

ACTIVE FAULT ACCOMMODATION OF A THREE TANK SYSTEM VIA SWITCHING CONTROL

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Abstract: In this paper, a fault detection and isolation algorithm together with a switching control strategy are used to solve the problem of a fault accommodation in the three tank process. The nonlinear model of the process is linearized through state feedback. Then, a PI and a state feedback controller are designed to insure some predefined control performances. The fault-tolerant control strategy is based on an online detection of faults and the selection of an adequate controller. The experimental results on a three tank benchmark are presented and discussed.

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Keywords: Fault-tolerant control, fault detection and isolation, switching control, statechart.

1. INTRODUCTION

The objective of fault detection and isolation (FDI) is to make an inventory of all possible faults that can occur during the functioning of a system and to design an algorithm to detect and isolate them. This algorithm allows the determination of the instant, the amplitude, the type and the location of the fault. On the other hand, the objective of fault-tolerant control (FTC) consists in taking the adequate actions to reduce the effects of the detected fault on the system performances. Many studies have contributed to the development of FTC, (see, Patton, 1997; Blanke *et al.*, 2000). Therein, a basic literature review covers most areas of fault-tolerant control. Active fault-tolerant systems require either *a priori* knowledge of expected fault types or a mechanism for detecting and isolating unanticipated ones. In the latter case, decisions concerning the location and nature of faults are then used to reschedule the controller function.

In this paper, the use of an FDI algorithm together with a logic-based switching control strategy (Morse, 1996; Charbonnaud *et al.*, 2001), allows us to perform active fault-tolerant control. Whereas in these papers the considered systems are linear, in the present paper an input affine nonlinear system is considered with a different FDI strategy. Our work deals with an active FTC method in the sense that an adequate controller, belonging to a set of precomputed ones, is activated whenever its corresponding fault is detected.

The paper is organized as follows. The problem statement of FTC is introduced in section 2. The strategy of the active fault accommodation via switching control is presented in section 3. The FDI scheme, introduced in this section, is based on the design of a bank of linear observers where each one is dedicated to one process output. The linear observer design is based on the exact feedback linearization of the process. A such technique gives satisfactory results and allows the isolation of faults (Ding *et al.*, 1999). The residual is con-

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tinuously compared to a set of thresholds where each one describes the fault magnitude or severity. In a first attempt, a set of fixed thresholds is used, knowing that adaptive ones can be applied also. The switching control strategy consists in activating the controller associated to the detected fault magnitude. Here, the multi-controller stands for the controller itself associated to the set of static state feedback. This accommodation strategy is applied to a nonlinear MIMO system, i.e., a three tank process. The experimental results of the real-time implementation of this strategy applied to a three tank process are discussed in section 4.

2. PROBLEM STATEMENT

Let us consider a process whose dynamics are modeled by the following nonlinear input affine model:

$$\begin{cases} \dot{x} = f(x) + g(x) u + EF_d(x) \\ y = h(x) \end{cases} \quad (1)$$

where $u \in \mathbb{R}^m$, $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^p$ are, respectively, the control input, the state and the output of the plant. This model describes the dynamic of the process under abnormal operating conditions. The vector fields $f(x)$, $g(x)$ and $h(x)$ are nonlinear. $E \in \mathbb{R}^{n \times q}$ is the dynamic fault distribution matrix. $F_d(x)$ is a vector field of dimension q , representing the dynamic fault. In our approach, $F_d(x)$ belongs to a finite set of vectors fields \mathcal{F} . Each element of $F_d(x)$, corresponding to a fault occurring on a component of the state vector, is associated to an element of \mathcal{F} which corresponds to a severity degree of this fault. The problem of FTC is to design a residual generator which can detect F_d and to take the appropriate actions to minimize the effects of this fault on the system performances.

The relation (1) describes a model of systems that are very usual in real world. In this paper, the proposed active fault accommodation is applied on the three tank process. The latest is shown on figure (1). It consists of three plexiglass cylinders T_1 , T_2 and T_3 , with the cross section $s_t = 0.0154 \text{ m}^2$.

