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LINEAR PROGRAMMING BASED GAIN SCHEDULING FOR LPV AND PL SYSTEMS

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Abstract: Parametric linear constant and gain-scheduled control systems design are studied for a class of linear parameter varying (LPV) systems and piecewise linear (PL) systems based on stabilization via convexity restrictions. Specifically, the maximum row sum (matrix) norm is utilized to obtain a computationally attractive state space method for the scheduling of simple linear controllers under incomplete modelling information. Furthermore, certain close connections of the studied framework with fuzzy gain scheduling are discussed.

Keywords: control system analysis, stability criteria, robustness

1. INTRODUCTION

Gain scheduling is an important type of nonlinear control in many application areas, such as in flight controls, jet engine control, missile control, vehicular engine control and process control (D. Gangsaas *et al.* 1986). The present work deals mostly with the scheduling of linear controllers for linear parameter varying (LPV) and piecewise linear (PL) systems. A nice discussion of many issues in gain scheduling is given in (Rugh 1991).

In applications it is typically desired that the scheduling system is as simple as possible, without sacrificing performance significantly, and so it is of importance that the scheduled controllers are themselves simple. This is possible to achieve by scheduling methods for reduced, or low, order linear controllers. Based on considerable experience from especially linear quadratic gaussian (LQG) control, it is possible to state that often a well-tuned, simple, reduced order controller attains about the same performance as the optimal full-order (LQG) controller (Mäkilä and Toivonen 1987, Sandelin *et al.* 1991). There is similar experience from reduced order optimal H_∞ control (Nyström *et al.* 1999).

The scheduling of linear controllers is studied here for LPV and PL systems. LPV systems have been quite popular in control design studies, see (Ridgely and McFarland 1999) and the references therein. PL systems are popular for example in the modelling of electronic circuits and systems (Leenaerts and van Bokhoven 1998), and also increasingly in control studies (Pettit and Wellstead 1995).

In the present work discrete-time LPV systems and PL systems are considered with special emphasis on the use of only partial and inaccurate modelling information about the system dynamics in the gain scheduling. This reveals also the close connections of the studied system modeling and gain scheduling concepts with certain fuzzy computing approaches (Årzén and Johansson 1999, Viljamaa 2000). In the present work, we use reduced-order, or reduced complexity, controllers. In addition a special feature is the use of sparse scheduling of such simple controllers, i.e. the number simple controllers that need to be scheduled may be much smaller than the number of dynamical models required to describe the full LPV or PL system dynamics. The controller design approach studied here is based on a certain simple convex design problem that can be solved via linear programming techniques (Rao 1978).

The proofs are omitted due to space limitations, but are available on request (from the authors).

2. PARAMETRIC DESIGN FOR LPV SYSTEMS

It is convenient to start by discussing constant linear controller design for LPV systems.

2.1 *Parametric Linear Controller Design*

Consider the following class of discrete linear parameter varying (LPV) systems

$$x(t+1) = \sum_{k=1}^{K} \theta_k(t)\left(A_k x(t) + B_k u(t)\right) + w(t) \quad (1)$$

$$z(t) = Dx(t) + v(t), \quad (2)$$

where x is the n dimensional state vector, u is the p dimensional input vector, z is an m dimensional output vector, and w and v are disturbance terms. Furthermore, $\{\theta_k(t)\}_{k=1}^{K}$ are possibly time-varying parameters satisfying

$$\sum_{k=1}^{K} \theta_k(t) = 1,\ \theta_k(t) \geq 0,\ k = 1, \ldots, K. \quad (3)$$

This means that x evolves as a, possibly time-varying, convex combination of K ordinary linear state space models.

Consider first a constant (fixed) controller of the form

$$u(t) = Fz(t). \quad (4)$$

This control law does not feed back the parameter values $\theta_k(t)$, and so it does not depend on the availability of these parameter values.

We are here mostly interested in issues of stabilization, so the first natural question is: *Does there exists a controller of the form (4) that can stabilize the LPV system (1) for an arbitrary sequence* $\{\theta_k(t)\}_{k=1}^{K}$, $t = 0, 1, \ldots,$ *satisfying (3)?*

We shall not try to give a general answer to this very difficult question. However, even so there are several interesting things that can be said about this problem. Clearly a necessary condition for a controller of the form (4) to be stabilizing is that it must stabilize each of the constant pairs (A_k, B_k). That is, the matrices $A_k + B_k FD$, $k = 1, \ldots, K$, should all have their eigenvalues strictly inside the unit circle.

