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DESIGN OF ROBUST TRACKING CONTROLLERS FOR PERTURBED INTERCONNECTED SYSTEMS

Jia-Ming Hsiao **Chih-Chiang Cheng** [1]

Department of Electrical Engineering, National Sun Yat-Sen University, No. 70, Lien-Hai Road, Kaohsiung, 804, Taiwan, Republic of China

Abstract: This paper is concerned with the design of a model reference adaptive variable structure control (MRAVSC) scheme for a class of perturbed large-scale systems with mismatched uncertainties and time-varying delay interconnections in order to solve robust state tracking problems. This control scheme with an adaptive mechanism embedded and a perturbation estimation process can achieve robust state tracking, neither the knowledge of upper-bound of mismatched perturbations nor the knowledge of exact function of time-delay in the interconnections is required. Furthermore, the overall controlled large-scale system is uniformly ultimately bounded. Finally, an example is demonstrated to show the feasibility of the proposed control scheme. *Copyright © 2002 IFAC*

Keywords: Variable Structure Control, Adaptive Control, Interconnected Systems, Time delay

1. INTRODUCTION

It has been shown that variable structure control (VSC) offers the robustness of stability and the property of insensitivity to matched system parameter variations and external disturbances (Yeung *et al.*, 1993; Żak and Hui, 1993; Walcott and Żak, 1988; Slotine and Sastry, 1983; Drazenovic, 1969). If mismatched perturbation exists, however, the desired tracking precision of traditional VSC in general may not be always achieved. On the other hand, the traditional VSC pays the price of chattering phenomenon and the requirement of knowledge of upper-bounds of perturbations in order to preserve the advantage of robustness against perturbations (Yeung *et al.*, 1993; Żak and Hui, 1993; Walcott and Żak, 1988; Slotine and Sastry, 1983; Drazenovic, 1969). In the steady state, chattering phenomenon serves a source of exciting the un-modeled high-frequency dynamics of system (Hung *et al.*, 1993; Slotine and Sastry, 1983). Since the knowledge of upper-bounds of perturbations is not available or too expensive to access in many practical applications (Żak and Hui, 1993), the requirement of these information may become a serious problem in the application of conventional VSC. Recently, a perturbation estimation scheme was proposed by Cheng *et al.* (2001) so that the knowledge of upper-bounds of perturbations is not required.

For large-scale systems, a few research results in which VSC technique is employed exist due to the complexity of control systems and the effects of interconnections. In such cases, the main challenge is in handling the interconnected terms among each subsystem. Matthews *et al.* (1988) and Richter *et al.* (1982) applied VSC technique to control a large-scale system with the requirement of exact knowledge of all local systems' parameters. Hsu (1997; 1998) and Wang *et al.* (1993) proposed a robust decentralized VSC for large-

[1] Corresponding author.
E-Mail: chengcc@ee.nsysu.edu.tw
Tel: +886-7-5252000 ext.4133.

scale systems with the requirement of information of upper-bound of perturbations. Chiang (1995) presented a model reference control for a large-scale variable structure adaptive control system. It is noted that all the approaches mentioned above do not consider time-delay interconnections. Chou and Cheng (2000) proposed a decentralized model following VSC scheme for perturbed large-scale system with time-delay interconnections, however, the knowledge of upper-bound of perturbation is needed.

Time-delay commonly exists in various practical engineering systems due to mechanism or the finite speed of signal processing, and its existence is frequently a source of instability. In past years, many different approaches are investigated to solve the problem of robust stabilization or tracking for dynamic system with time-delay, for example: Yu (1983), Cheres *et al.* (1989), Luo and Sen (1993).

The objective of this paper is to present a design method of decentralized robust state tracking controllers for large-scale systems with time-varying delay interconnections and mismatched uncertainties. Both a perturbation estimation process and an adaptive control mechanism are embedded in the controller, so that the proposed control scheme is designed without the requirement of the knowledge of upper-bounds of perturbations, and the chattering phenomenon can also be reduced. The resultant control scheme can achieve the property of uniformly ultimate boundness.