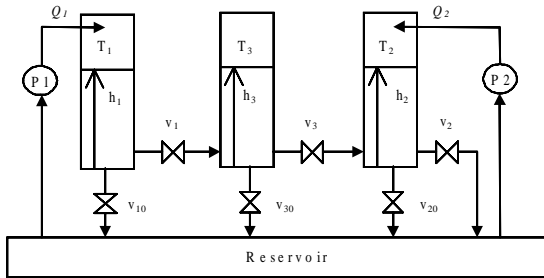


Fig. 1. Three tank system.

They are serially connected with each other by cylindrical pipes with cross section $s_p = 5 \times$

10^{-5} m^2 . The outflowing liquid from the second tank is collected in a reservoir which supplies the pumps P1 and P2. Q_1 and Q_2 are the flow rates of the pumps P1 and P2 driven by a DC motor. For the purpose of simulating clogs or operating errors, the connecting pipes and the nominal outflow are equipped with manually adjustable ball valves v_i ($i = 1, 2, 3$), which allow to close the corresponding pipe. For the purpose of simulating leaks, each tank has a circular opening with the cross section s_p and a manually adjustable ball valve v_{i0} ($i = 1, 2, 3$). The pump flow rates Q_1 and Q_2 denote the input signals. The liquid levels of T_1 , T_2 and T_3 denoted, respectively, h_1 , h_2 , and h_3 , are the output signals.

The plant can be modeled by the following nonlinear relations using the mass balance equation:

$$\begin{cases} s_t \frac{dh_1}{dt} = Q_1 - (Q_{13_n} - Q_{13_f}) - Q_{10_f} \\ s_t \frac{dh_2}{dt} = Q_2 + (Q_{32_n} - Q_{32_f}) - (Q_{out_n} - Q_{out_f}) - Q_{20_f} \\ s_t \frac{dh_3}{dt} = (Q_{13_n} - Q_{13_f}) - (Q_{32_n} - Q_{32_f}) - Q_{30_f} \end{cases} \quad (2)$$

where Q_{ij_n} and Q_{ij_f} are, respectively, the nominal and the faulty flow rate from tank i to j . Q_{out_n} and Q_{out_f} are, respectively, the nominal and faulty outflow rate of the water issued from tank T_2 . Q_{i0_f} is the leak flow rate of tank i simulated by opening the valve v_{i0} . Using the Torricelli rule, these flow rates are expressed by:

$$\begin{cases} Q_{ij_n} = v_{i_n} s_p \text{sign}(h_i - h_j) \sqrt{2 g |h_i - h_j|} \\ Q_{out_n} = v_{2_n} s_p \sqrt{2 g h_2} \\ Q_{ij_f} = v_{i_f} s_p \text{sign}(h_i - h_j) \sqrt{2 g |h_i - h_j|} \\ Q_{out_f} = v_{2_f} s_p \sqrt{2 g h_2} \\ Q_{i0_f} = v_{i0_f} s_p \sqrt{2 g h_i} \end{cases} \quad (3)$$

where v_{i_n} , v_{i0_n} are the nominal coefficients of the valves opening. The model given by equations (2) and (3), can be rewritten in the form given by (1), where $x = [h_1 \ h_2 \ h_3]^T$ is the state vector, $u = [Q_1 \ Q_2]^T$ is the control input, $y = [h_1 \ h_2 \ h_3]^T$ is the output vector, f , g , F_d and E are given, respectively, by:

$$\begin{aligned} f(x) &= \begin{bmatrix} -\frac{1}{s_t}(Q_{13_n}) \\ \frac{1}{s_t}(Q_{32_n} - Q_{out_n}) \\ \frac{1}{s_t}(Q_{13_n} - Q_{32_n}) \end{bmatrix}, \quad g(x) = \begin{bmatrix} \frac{1}{s_t} & 0 \\ 0 & \frac{1}{s_t} \\ 0 & 0 \end{bmatrix}, \\ F_d(x) &= [Q_{10_f} \ Q_{20_f} \ Q_{30_f} \ Q_{13_f} \ Q_{32_f} \ Q_{out_f}]^T \\ E &= \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{bmatrix} \end{aligned} \quad (4)$$

3. FAULT ACCOMMODATION VIA SWITCHING CONTROL

In this section, the control accommodation strategy will be presented. The nonlinear system is feedback linearized. Then, a state feedback and PI controllers are designed to achieve some predefined control performances. Finally, the FDI technique and the fault accommodation strategy will be developed.