The LPV system (1) contains many interesting special cases. One such special case is obtained when $\theta_k(t) = \theta_k(t + K)$ for any k and $t \geq 0$, and $\theta_1(0) = 1$, $\theta_2(1) = 1$, $\ldots$, $\theta_K(K-1) = 1$. This choice corresponds to a simple periodic time-varying system. Let us consider an example.

Example 1. Let us have the matrices

$$A_1 = \begin{pmatrix} -a & -b \\ 0 & -a \end{pmatrix},\ A_2 = \begin{pmatrix} -a & 0 \\ -b & -a \end{pmatrix},$$

where $0 < a < 1$ and $b > 0$. Consider the linear 2-periodic system

$$x(t+1) = A(t)x(t),$$

where $A(2t) = A_1$, $A(2t+1) = A_2$, for any $t = 0, 1, \ldots$.

Take $x(0) = [1\ 1]^T$. It is then easy to compute that

$$[x_1(2t)\ \ x_2(2t)] > [a(a+b)]^t [1\ \ 1],$$

and so the 2-periodic system is certainly unstable if $a(a+b) > 1$ (recall that $0 < a < 1$ and $b > 0$). Take for example $a = 0.1$ and $b = 10$. Then the 2-periodic system is unstable although both A_1 and A_2 are stable matrices (and the eigenvalues of A_1 and A_2 are on a circle of radius 0.1 only, i.e. close to the origin).

It is clear from this example that controller design methods that are based on the locations of the eigenvalues of the matrices $A_k + B_k FD$, $k = 1, \ldots, K$, do not provide a natural approach to address stabilization problems for LPV systems (1). Such methods would be also necessarily very difficult computationally. We shall next state a sufficient stabilization condition for (1) using controllers of the form (4).

Theorem 2. Let an LPV system of the form (1) be given. Let the system be controlled with a parametric controller of the form (4). Then a sufficient condition for a feedback gain matrix F to be stabilizing for an arbitrary $\{\theta_k(t), k = 1, \ldots, K\}_{t\geq 0}$ sequence satisfying (3), is that there exists an invertible $n \times n$ matrix T and an induced matrix norm $\|\cdot\|$ such that

$$\max_{k=1,\ldots,K} \|T(A_k + B_k FD)T^{-1}\| < 1. \quad (5)$$

Remark 3. Clearly the above result applies to a generalized form of the LPV system (1) obtained by relaxing conditions (3) to

$$0 < \sum_{k=1}^{K} \theta_k(t) \leq 1,\ \theta_k(t) \geq 0,\ k = 1, \ldots, K. \quad (6)$$

The notation is in the next result as in the previous one.

Proposition 4. Let T be an invertible matrix. Let

$$S_F(T,\rho) \equiv \{F \mid \|T(A_k + B_k F D)T^{-1}\| < \rho\}, \quad (7)$$

where $k = 1, \dots, K$ and $\|\cdot\|$ is any induced matrix norm and ρ is any positive number. Introduce the function

$$\ell(F) \equiv \max_{k=1,\dots,K} \|T(A_k + B_k F D)T^{-1}\|. \quad (8)$$

Then $S_F(T,\rho)$ is a convex set and $\ell(F)$ is a convex function in the set of all (real) matrices of the same size as F.

Corollary 5. Let the set $S_F(T,1)$ be defined as above and let this set be non-empty. Furthermore, let Λ_k denote the set of feedback gain matrices F such that $\max_i |\lambda_i(A_k + B_k F D)| < 1$, $k = 1, \dots, K$. Let $\cap_{k=1}^{K} \Lambda_k$ consist of at least two disjoint subsets, each of the subsets being a connected set. Then $S_F(T,1)$ belongs to exactly one of these disjoint subsets.

Hence by looking at some induced matrix norm in place of the spectral radius (the largest eigenvalue in absolute value) of the matrices $A_k + B_k F D$, $k = 1, \dots, K$, allows a geometrically much easier approach for addressing stabilization of the LPV system (1).

Let $\|A\|_1 \equiv \max_i \sum_j |A_{ij}|$ denote the (maximum) row sum norm of the square matrix A. Then it is well-known that

$$\|A\|_1 = \sup_{x \neq 0} \frac{\|Ax\|_\infty}{\|x\|_\infty}, \quad (9)$$

where $\|x\|_\infty = \sup_i |x_i|$ denotes the supremum (maximum) norm of a vector. Thus $\|\cdot\|_1$ is indeed an induced matrix norm.

