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INTERVAL OBSERVERS FOR INTERCONNECTED BIOTECHNOLOGICAL SYSTEMS

J. Harmand[1], A. Rapaport[2]

[1] *Laboratoire de Biotechnologie de l'Environnement,*
INRA, Avenue des étangs, 11100 Narbonne, France.
e-mail: harmand@ensam.inra.fr
Tel : +33 (0) 468.425.159, Fax : +33 (0) 468.425.160
[2] *Laboratoire d'Analyse des Systèmes et Biométrie*
INRA, 2, Place Viala, 34000 Montpellier, France.
e-mail: rapaport@ensam.inra.fr
Tel : +33 (0) 499.612.652, Fax : +33 (0) 467.521.427

Abstract : In this paper, the design of interval observers for the estimation of biomass concentrations in complex biotechnological systems (interconnected biological systems) is described. In particular, the design of such observers for tanks in cascade (including a recirculation) is considered. The design procedure as well as a sufficient condition for the stability of the observer are established. The proposed approach is robust with respect to unknown kinetics and bounded disturbances on the inputs. *Copyright © 2002 IFAC.*

Keywords: cascade of bioreactors, estimation, unknown inputs, interval observers.

1. INTRODUCTION

Today, biological systems are used in a large number of production systems. If a number of sensors are available in high added value industries such as pharmaceutical or food, biological processes suffer of a systematic lack of sensors when used as Treatment Processes. In such processes, due to the natural biodiversity, the number of present species can be extremely large and it is not possible to closely monitor the depleting ecosystem at the opposite of what happens in pharmaceutical industries where usually only one well known species is used.

When the inputs of biological processes are known or can be monitored, classical observation and control tools can be used to estimate unmeasured variable and to optimize the process (see (Bastin and Dochain, 1990)). However, treatment processes are subject to external disturbances such as variations of both the quantity and the quality of the liquid or solid wastes to be treated and these disturbances are actually not measurable on-line. As a consequence, since it is not possible to measure all inputs of the process, classical observation and control methods cannot be applied. Of course, a number of hypotheses can be posed in order to simplify the problem but results can then be quite conservative. To rigorously solve this problem, an alternative approach has been proposed (See (Gouzé *et al*., 2000)). From known lower and upper bounds in between which the inputs are assumed to lie, this approach allows the user to on-line estimate guaranteed lower and upper bounds on the unmeasured variables. Since the actual value is not uniquely reconstructed, this approach is called the "interval based observer". Recently, the method has been experimentally validated on a 1m3 fixed bed reactor ((Alcaraz-Gonzalez *et al*. 1999)). Its use for the design of robust control laws has also been investigated (see (Rapaport and Harmand, 2001)). However, all the previously cited studies were related to single biological units and none were devoted to the interaction of systems. Yet, in reality - and in particular when used as treatment processes - biological tanks are interconnected. In this case, the above mentioned theory are not systematically and directly applicable and further investigation is necessary (the reader can refer to (Chen, 1992) for studies about interconnected biological systems).

This paper presents the use of interval observers approach for complex biological systems composed of several interconnected biological reactors. The paper is organized as follows. We first present the system under consideration. Then, we propose an asymptotic observer for unknown kinetics but with known exogenous inputs. In the next section, interval observers are derived. Finally, simulation results on a simple example are presented.

2. THE PROCESS UNDER CONSIDERATION

The complex biological system considered in the present study is represented as a succession of N tanks connected in the following way :