Logic-based switching algorithms have been used extensively in control of automated processes. The switching arises when no single controller is capable to insure the predefined performances when connected to a process with large parametric uncertainties and modelling errors or when it has many operating modes. The strategy which is developed in this paper consists in combining the design of an FDI scheme, to detect and isolate a fault belonging to a set of *a priori* known ones, and of a bank of controllers where each one is associated to a fault severity degree. Figure 2 describes the multi-controller structure which is used to select an adequate controller when its associated fault severity degree is detected. Here the controller stands for the connection of the fixed controller with a given linearizing static feedback. The fixed

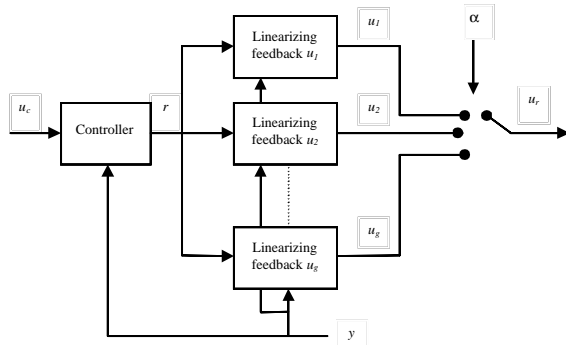


Fig. 2. Multi-controller structure.

controller is designed to achieve predefined control performances on the basis of the nominal feedback linearized model of the plant. When a fault occurs, the nominal linearizing control input is no longer valid. Each linearizing feedback u_j , $j = 1, \dots, g$ (g is not to be confused with the vector field), corresponds to a fault severity degree. The accommodation signal α selects the adequate control input u_j , associated to the detected fault severity degree. The signals u_c and r denote, respectively, the reference input and the control input issued from the fixed controller.

3.1 Exact Feedback Linearization

The class of nonlinear systems modeled by relation (1), can be linearized via exact linearization

through state feedback. This method consists in designing a new control input w , such that:

$$u = \beta(x) + \gamma(x) w, \quad (5)$$

where $\beta(x)$ (invertible) and $\gamma(x)$ are nonlinear vector fields, to linearize the model given by relation (1). Since the conditions given in (Isidori, 1995) are satisfied, the model can be linearized through the following control input, (Ding *et al.*, 1999; Ponsart *et al.*, 1999):

$$u = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} Q_{13} + Q_{10} \\ Q_{out} - Q_{32} + Q_{20} \end{bmatrix} \quad (6)$$

$$+ \begin{bmatrix} s_t & 0 \\ 0 & s_t \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad (7)$$

Then, the nonlinear system given by (1) and (4) becomes:

$$\dot{x} = \begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad (8)$$

This linear system can be rewritten in the following form:

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -k_{11} & 0 \\ 0 & -k_{22} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} k_{11} & 0 \\ 0 & k_{22} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \quad (9)$$

A state feedback or a PI controllers can be designed to control this MIMO linear system.

3.2 Controller Design

The three tank process has slow dynamics. For a time constant of 20 seconds, i.e., $k_{11} = k_{22} = 0.05$, the linearized system behaves in a satisfactory manner. The state space representation of the system with $k_{11} = k_{22} = 0.05$, is as follows:

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -0.05 & 0 \\ 0 & -0.05 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \quad (10)$$

The process can be controlled via a PI controller so that the closed loop system behaves as a second order system with a damping factor $\zeta = 1$ and a natural frequency $\omega_0 = 0.07 \text{ rad} \cdot \text{s}^{-1}$. The transfer function of a PI controller which achieves these objectives is given by:

$$C_{1,2}(s) = \frac{r_{1,2}(s)}{h_{ref_{1,2}}(s) - h_{1,2}(s)} = 2 + \frac{0.1}{s} \quad (11)$$

A state feedback controller can be designed to ensure the same performances. This is achieved by computing a state feedback gain K , such that:

$$\begin{bmatrix} \dot{r}_1 \\ \dot{r}_2 \end{bmatrix} = -K \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + L \begin{bmatrix} u_{c1} \\ u_{c2} \end{bmatrix} \quad (12)$$

where $K = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}$ and $L = \begin{bmatrix} 7/5 & 0 \\ 0 & 7/5 \end{bmatrix}$.

