley is an open frame, as this allows dispensing systems and instruments to be exchanged when required. A long belt supplies the linear motion, rotary motion being provided by an identical induction motor/belt drive system as for the conveyor.

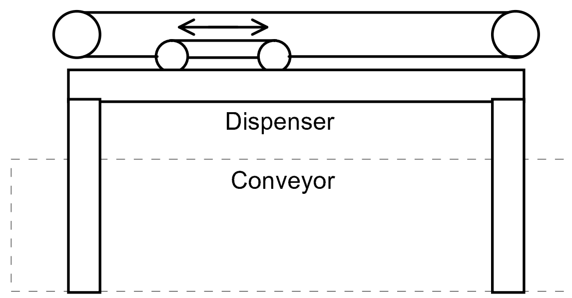

Fig. 2. Schematic of Conveyor Construction.

Frequency-Domain Model

The models of the conveyor and the dispenser used are linear approximations, which were developed for simulation purposes. These were obtained by driving the conveyor and the dispenser with a variable frequency sinusoid, provided by a dc motor drive and recording the frequency response. The motor velocity was measured by using a tachometer, and then scaled to give a response relating input voltage to output velocity in counts/seconds. From the resulting Bode plots linear approximations were derived for the conveyor and dispenser respectively as

$$G_{\text{conv}}(s) = \frac{615.06 \times 10^6}{(s^2 + 49s + 35^2)(s^2 + 54s + 180^2)} \tag{25}$$

$$G_{\text{disp}}(s) = \frac{6.47 \times 10^6}{(s + 35)(s^2 + 99s + 110^2)} \tag{26}$$

Control Implementation

A PC controls the system that includes the DEVA interface card. The card has two 14-bit Digital to Analogue Converters ($D\backslash A$) for speed demand output and two optional-isolated digital outputs for axes enabling. A programmable interrupt controller is provided to produce regularly timed processor interrupts suitable for running discrete controllers. As the inverters are unable to accept a -10V to +10V speed demand, a positive analogue speed signal is provided, with a single line of the parallel port linked to the direction setting pin of the inverter to provide direction control. A program, written in C, provides a user interface to the hardware and also implements the controller.

System Simulation

In order to begin the evaluation of the performance of norm optimal ILC designs in this area, including the relative advantages/disadvantages of predictive action, a simulation of the system operating in synchronous mode has been constructed in MATLAB/SIMULINK. Also a range of controllers have been designed. For example, Figure 3 shows a sample design (for the dispenser transfer function) where the reference signal used is the same as that used in the previous work on the use of PID ILC for this application (Barton *et al.*, 2000).

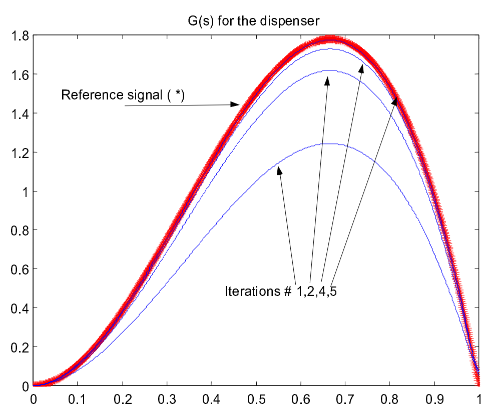

Fig. 3. Illustrative Design.

4. CONCLUSIONS

The goal of the research programme on which this paper is based is to evaluate the performance (both stand alone and comparative terms) of norm optimal based ILC schemes in the 'real world' operating domain. The testbed chosen for this is a chain conveyor system. To date, the experimental testbed has been constructed and the relevant parts of its dynamics approximated by linear models, in the form of transfer functions constructed from measured frequency domain data, obtained. Also a range of controllers based on both the norm optimal and predictive norm optimal ILC designs have been completed (a sample norm optimal design has been included here).

This paper has described the necessary background development to undertake an extensive range of experimental tests which will be used to address the following key questions (and others).

(1) How do normal optimal and predictive norm optimal ILC compare against alternatives (from, in the main, (Barton *et al.*, 2000) and the relevant cited references).
(2) Are there any benefits to be obtained by using predictive norm optimal ILC, i.e. a higher order learning law, against just norm optimal ILC.
(3) If norm optimal ILC does indeed give improved (relative) performance, how can this be quantified in terms of the key extra parameters in these control laws, i.e. the prediction horizon N and the weighting factor λ?
(4) What are the general messages from this study in terms of the theme of this special session, i.e. the relative merits of higher order ILC.

Early results from the experimental programme can be found in (Al-Towaim *et al.*, 2002).

REFERENCES

Al-Towaim, T., P.L. Lewin and E. Rogers (2002). Experimental implementation of norm optimal ilc. *Research Report, Department of Electronics and Computer Science, University of Southampton, UK.*

Amann, N., D.H. Owens and E. Rogers (1996). Iterative learning control using optimal feedback and feedforward actions. *International Journal of Control* **65(2)**, 277–294.

Amann, N., D.H. Owens and E. Rogers (1998). Predictive optimal iterative learning control. *International Journal of Control* **69(2)**, 203–226.

Arimoto, S., S. Kawamura and F. Miyazaki (1984). Bettering operations of robots by learning. *Journal of Robotic Systems* **1**, 123–140.

Barton, A.D., P.L. Lewin and D.J. Brown (2000). Practical implementation of a real-time iterative learning controller. *International Journal of Control* **73(10)**, 992–999.

Moore, K.L. (1993). *Iterative Learning Control for Deterministic Systems.* Springer-Verlag. London.

Rogers, E. and D.H. Owens (1992). *Stability Analysis for Linear Repetitive Processes, LNCIS Series, Vol 175.* Springer-Verlag. Berlin.