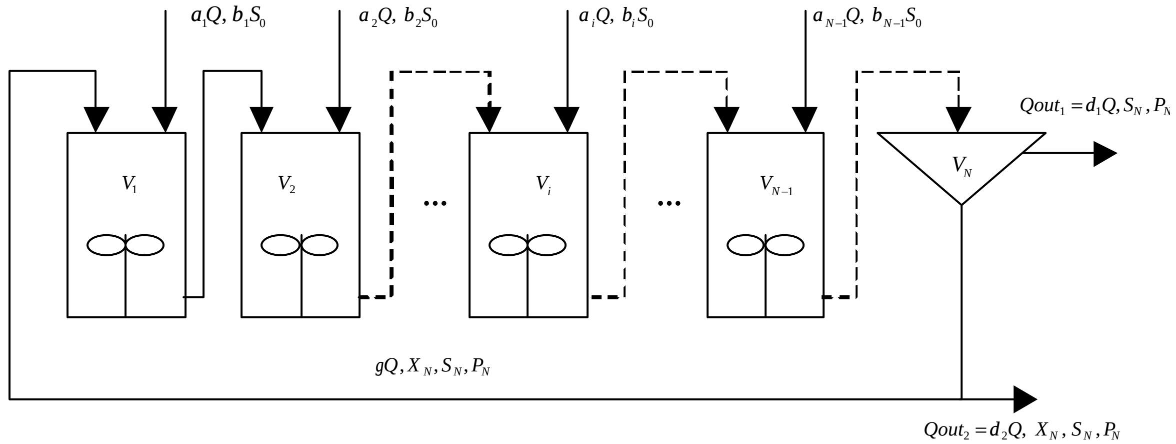

Figure 1 : *System considered*

$\alpha_i Q$ and $b_i S_0$ represent respectively the flow rate and the concentration of the input substrate in tank i (i=1...N-1). d_1 and d_2 are the part of the total input flow rate going out through $Qout_1$ and $Qout_2$ respectively and thus verify $d_1+d_2=1$. α_i and b_i are functions of time belonging to [0,1] and $\sum_{k=1}^{N-1} a_k = 1$. Finally, γQ is the recirculation flow rate (and thus γ is a non negative function of time).

Hypotheses :

- Tanks 1 to N-1 are assumed to be perfectly mixed reactors;
- The settler (tank N) is assumed to be perfect : the particulate matter (the biomass) is completely settled and none of it leaves the process by the way of $Qout_1$.

The model of the process is given hereafter, where X_i, S_i and P_i are the concentrations, respectively, of biomass, substrate and product in the i^{th} tank. V_i are the volume of each tank. We consider the same biological reaction in each tank i (i=1…N) involving three concentrations X_i, S_i and P_i. Y_1 is the yield coefficient of this reaction, identical in each tank i.

For the first tank, a mass balance between the input and the output is written as :

$$\begin{cases} \dfrac{dX_1}{dt} = g(t)\dfrac{Q}{V_1}X_N + m_1(t)X_1 - (a_1(t)+g(t))\dfrac{Q}{V_1}X_1 \\ \dfrac{dS_1}{dt} = a_1(t)\dfrac{Q}{V_1}b_1(t)S_0 + g(t)\dfrac{Q}{V_1}S_N - \dfrac{m_1(t)X_1}{Y_1} - (a_1(t)+g(t))\dfrac{Q}{V_1}S_1 \\ \dfrac{dP_1}{dt} = g(t)\dfrac{Q}{V_1}P_N + \dfrac{m_1(t)X_1}{Y_2} - (a_1(t)+g(t))\dfrac{Q}{V_1}P_1 \end{cases} \quad (1)$$

For the i^{th} (i=2...N-1) reactor, we have :

$$\begin{cases} \dfrac{dX_i}{dt} = h_{i-1}(t)\dfrac{Q}{V_i}X_{i-1} + m_i(t)X_i - h_i(t)\dfrac{Q}{V_i}X_i \\ \dfrac{dS_i}{dt} = h_{i-1}(t)\dfrac{Q}{V_i}S_{i-1} + a_i(t)\dfrac{Q}{V_i}b_i(t)S_0 - \dfrac{m_i(t)X_i}{Y_1} - h_i(t)\dfrac{Q}{V_i}S_i \\ \dfrac{dP_i}{dt} = h_{i-1}(t)\dfrac{Q}{V_i}P_{i-1} + \dfrac{m_i(t)X_i}{Y_2} - h_i(t)\dfrac{Q}{V_i}P_i \end{cases} \quad (2)$$