3.3 Fault Detection, Isolation and Accommodation

The state space realization of the open-loop system in the presence of a dynamic fault is written as:

$$\begin{cases} \dot{x} = A x + B r + E F_d(x) \\ y = C x \end{cases} \quad (13)$$

The purpose of FDI is to generate a set of residuals that can reveal an occurrence of a fault during the system functioning. This can be achieved by an adequate design of a Luenberger observer with the following structure:

$$\begin{cases} \dot{\hat{x}} = A \hat{x} + B r + M (y - \hat{y}), \\ \hat{y} = C \hat{x}, \end{cases} \quad (14)$$

and by the evaluation of the residual generator which is expressed by $\varepsilon(t) = y(t) - \hat{y}(t)$.

Since the operations of control and supervision have to be implemented numerically, one has to use discrete time observers to accomplish the FDI tasks. Taking 0.5 s as a sampling period, the discrete time state space realisation equivalent to the one given by relation (9) is as follows:

$$\begin{cases} h_{1,2}(k+1) = 0.9753 h_{1,2}(k) + r_{1,2}(k) \\ y_{1,2}(k) = 0.02469 h_{1,2}(k) \end{cases} \quad (15)$$

The Luenberger observer associated to this model is given by:

$$\begin{cases} \hat{h}_{1,2}(k+1) = 0.9753 \hat{h}_{1,2}(k) + r_{1,2}(k) \\ \quad + M (y_{1,2}(k) - \hat{y}_{1,2}(k)) \\ \hat{y}_{1,2}(k) = 0.02469 \hat{h}_{1,2}(k) \end{cases} \quad (16)$$

The observer gain M is calculated by placing the eigenvalue of $(0.9753 - M 0.02469)$ at $z = 0.9513$, thus $M = 0.97205$.

A suitable observer of the water level in the tank T_3 is given by:

$$\dot{\hat{h}}_3 = M_3 (h_3 - \hat{h}_3) + \frac{1}{s_t} (Q_{13} - Q_{32}) \quad (17)$$

This relation is implemented numerically by a discrete time integrator using the trapezoidal method. The gain $M_3 = 2$ shows satisfactory dynamics for the observer.

The residuals associated to the three levels h_1 , h_2 and h_3 are given by $\varepsilon_i = h_i - \hat{h}_i$, $i = 1, 2, 3$. In (Ding *et al.*, 1999) a residual evaluation stage is carried out to reduce the effects of measurement noise on the fault isolation stage. The problem of distinguishing sensor faults from dynamic ones is also solved. In our study, we are interested in isolating the leaks and pluggings that could occur in each tank. The residuals ε_i can be processed by a moving average filter with a sliding window of

size N . For $i = 1, 2, 3$, the residuals are evaluated by:

$$\varepsilon_{i_{avg}}(k) = \frac{1}{N} \sum_{j=1}^N [h_i(k-j) - \hat{h}_i(k-j)], \quad (18)$$

Under the assumption that no sensor faults occur and through the filtering given above, it is possible to use the following statechart to describe the fault accommodation strategy (see Fig. 3).

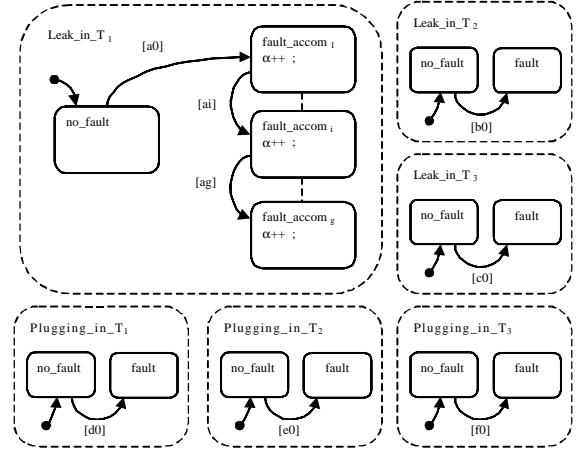


Fig. 3. Statechart representation of the fault accommodation strategy via switching control in the case of a leak in T_1 .