By Theorem 2 the existence of a feedback gain matrix F such that $\max_k \|A_k + B_k F D\|_1 < 1$ implies stability of the LPV system (1) for an arbitrary $\{\theta_k(t), k = 1, \dots, K\}$ sequence satisfying (3). As this condition can be tested as a linear programming feasibility problem (for real matrices A_k, B_k,($k = 1, \dots, K$,) F, and D), it is a convenient condition to use.

2.2 *Gain Scheduling for LPV Systems*

Let us define a LPV controller by generalizing the control law (4) as follows

$$u(t) = F(\varphi(t))z(t), \quad (10)$$

where

$$F(\varphi(t)) = \sum_{\ell=1}^{L} \varphi_\ell(t) F_\ell. \quad (11)$$

Here $\{F_\ell\}_{\ell=1}^{L}$ are constant matrices, $\varphi(t) = \{\varphi_\ell(t)\}_{\ell=1}^{L}$, and the measurable (scheduling) parameters satisfy

$$\sum_{\ell=1}^{L} \varphi_\ell(t) = 1, \ \varphi_\ell(t) \geq 0, \ \ell = 1, \dots, L. \quad (12)$$

Proposition 6. Consider the LPV system (1) satisfying (6) and controlled with (10)–(11). Let there exist an induced matrix norm $\|\cdot\|$ such that

$$\max_{k,\ell} \|A_k + B_k F_\ell D\| < 1. \quad (13)$$

Then the closed loop system is (asymptotically) stable.

We have so far made no assumptions on the interconnection structure of the parameters $\{\theta_k(t)\}$. For practical applications it is relevant to assume certain sparseness properties about which of the parameters $\{\theta_k(t)\}$ can be simultaneously non-zero. This allows interesting gain scheduling design and analysis.

2.3 *Gain Scheduling for Localized LPV Systems*

Let us define the $K \times K$ symmetric matrix Γ so that its entries can take the (binary) values 0 or 1 only. Define $\Gamma_{ii} = 1$ for any i. Furthermore, for $i \neq j$, $\Gamma_{ij} = 0$ means that $\theta_i(t)$ and $\theta_j(t)$ can not be simultaneously non-zero whilst $\Gamma_{ij} = 1$ means that they can. We shall call Γ *the interconnection topology matrix* of the LPV system (1).

Let $q_1(i) = \min j$ such that $\Gamma_{ij} = 1$. Furthermore let $q_2(i) = \max j$ such that $\Gamma_{ij} = 1$. Introduce the quantity

$$\text{band}(\Gamma) = \max_i \{q_2(i) - q_1(i) + 1\}. \quad (14)$$

If $\text{band}(\Gamma) \ll K$ then clearly Γ is a sparse band matrix. In this case only a small set of neighbouring index values k can have non-zero parameter values $\theta_k(t)$ at any given time t. We say that the LPV system (1) is then a *localized* system.

Let $r(t)$ define an integer valued indicator function with the property that it gives as its value the index k of ONE non-zero $\theta_k(t)$ parameter at time t. Let now $\beta \leq K$ define an upper bound for $\text{band}(\Gamma)$. Hence any index ℓ such that $\theta_\ell(t)$ is non-zero at time t satisfies

$$\ell \in P(t) \equiv \big\{\max\{1, r(t) - \beta + 1\}, \dots, \\ \min\{r(t) + \beta - 1, K\}\big\}. \quad (15)$$

We can interpret the index set $P(t)$ of consecutive integers as the information available about the active local dynamics of the LPV system (1) at time t. That is (cf. (1))

$$x(t+1) = \sum_{k \in P(t)} \theta_k(t)(A_k x(t) + B_k u(t)) + w(t). \quad (16)$$

The assumption that the LPV system is localized, motivates a sparse, local, gain scheduling design as follows.