2. SYSTEM DESCRIPTIONS AND ASSUMPTIONS

Consider a class of perturbed large-scale systems, which are composed of N-linked subsystems with time-varying delay in the interconnections. The dynamic equation of each subsystem is described as

$$\begin{aligned}\dot{\mathbf{x}}_i(t) = &\mathbf{A}_i\mathbf{x}_i(t) + \mathbf{B}_i\mathbf{u}_i(t) + \sum_{j=1,j\neq i}^{N} \mathbf{A}_{i,j} \times \\ &\mathbf{x}_j(t-h_{ij}(t)) + \mathbf{d}_i(t,\mathbf{x}_i), \qquad (1)\\ \mathbf{x}_i(t) = &\boldsymbol{\varphi}_i(t),\ t\in[-\bar{h},0],\ i=1,2,\cdots,N,\end{aligned}$$

where $\mathbf{x}_i(t) \in \Re^{n_i}$, $\mathbf{u}_i(t) \in \Re^{m_i}$ are the state variable and control input vector of the i-th subsystem, respectively. The vector $\mathbf{d}_i(t,\mathbf{x}_i)$ is the unknown, lumped perturbation due to system's parameter variations, e.g., $\Delta\mathbf{A}_i(t,\mathbf{x}_i)\mathbf{x}_i(t)$, nonlinearity, modeling inaccuracy and external disturbances. $\mathbf{A}_i$, $\mathbf{A}_{ij}$, $\mathbf{B}_i$ are known and real matrices with appropriate dimensions. The unknown scalar function $h_{ij}(t)$ denotes unknown non-negative, continuous and bounded time-varying delay satisfying

$$0 \le h_{ij}(t) \le \bar{h} < \infty,\ \forall i,j,\ i\neq j,$$

where $\bar{h}$ is a known positive constant. $\boldsymbol{\varphi}_i(t)$ is an arbitrarily known continuous state vector-valued function for specifying initial conditions. For the purpose of model reference, the local reference model of the i-th subsystem is given by

$$\dot{\mathbf{x}}_{mi}(t) = \mathbf{A}_{mi}\mathbf{x}_{mi}(t) + \mathbf{B}_{mi}\mathbf{r}_i(t), \qquad (2)$$

$i = 1,\cdots,N$, where $\mathbf{x}_{mi}(t) \in \Re^{n_i}$ is the state vector of the i-th reference model, $\mathbf{r}_i(t) \in \Re^{m_i}$ is the piecewise continuous and bounded reference input vector of the i-th reference model. $\mathbf{A}_{mi}$, $\mathbf{B}_{mi}$ are real constant matrices with appropriate dimensions, and $\mathbf{A}_{mi}$ is stable. It is also assumed that the following assumptions are valid:

A1. $(\mathbf{A}_i,\mathbf{B}_i)$ is a controllable pair.
A2. The state vector $\mathbf{x}_i$ of the i-th subsystem is locally measurable for all time.
A3. $\mathbf{d}_i(t,\mathbf{x}_i)$ is continuously differentiable in $\mathbf{x}_i$ and piecewise continuous in t. There exist two unknown constant q_{i0} and q_{i1} such that $\|\mathbf{d}_i(t,\mathbf{x}_i)\| \le q_{i0} + q_{i1}\|\mathbf{x}_i\|$.
A4. There exist two unknown constant g_{i0} and g_{i1} for all $h_{ij}(t) \in [0,\bar{h}]$ such that $\|\mathbf{x}_{ij}(t-h_{ij})\| \le g_{i0} + g_{i1}x_{i,sup}$, $i\neq j$, where $x_{i,sup} \triangleq \sup_{t-\bar{h}<\tau<t}\|\mathbf{x}_i(\tau)\|$.

The main objective of this paper is to design a decentralized robust tracking controller so that the state variable $\mathbf{x}_i$ of the i-th subsystem can track the state variable $\mathbf{x}_{mi}$ of the i-th reference model in spite of perturbations, and the robustness of overall system's stability can be guaranteed.