The statechart consists of six parallel superstates with dash-dotted contour that represent a concurrent mode of operation. Each superstates has a label which indicates its function. The superstates $\text{Leak_in_}T_1$ and $\text{Plugging_in_}T_1$ detect, respectively, a leak and a plugging in the first tank. Since we are interested only in the accommodation of a leak in the first tank, the superstates $\text{Leak_in_}T_i$, $i = 2, 3$, and $\text{Plugging_in_}T_i$, $i = 1, 2, 3$, do consist of two states that indicate a normal and an abnormal functioning. The change of states occurs when the conditions marked on the transitions are satisfied. These transitions are labeled $b0$, $c0$, $d0$, $e0$ and $f0$, where:

$$\begin{aligned} [b0] &\triangleq [\varepsilon_{2_{avg}}(k) < S_b] \\ [c0] &\triangleq [\varepsilon_{3_{avg}}(k) < S_c] \\ [d0] &\triangleq [\varepsilon_{1_{avg}}(k) > S_d] \\ [e0] &\triangleq [\varepsilon_{2_{avg}}(k) > S_e] \\ [f0] &\triangleq [\varepsilon_{3_{avg}}(k) > S_f] \end{aligned} \quad (19)$$

Here, S_b , S_c , S_d , S_e and S_f are fixed thresholds that have to be determined such that to reduce the false alarms rate and to minimize the effect of measurement noise on the decision stage. In figure (3), the super state “ $\text{Leak_in_}T_1$ ” consists of one “no_fault” state and g “fault_accom” states that correspond to the degrees of the leak severity. The condition $a1$ is equivalent to $\varepsilon_{1_{avg}}(k) < S_{a1}$. The variable $\alpha \in \{1, \dots, g\}$ is a flag which indicates whether a given state has been enabled or not. This flag is initialized to one and incremented

when the fault severity increases. It is used also as the accommodation function which selects the adequate controller when the transitions ai , $i = 1, \dots, g$, are validated. The transitions $a0$ and ai are equivalent to:

$$\begin{aligned} [a0] &\triangleq [\varepsilon_{1avg}(k) < S_{a0}] \\ [ai] &\triangleq [\varepsilon_{1avg}(k) < S_{ai}] \wedge [\alpha = i - 1] \end{aligned} \quad (20)$$

Here, the set of thresholds $\{S_{a0}, S_{a1}, \dots, S_{ag}\}$ are determined such that to characterize the leak severity. This is due to the fact that the residual is proportional to the fault severity. Each element of this set is associated to a fault amplitude or a degree of severity $F_d \in \mathcal{F} = \{F_{d1}, F_{d2}, \dots, F_{dg}\}$. If the fault severity F_d can be approximated to one of the faults F_{dj} belonging to \mathcal{F} , then the accommodation signal α activates the linearizing control input u_j , $j \in \{1, 2, \dots, g\}$ such that the real control input signal u_r satisfies:

$$u_r = u_j = \beta_j(x) + \gamma_j(x) w \quad (21)$$

By this control strategy, the fault effect on the system is reduced. Therefore, the accommodation strategy consists in detecting the fault type by the rules given by relations (19), in characterizing its magnitude by the comparison of the residual with a set of thresholds, and in switching to the feedback linearizing control input to compensate the fault effects on the system as shown on figure (2).

Without loss of generality, in a case of a leak in T_1 , the i th linearizing feedback is expressed by:

$$u_i = Q_{13} + Q_{10_{f_i}} + s_t w_1 \quad (22)$$

where $Q_{10_{f_i}}$, associated to the i th threshold S_i , denotes the amount of leak to be compensated. The accommodation function α is given by:

$$\alpha(k) = \{i \mid [\varepsilon_{1avg} < S_{ai}] \wedge [\alpha(k-1) = i-1]\} \quad (23)$$

The experimental results of this fault-tolerant control strategy on the three tank benchmark will be discussed in the section below.