For simplicity we shall only consider gain scheduling design with as few design parameters as possible. Hence we restrict us to *uniform*, 1-neighbour overlapping, designs by considering the quantity

$$\rho_M(W) = \max_{\substack{i=1+(j-1)W/2 \\ j=1,\dots,J}} \min_{\|F_j\|_1 \le M} \max_{i \le k \le i+W-1} \|A_k + B_k F_j D\|_1. \quad (17)$$

Here $W(< K)$ is an even positive integer (for the sake of notational simplicity) and the maximum value, J, of j is the smallest integer $j \ge 1$ such that $(j+1)W/2 \ge K$. Furthermore, the largest value for k that is allowed above, is K (for notational simplicity we have not included this restriction in the expression for $\rho_M(W)$). Introduce the notation

$$I(j) = \{k \in \mathcal{Z}_+ \mid 1 + (j-1)(W/2) \le k \le (j+1)(W/2)\},\ 1 \le j < J,$$
$$I(J) = \{k \in \mathcal{Z}_+ \mid 1 + (J-1)(W/2) \le k \le K\},$$

where $\mathcal{Z}_+$ denotes the (usual ordered) set of all positive integers. The set of consequtive k values from 1 to K is thus divided, when computing $\rho_W(M)$, into J sets of consequtive integers $I(j)$, $j = 1, \dots, J$. The set $I(j)$ has nonzero intersections only with its left neighbour $I(j-1)$ for $1 < j \le J$ and with its right neighbour $I(j+1)$ for $1 \le j < J$. Each feedback gain matrix F_j is designed locally for k values in $I(j)$ and each $I(j)$ has W elements, except possibly the set $I(J)$ may have fewer. Each k value in $I(j)$, $1 < j < J$, belongs either to the left neighbour $I(j-1)$ or to the right neighbour $I(j+1)$ of $I(j)$, but not to both of them. That is, the intersection $I(j-1) \cap I(j+1) = \emptyset$ is empty for $1 < j < J$. Hence the terminology uniform 1-neighbour overlapping design.

Note that $\rho_M(W)$ can be computed by solving a set of linear programming problems for each given M and W value. The task is to find, if possible, a small M value and a large W value such that

$$\rho_M(W) \le \rho < 1, \quad (18)$$

where $\rho < 1$ is a user chosen design parameter. The smaller M and ρ values can be found, the better robustness properties are implied. A larger W value implies that a simpler gain scheduling controller is obtained. Let $\{F_j\}_{j=1}^J$ define the local controllers obtained whilst computing a satisfactory $\rho_M(W)$ value.

Note that $I(j)$ is the set of consecutive index values k for which the pair $A_k + B_k F_j D$ is known (designed) to be a stable matrix when $\rho_M(W) < 1$.

The scheduled gain matrix $F(t)$ in the control law

$$u(t) = F(t)z(t) \quad (19)$$

is defined as follows. (Recall the definition of $z(t)$ in (1).)

(1) If $P(t) \subset I(1)$ and $P(t) \not\subset I(2)$ then $F(t) = F_1$.
(2) If $P(t) \subset I(j)$ and $P(t) \not\subset I(j+1)$ for some $1 < j < J$ then $F(t) = \lambda_t F_j + (1-\lambda_t) F_{j+1}$ for any $0 \le \lambda_t \le 1$.
(3) If $P(t) \subset I(J)$ and $P(t) \not\subset I(J-1)$ then $F(t) = F_J$.

It is readily seen that the gain scheduling controller (19) stabilizes the localized LPV system if $\rho_M(W) < 1$ AND $\beta \le [(W/4)+1]_-$, where $[y]_-$ denotes the largest integer j satisfying $j \le y$ (y is here a real number). We shall summarize some properties of the gain scheduling system in the form of a robustness result. The notation is as in the above construction of the gain scheduling controller (19).

Theorem 7. Consider a localized LPV system of the form (1). Let a gain scheduled controller (19) be designed for this system as described above. Let $\rho_M(W) < 1$ and $\beta < [(W/4)+1]_-$. The designed controller will then also stabilize any LPV system obtained from the system (1) by replacing the pairs (A_k, B_k) with any pairs $(A_k + \triangle A_k, B_k + \triangle B_k)$, $k = 1, \dots, K$, satisfying

$$\sup_k \|\triangle A_k\|_1 + \sup_k \|\triangle B_k\|_1 M \|D\|_1 < 1 - \rho_M(W). \quad (20)$$

If the LPV system (1) satisfies in addition the conditions

$$\|A_{k+1} - A_k\|_1 \le \epsilon_A \quad (21)$$
$$\|B_{k+1} - B_k\|_1 \le \epsilon_B, \quad (22)$$

for some small positive numbers ϵ_A and ϵ_B, then it can be often expected that a simple uniform gain scheduling controller can be designed for the localized system.