and for the N^{th} reactor (the settler), the mass balance is written as (3) :

$$\begin{cases} \dfrac{dX_N}{dt} = h_{N-1}(t)\dfrac{Q}{V_N} X_{N-1} - (g(t) + d_2(t))\dfrac{Q}{V_N} X_N \\ \dfrac{dS_N}{dt} = h_{N-1}(t)\dfrac{Q}{V_N} S_{N-1} - (g(t) + d_1(t) + d_2(t))\dfrac{Q}{V_N} S_N \\ \dfrac{dP_N}{dt} = h_{N-1}(t)\dfrac{Q}{V_N} P_{N-1} - (g(t) + d_1(t) + d_2(t))\dfrac{Q}{V_N} P_N \end{cases} \quad (3)$$

In these equations, $h_i(t) = g(t) + \sum_{k=1}^{i} a_k(t)$ and the terms $m_i(t)$ represent the realizations of the growth functions, i.e. $m_i(t)=m_i(t,X(t),S(t))$, which are unknown functions of the concentrations of biomasses and substrates.

3. SYNTHESIS OF AN UNKNOWN KINETICS OBSERVER

In the following we assume that for any tank i (i=1...N), either the substrate concentration S_i or the product concentration P_i is measured on-line. The objective is to estimate the biomass concentrations X_i, without the knowledge of the kinetics $m_i(.)$. Following the design procedure proposed in (Bastin and Dochain, 1990), the auxiliary variables $Z_i=X_i/Y_1+S_i$ and $W_i=X_i/Y_2-P_i$ (i=1...N) are introduced. Their dynamics are given by (for simplicity time dependence of parameters are not written) :

$$\begin{cases} \dot{Z}_i = h_{i-1}\dfrac{Q}{V_i} Z_{i-1} - h_i \dfrac{Q}{V_i} Z_i + a_i \dfrac{Q}{V_i} b_i S_0 \\ \dot{Z}_N = h_{N-1}\dfrac{Q}{V_N} Z_{N-1} - (g + d_2)\dfrac{Q}{V_N} Z_N - d_1 \dfrac{Q}{V_N} S_N \end{cases} \quad (4)$$

with the convention $Z_0=Z_N$ and

$$\begin{cases} \dot{W}_i = h_{i-1}\dfrac{Q}{V_i} W_{i-1} - h_i \dfrac{Q}{V_i} W_i \\ \dot{W}_N = h_{N-1}\dfrac{Q}{V_N} W_{N-1} - (g + d_2)\dfrac{Q}{V_N} W_N + d_1 \dfrac{Q}{V_N} P_N \end{cases} \quad (5)$$

with the convention $W_0=W_N$. This leads to the construction of two observers (6) and (7):

$$\begin{cases} \dot{\hat{Z}}_i = h_{i-1}\dfrac{Q}{V_i}\hat{Z}_{i-1} - h_i\dfrac{Q}{V_i}\hat{Z}_i + a_i\dfrac{Q}{V_i} b_i S_0 \\ \dot{\hat{Z}}_N = h_{N-1}\dfrac{Q}{V_N}\hat{Z}_{N-1} - (g + d_2)\dfrac{Q}{V_N}\hat{Z}_N - d_1\dfrac{Q}{V_N} S_N \\ \hat{X}_i^{\#1} = Y_1(\hat{Z}_i - S_i) \end{cases} \quad (6) \text{ and } \begin{cases} \dot{\hat{W}}_i = h_{i-1}\dfrac{Q}{V_i}\hat{W}_{i-1} - h_i\dfrac{Q}{V_i}\hat{W}_i \\ \dot{\hat{W}}_N = h_{N-1}\dfrac{Q}{V_N}\hat{W}_{N-1} - (g + d_2)\dfrac{Q}{V_N}\hat{W}_N + d_1\dfrac{Q}{V_N} P_N \\ \hat{X}_i^{\#2} = Y_2(\hat{W}_i - P_i) \end{cases} \quad (7)$$