4. EXPERIMENTAL RESULTS

The control and supervision operations are implemented on a pentium PC connected to the plant via a 12 bits A/D converter (AD RTI 815). Only one fault accommodation, in the case of a leak in the first tank, is considered. The experimental results of the proposed accommodation strategy are presented. The control objective was to maintain the levels of T_1 and T_2 , respectively, at 0.3 m and 0.2 m. The sampling period is 0.5 second. To achieve these specifications, the computed controllers given in section 3 were used. Figures 4 and 5 show the real-time evolution of the levels (Fig. 4.a and 5.a), the pumps flow rate

(Fig. 4.b and 5.b), the control input (Fig. 4.c and 5.c), the residuals (Fig. 4.d and 5.d) and the accommodation function (Fig. 5.e), during 250 seconds. When using a state feedback controller, the control objective is reached in the presence of modeling uncertainties and measurement noise. As expected, the residuals tend to zero. However, when a leak fault (valve opening at 50%) occurs in T_1 at $t \simeq 137.5$ seconds, the level of T_1 drops to 0.27 m and affects also the one of T_3 . Hence, the control performances are no more insured. The loss of performances is worst when the leak fault is more severe. This can be observed on the level of T_1 when the leak valve is opened at 100%, at $t \simeq 209$ seconds. The use of the accommodation strategy allows keeping the control objectives and performances in the sense that the level of T_1 is kept at 0.3 m when the leak fault occurs and when its severity becomes higher (see Fig. 5.a). Here, the thresholds corresponding to the two severity degrees are fixed at -0.005 and at -0.01 by taking into account the noise variance. It is also relevant to analyze the results of the experimentation when using a PI controller. The fault rejection is very limited using this controller (see Fig. 6.a). When the fault occurs, the settling time of the level of T_1 is approximately 5 seconds. The use of this PI controller together with the accommodation strategy insures the reduction of this settling time and maintains the control objectives and performances (see Fig. 7.a).

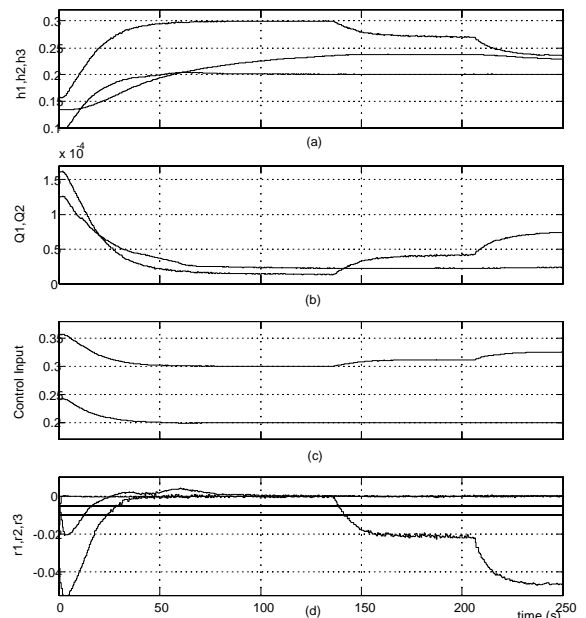


Fig. 4. Plant data with a state feedback controller, without accommodation in presence of two faults of a leak type, v_{10} is opened at 50% , $t \simeq 137.5$ seconds and then at 100%, $t \simeq 209$ seconds.