Note that there is no advantage in trying to write the controller $F(t)$ in (19) as some function $F(t) = G(P(t))$ of the information $P(t)$. In fact, we could even choose the parameter λ_t in a fuzzy, or alternatively random, manner so that $F(t)$ need not even be (mathematically speaking) an ordinary function. Hence we see that if-then rule based controllers arise naturally in gain scheduling for localized LPV systems. We see thus that fuzzy computing type ideas arise naturally in this context.

3. PIECEWISE LINEAR SYSTEMS

Piecewise linear (PL) systems are especially popular in the modelling of circuits. It is convenient to consider here discrete-time PL systems defined in state space form as

$$x(t+1) = A(y)x(t) + B(y)u(t) + w(t) \quad (23)$$
$$y(t) = Cx(t), \quad (24)$$

where x is the n dimensional state vector, u is the p dimensional input vector, y is a scalar variable, w is a disturbance term, and

$$\begin{aligned} &y(t) \leq y_1 \Rightarrow A(y) = A_1,\ B(y) = B_1, \\ &y(t) \in (y_{k-1}, y_k] \Rightarrow A(y) = A_k,\ B(y) = B_k, \\ &\qquad k = 2, \ldots, K-1 \\ &y(t) > y_{K-1} \Rightarrow A(y) = A_K,\ B(y) = B_K. \quad (25) \end{aligned}$$

Here $K \geq 2$ is an integer and (A_k, B_k), $k = 1, \ldots, K$, are matrices of appropriate dimensions and $\{y_k\}_{k=1}^{K-1}$ are real numbers satisfying $y_k < y_{k+1}$ for $k = 1, \ldots, K-2$. (Clearly for $K = 2$ only the first and last rules above for defining $A(y)$ and $B(y)$ are present.)

The dynamics of PL systems can be very complex. PL systems are clearly closely related to LPV systems (1). Furthermore there is a close connection between LPV systems (1) and PL systems from the point of view of convex parametric design. Let us start with a simple example.

Example 8. Consider the PL system

$$x(t+1) = A(y)x(t)$$
$$y(t) = [1\ \ 1]x(t),$$

where

$$y(t) \leq 0 \Rightarrow A(y) = A_1$$
$$y(t) > 0 \Rightarrow A(y) = A_2,$$

where A_1 and A_2 are the 2×2 matrices given in Example 1.

Take $x(0) = [1\ \ 1]^T$. Then as in Example 1

$$[x_1(2t)\ \ x_2(2t)] > [a(a+b)]^t[1\ \ 1].$$

Hence the PL system of this example is certainly unstable if $a(a+b) > 1$ although A_1 and A_2 are stable matrices (recall that $0 < a < 1$ by assumption).

Let the PL system (23)–(25) be controlled with a parametric controller of the form (4). That is, let

$$u(t) = Fz(t), \quad (26)$$

where

$$z(t) = Dx(t) + v(t). \quad (27)$$

Note that the scalar variable y in the definition of the PL system could be a component in z.

Clearly due to the formal similarities between LPV systems and PL systems, it is quite easy to derive analogous results to those in Section 2 for PL systems.

We shall next consider gain scheduling design for a PL system based on a (typically small) set of approximate models $(\hat{A}(\sigma_i), \hat{B}(\sigma_i))$, $i = 1, \ldots, L$. Here $\{\sigma_i\}$ are real numbers satisfying $\sigma_i < \sigma_{i+1}$ for $i = 1, \ldots, L-1$. Let $L < K$.

Let us design L gain matrices F_i, each F_i obtained as a minimizer to

$$g_i(M) \equiv \min_{\|F\|_1 \leq M} \|\hat{A}(\sigma_i) + \hat{B}(\sigma_i)FD\|_1, \quad (28)$$

where $i = 1, \ldots, L$ and $M > 0$ is a design parameter. Let $\sup_i g_i(M) < 1$ and

$$\max_i \|\hat{A}(\sigma_i) - A_{k(i)}\|_1 \leq \delta_A \quad (29)$$
$$\max_i \|\hat{B}(\sigma_i) - B_{k(i)}\|_1 \leq \delta_B, \quad (30)$$

where $k(i)$ is a minimizer of $\min_k |\sigma_i - y_k|$ and δ_A and δ_B are non-negative numbers.