Define the matrix $A(t) = \begin{bmatrix} -(g(t)+a_1(t))\dfrac{Q}{V_1} & 0 & 0 & 0 & g(t)\dfrac{Q}{V_1} \\ \ddots & \ddots & 0 & 0 & 0 \\ 0 & h_{i-1}\dfrac{Q}{V_i} & -h_i\dfrac{Q}{V_i} & 0 & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & h_{N-1}\dfrac{Q}{V_N} & -(g(t)+d_2(t))\dfrac{Q}{V_N} \end{bmatrix}$

then we have $\dot{\hat{Z}} = A(t)\hat{Z} + \begin{pmatrix} a_1(t)\dfrac{Q}{V_1} b_1(t) S_0 \\ \vdots \\ a_i(t)\dfrac{Q}{V_i} b_i(t) S_0 \\ \vdots \\ -d_1(t)\dfrac{Q}{V_N} S_N \end{pmatrix}$ and $\dot{\hat{W}} = A(t)\hat{W} + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ d_1(t)\dfrac{Q}{V_N} P_N \end{pmatrix}$.

A sufficient condition of stability of these observers is now given when $b_i(.)$ and $d_1(.)$ are known.

Proposition 1: Assume that at any time, $a_i(t)$ and $g(t)$ belongs respectively to [$\underline{a}_i$, $\overline{a}_i$] and [0, $\overline{g}$] with (i=1...N), and $d_2(t)$ belongs to [$\underline{d}_2$, $\overline{d}_2$], where $\underline{a}_i$, $\overline{a}_i$, $\overline{g}$, $\underline{d}_2$ and $\overline{d}_2$ are positive numbers. If $\hat{X}^{\#1}$ and $\hat{X}^{\#2}$ are initialized such that $\hat{X}_i^{\#1}(0) \le X_i(0)$ and

$\hat{X}_i^{\#2}(0) \geq X_i(0)$ then a sufficient condition to guarantee that (6) and (7) are asymptotic observers is given by :

$$1 < \frac{\underline{d}_2}{\bar{g} + \sum_{k=1}^{N-1} \bar{a}_k} \prod_{m=1}^{N-1} \left(\frac{\sum_{k=1}^{m} \underline{a}_k}{\bar{g} + \sum_{k=1}^{m-1} \bar{a}_k} \right) \qquad (8)$$

Proof : Consider the error vectors of the two observers: $E_1(t) = \hat{X}^{\#1} - X(t)$ and $E_2(t) = \hat{X}^{\#2} - X(t)$, we have: $E_1(t) = \hat{Z}(t) - Z(t)$ and $E_2(t) = \hat{W}(t) - W(t)$ which dynamics are $\dot{E}_1 = A(t)E_1$ and $\dot{E}_2 = A(t)E_2$. Consider now the two following matrices :

$$A^+ = \begin{bmatrix} -\underline{a}_1 \frac{Q}{V_1} & 0 & 0 & 0 & \bar{g}\frac{Q}{V_1} \\ \ddots & \ddots & 0 & 0 & 0 \\ 0 & \left(\bar{g} + \sum_{k=1}^{i-1} \bar{a}_k\right)\frac{Q}{V_i} & -\sum_{k=1}^{i} \underline{a}_k \frac{Q}{V_i} & 0 & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \left(\bar{g} + \sum_{k=1}^{N-1} \bar{a}_k\right)\frac{Q}{V_N} & -\underline{d}_2 \frac{Q}{V_N} \end{bmatrix}$$

$$A^- = \begin{bmatrix} -(\bar{g} + \bar{a}_1)\frac{Q}{V_1} & 0 & 0 & 0 & \bar{g}\frac{Q}{V_1} \\ \ddots & \ddots & 0 & 0 & 0 \\ 0 & \sum_{k=1}^{i-1} \underline{a}_k \frac{Q}{V_i} & -\left(\bar{g} + \sum_{k=1}^{i} \bar{a}_k\right)\frac{Q}{V_i} & 0 & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \sum_{k=1}^{N-1} \underline{a}_k \frac{Q}{V_N} & -(\bar{g} + \bar{d}_2)\frac{Q}{V_N} \end{bmatrix}$$