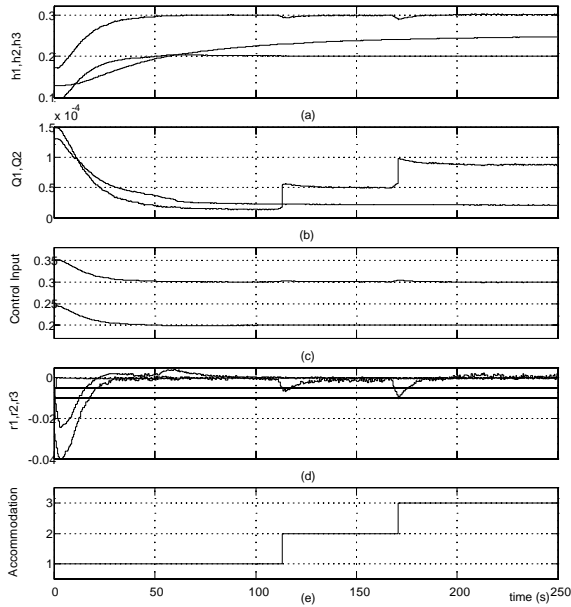


Fig. 5. Plant data with a state feedback controller, with the accommodation of two faults of a leak type, v_{10} is opened at 50%, $t \simeq 112$ seconds and then at 100%, $t \simeq 168$ seconds.

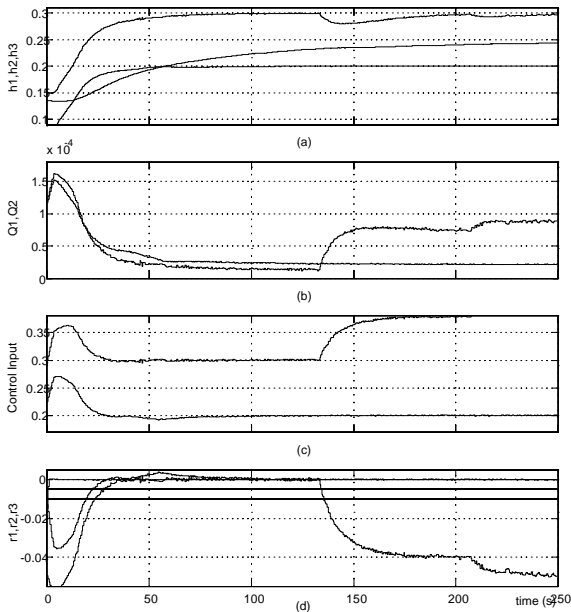


Fig. 6. Plant data with a PI controller, without accommodation in presence of two faults of a leak type, v_{10} is opened at 50%, $t \simeq 133$ seconds and then at 100%, $t \simeq 208$ seconds.

5. CONCLUSION

In this paper, a multi-controller structure, with a fixed controller and multiple linearizing static feedbacks, was introduced to deal with a fault-tolerant control strategy of a nonlinear system. It was applied successfully on the three tank benchmark in real-time, and has given satisfactory results since the effects of the considered fault on the system performances have been attenuated. This FTC method is based on a joint design of

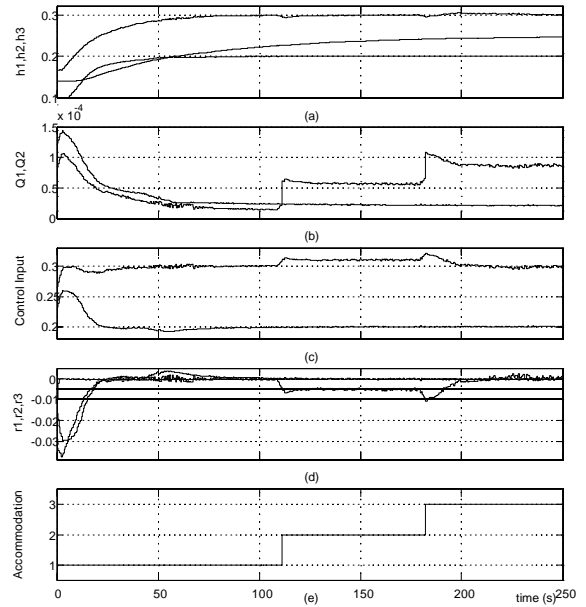


Fig. 7. Plant data with a PI controller, with the accommodation of two faults of a leak type, v_{10} is opened at 50%, $t \simeq 109$ seconds and then at 100%, $t \simeq 180$ seconds.

an FDI algorithm to detect *a priori* known faults and the design of a switching control strategy to achieve a fast and an accurate accommodation in comparison with other adaptive techniques.

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