Furthermore let

$$\max_k \frac{\|A_{k+1} - A_k\|_1}{|y_{k+1} - y_k|} \leq \xi_A \quad (31)$$
$$\max_k \frac{\|B_{k+1} - B_k\|_1}{|y_{k+1} - y_k|} \leq \xi_B \quad (32)$$

for some non-negative number ξ_A and ξ_B. Here y_K is a positive number satisfying $y_K > y_{K-1}$.

Let us now determine for each i the range of k values for which it is guaranteed that $\|A_k + B_kF_iD\|_1 < 1$, $i = 1, \ldots, L$. We get that $\|A_k + B_kF_iD\|_1 < 1$ if

$$|y_k - y_{k(i)}| < \frac{1 - g_i(M) - \delta_A - \delta_B M\|D\|_1}{\xi_A + \xi_B M\|D\|_1} \equiv \Sigma_i. \quad (33)$$

Let

$$\max_{1 \leq k \leq K-1} |y_{k+1} - y_k| \leq \epsilon_y, \quad (34)$$

where $\epsilon_y > 0$.

Let us define the numbers $\{a_i, b_i\}_{i=1}^{L}$ so that $a_i < a_{i+1}$, $b_i < b_{i+1}$ for $1 \leq i \leq L-1$, and $a_i < \sigma_i < b_i$, $i = 1, \ldots, L$. Furthermore, let $a_i < b_{i-1} < \sigma_i < a_{i+1}$, $1 < i < L$. Let us assume that $a_1 \le y_1 < \sigma_1$ and $\sigma_L < y_{K-1} \le b_L$. Furthermore, it assumed that $\sigma_i - a_i < \Sigma_i - c \times \epsilon_y/2$ and $b_i - \sigma_i < \Sigma_i - c \times \epsilon_y/2$ for $i = 1, \dots, L$, where $c > 0$. We shall later try to choose a value for c based on stabilization considerations for the gain scheduled closed loop system. Recall that the quantities $\{\Sigma_i\}_{i=1}^{L}$ are defined in (33).

The gain-scheduled controller will be of the form

$$u(t) = F(y(t))z(t). \quad (35)$$

The gain scheduling of $F(t)$ is done as follows. (We assume below that $L \ge 2$.)

(1) If $y(t) < a_2$ then $F(y(t)) = F_1$.
(2) If $y(t) \in [a_{i+1}, b_i)$ $(i < L)$ then $F(y(t)) = \lambda_t F_i + (1-\lambda_t)F_{i+1}$ for any $0 \le \lambda_t \le 1$.
(3) If $y(t) \in [b_{i-1}, a_{i+1})$ $(1 < i < L)$ then $F(y(t)) = F_i$.
(4) If $y(t) \ge b_{L-1}$ then $F(y(t)) = F_L$.

Note that the PL system (23) can be naturally interpreted as a localized system as only one of the pairs (A_k, B_k), $k = 1, \dots, K$, is active at any given time t.

We can now state the following theorem.

Theorem 9. Consider the PL system (23). Let the models $(\hat{A}(\sigma_i), \hat{B}(\sigma_i))$, $i = 1, \dots, L$, be used to design the gain-scheduled controller $u(t) = F(y(t))z(t)$ to control the PL system (23). (Here $L < K$.) Let (29)–(32) hold for given non-negative numbers δ_A, δ_B, ξ_A and ξ_B. Let (34) hold for a given non-negative number ϵ_y. Let $M > 0$ be a given number. Let the numbers (a_i, b_i), $i = 1, \dots, L$ be defined as above and let them satisfy also the conditions

$$\sigma_i - a_i < \Sigma_i - (3/2)\epsilon_y \text{ and}$$
$$b_i - \sigma_i < \Sigma_i - (3/2)\epsilon_y, \; i = 1, \dots, L, \quad (36)$$

where Σ_i is defined by (33).

Then clearly the gain-scheduled controller $u(t) = F(y(t))z(t)$ stabilizes asymptotically the PL system (23).

4. CONCLUSIONS

The design of gain-scheduled controllers from simple linear controllers has been studied for linear parameter varying (LPV) systems and for piecewise linear (PL) systems. Via suitably augmented state space models, it is possible to consider with the studied approach gain scheduling of PID controllers and of arbitrary linear dynamic state space controllers, cf. parametric design of arbitrary linear controllers (Mäkilä and Toivonen 1987). Hence the studied approach provides a reasonably unified approach for gain scheduling. This approach is based on a computationally attractive linear programming design technique.

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