Consider the systems $\dot{E}_i^- = A^- E_i^-$, $E_i^-(0) = E_i(0) \geq 0$ (noted L) and $\dot{E}_i^+ = A^+ E_i^+$, $E_i^+(0) = E_i(0) \geq 0$ (noted U) for i=1,2. Matrices A^+ and A^- being cooperative (*i.e.* all nondiagonal elements are non positive) and such that $A^- \leq A(t) \leq A^+$, whatever t, we can deduce that the solution of (L) and (U) are such that $0 \leq E_i^-(t) \leq E_i(t) \leq E_i^+(t)$ whatever t (see for instance (Smith, 1995)). The convergence of the two observers is guaranteed.as soon as A^+ is Hurwitz. Recall the following result in matrix analysis (see (Newman, 1959) and (Laddle, 1976)) : a matrix M with negative main diagonal, for which there exists b_i>0 and a>0 such that $|m_{ii}| - \sum_{j=1, j\neq i}^{n} \frac{b_j}{b_i}|m_{ij}| > a$ (i=1…n) is stable.

A sufficient condition for the stability of A^+ is that there exists positive $l_1 \ldots l_n$ such that : $l_N < l_1 \frac{\underline{a}_1}{\bar{g}}$, $l_1 < l_2 \frac{\underline{a}_1 + \underline{a}_2}{\bar{g} + \bar{a}_1}$,..., $l_{i-1} < l_i \frac{\sum_{k=1}^{i} \underline{a}_k}{\bar{g} + \sum_{k=1}^{i-1} \bar{a}_k}$,..., $l_{N-2} < l_{N-1} \frac{\sum_{k=1}^{N-1} \underline{a}_k}{\bar{g} + \sum_{k=1}^{N-2} \bar{a}_k}$, $l_{N-1} < l_N \frac{\underline{d}_2}{\bar{g} + \sum_{k=1}^{N-1} \bar{a}_k}$ which is equivalent to

$$1 < \frac{\underline{d}_2}{\bar{g} + \sum_{k=1}^{N-1} \bar{a}_k} \prod_{m=1}^{N-1} \left(\frac{\sum_{k=1}^{m} \underline{a}_k}{\bar{g} + \sum_{k=1}^{m-1} \bar{a}_k} \right)$$

which is exactly condition (8).

4. SYNTHESIS OF AN INTERVAL OBSERVER IN PRESENCE OF UNKNOWN INPUTS

When used as wastewater treatment processes, it is the rule rather than the exception that biological systems are subject to input disturbances both in flow rates and concentrations. When controlled, however, it is usually considered that the input flow rates are the control inputs and that the only disturbances are the input concentrations, that are the $b_i S_0$. In other words, the parameters b_i are not constant over the time. Furthermore, because of the presence of suspended solids in the input, it is usually not possible to measure on-line and directly these variations. As a consequence, the input concentrations have to be considered as unknown inputs. Notice that the observers (6) and (7) proposed here above can no longer be implemented since we precisely assumed that the input concentrations were measured on-line. Other approaches must be used. A first solution is to estimate these unmeasured inputs using the available output measurements, but then kinetics must be known. Another method consists in the synthesis of "interval observers" in the spirit described in (Gouzé *et al*, 2000): from known bounds on the inputs, we derive time varying bounds on the state variables to be estimated. With this approach, we no longer need the system to be observable for unknown inputs, as we do not require intervals to converge towards singletons, but at least we provide guaranteed bounds at any time. It is also very easy at the same time to take into uncertainties on $a_i(.)$, $g(.)$, $b_i(.)$ (i=1..N) and $d_j(.)$ (j=1,2), as it is formulated the following proposition:

Proposition 2: When $a_i(.)$, $g(.)$, $b_i(.)$ (i=1..N) and $d_j(.)$ (j=1,2) are unknown but bounded, respectively by $[\underline{a}_i, \overline{a}_i]$, $[0, \overline{g}]$, $[\underline{b}_i, \overline{b}_i]$, $[\underline{d}_2, \overline{d}_2]$, and unknown initial conditions $X_i(0)$ belongs to $[X_{i,0}^-, X_{i,0}^+]$ (i=1...N), then the interval observers (X^+,X^-) defined by $X_i^+ = Y_1(Z_i^+ - S_i)$ (resp $X_i^+ = Y_2(W_i^+ + P_i)$), $X_i^- = Y_1(Z_i^- - S_i)$ (resp. $X_i^- = Y_2(W_i^- + P_i)$) whose dynamics are :

$$\dot{Z}^+ = A^+Z^+ + \begin{pmatrix} \overline{a}_1 \frac{Q}{V_1} \overline{b}_1 S_0 \\ \vdots \\ \overline{a}_i \frac{Q}{V_i} \overline{b}_i S_0 \\ \vdots \\ -\underline{d}_1 \frac{Q}{V_N} S_N \end{pmatrix}, \quad \dot{W}^+ = A^+W^+ + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \underline{d}_1 \frac{Q}{V_N} P_N \end{pmatrix}, \quad \dot{Z}^- = A^-Z^- + \begin{pmatrix} \underline{a}_1 \frac{Q}{V_1} \underline{b}_1 S_0 \\ \vdots \\ \underline{a}_i \frac{Q}{V_i} \underline{b}_i S_0 \\ \vdots \\ -\overline{d}_1 \frac{Q}{V_N} S_N \end{pmatrix} \quad \dot{W}^- = A^-W^- + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \overline{d}_1 \frac{Q}{V_N} P_N \end{pmatrix}$$

with initial conditions $Z^+(0) = \frac{X^+(0)}{Y_1} + S(0)$, $W^+(0) = \frac{X^+(0)}{Y_2} - P(0)$, $Z^-(0) = \frac{X^-(0)}{Y_1} + S(0)$ and $W^-(0) = \frac{X^-(0)}{Y_2} - P(0)$ guarantee that $X_i^-(t) \le X_i(t) \le X_i^+(t)$ (i=1...N) for any time. Furthermore, when condition (C1) is fulfilled, the vector $X^+(t)$-$X^-(t)$ remains bounded.

Proof: Similarly to the proof of Proposition 1, the error vector is solution of a cooperative dynamical systems, whose solutions can be bounded by two autonomous lower and upper dynamical systems.

5. SIMULATIONS

We consider a digestion process that can be modeled by a cascade of three tanks followed by a settler. The detailed model of this process is presented in (Harmand *et al*. 2001). In particular, it is used to produce Volatile Fatty Acid, an easily biodegradable substrate that is useful in denitrification processes. The input loading rates of solid waste, soluble waste and VFA in the reactor i are respectively a_iQX_{sin}, a_iQS_{sin}, a_iQA_{in}. In the present study, we are interested in designing an interval observer for estimating the hydrolytic bacteria concentrations X_j (j=1…4), from the knowledge of lower and upper bounds on the input concentrations a_jX_{sin} (j=1…3) and from the on-line measurements of substrates S_j (j=1…4). The settler being assumed to be perfect, no biomass is present in the outlet. The variable X_4 is the Hydrolytic biomass concentration in the recirculation loop. Using the design procedure described above, it is straightforward to synthesis an interval based observer for the VFA production system. The simulations presented below have been obtained over 250 days. Arbitrary initial conditions have been chosen and an uncertainty on the input substrate concentration of 10 % has been considered. The following results have been obtained :

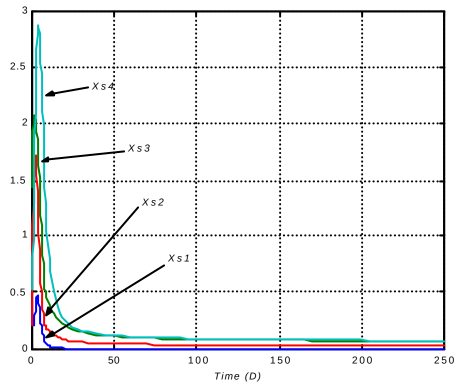

Figure 2a : *Available measurements*

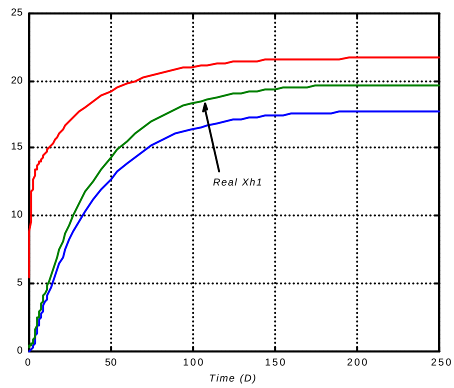

Figure 2b : *Lower/upper bounds and real value of* X_1

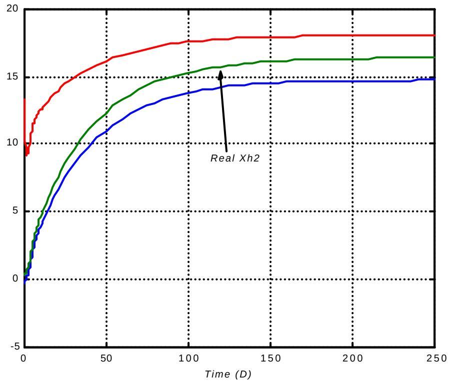

Figure 2c : *Lower/upper bounds and real value of* X_2

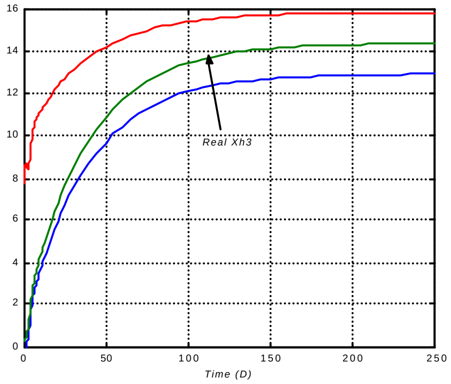

Figure 2d : Lower/upper bounds and real value of X_3

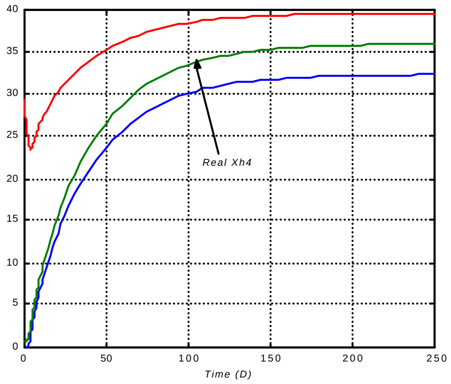

Figure 2e : Lower/upper bounds and real value of X_4

As shown on Figures 2, the approach allows us to estimate lower and upper bounds on unmeasured variables given a number of on-line measurements and the knowledge of lower and upper bounds on unknown inputs. Furthermore, no knowledge at all about the kinetics is necessary. However, the most important drawback we can notice is the low convergence properties of the observer that only depends on the flow rates (see (Bastin and Dochain 1990)). Tentative alternatives have been studied but all of them need for the knowledge - at least partial - of the kinetic terms of the reaction network.

Conclusions

In this paper, an interval observer for complex interconnected biological systems has been proposed. In particular, a sufficient condition for the stability of such an observer was established. The application of this new scheme for estimating biomass concentration in a complex anaerobic system was proposed.

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