3. DESIGN OF DECENTRALIZED ROBUST TRACKING CONTROLLERS

The design procedures of the proposed controller are described as follows.
Step 1: Design of sliding surface
For the state variable $\mathbf{x}_i$ of the i-th subsystem, the tracking error vector is defined as

$$\mathbf{e}_i \triangleq \mathbf{x}_i - \mathbf{x}_{mi},\ i=1,\cdots,N. \qquad (3)$$

From (1), (2) and (3), one can obtain

$$\begin{aligned}\dot{\mathbf{e}}_i = &\mathbf{A}_{mi}\mathbf{e}_i + (\mathbf{A}_i-\mathbf{A}_{mi})\mathbf{x}_i + \mathbf{B}_i\mathbf{u}_i\\ &-\mathbf{B}_{mi}\mathbf{r}_i + \sum_{j=1,j\neq i}^{N}\mathbf{A}_{i,j}\mathbf{x}_j(t-h_{ij}(t))\\ &+\mathbf{d}_i(t,\mathbf{x}_i,\mathbf{u}_i),\ i=1,\cdots,N. \qquad (4)\end{aligned}$$

For achieving the objective of state tracking, it is desired that $\mathbf{e}_i \to 0$ as $t\to\infty$. Therefore the sliding surface of each subsystem is designed as

$$\boldsymbol{\sigma}_i = \mathbf{C}_i\mathbf{e}_i - \int_0^t \mathbf{C}_i(\mathbf{A}_{mi}+\mathbf{B}_i\mathbf{K}_i)\mathbf{e}_i d\tau, \qquad (5)$$

where $\mathbf{C}_i \in \Re^{m_i \times n_i}$ is a constant matrix with full rank, and is chosen such that $\mathbf{C}_i\mathbf{B}_i$ is nonsingular as well as $\mathbf{C}_i$ contains no columns whose entries are all zero. The matrix $\mathbf{K}_i \in \Re^{m_i \times n_i}$ is designed to satisfy the inequality

$$\max\left[Re(\lambda(\mathbf{A}_{mi} + \mathbf{B}_i\mathbf{K}_i))\right] < 0. \tag{6}$$

Step 2: Controller design
Substituting (4) into the derivative of (5) yields

$$\dot{\boldsymbol{\sigma}}_i = \boldsymbol{\Psi}_i + \mathbf{C}_i\mathbf{B}_i\mathbf{u}_i + \Delta\mathbf{P}_i, \tag{7}$$

where

$$\boldsymbol{\Psi}_i \triangleq \mathbf{C}_i[(\mathbf{A}_i - \mathbf{A}_{mi})\mathbf{x}_i - \mathbf{B}_{mi}\mathbf{r}_i] - \mathbf{C}_i\mathbf{B}_i\mathbf{K}_i\mathbf{e}_i,$$

and

$$\Delta\mathbf{P}_i \triangleq \mathbf{C}_i\left[\sum_{j=1,j\neq i}^{N}\mathbf{A}_{i,j}\mathbf{x}_j(t - h_{ij}(t)) + \mathbf{d}_i(t,\mathbf{x}_i)\right] \tag{8}$$

is a new lumped perturbation vector. Now one can design the proposed controller as

$$\begin{aligned}\mathbf{u}_i \triangleq (\mathbf{C}_i\mathbf{B}_i)^{-1}(&-\boldsymbol{\Psi}_i - \mathbf{K}_{si}\boldsymbol{\sigma}_i \\ &-\Delta\mathbf{P}_{i,est} - \mathbf{u}_{i,adp}),\end{aligned} \tag{9}$$

where $\mathbf{K}_{si} \triangleq diag[k_{sil}] \in \Re^{m_i \times n_i}$, $l = 1, \cdots, m_i$, is a constant gain matrix, $k_{sil} > 0$, $\Delta\mathbf{P}_{i,est} \triangleq \left[\Delta p_{i1,est} \ \cdots \ \Delta p_{im_i,est}\right]^T$ is the estimation of $\Delta\mathbf{P}_i$. The vector $\mathbf{u}_{i,adp} \triangleq \left[u_{i1,adp} \ \cdots \ u_{im_i,adp}\right]^T$ is adaptive control effort used to overcome the unknown upper-bound of perturbation estimation error, and is designed as

$$\begin{aligned}u_{ij,adp} = \Bigg(&\hat{\gamma}_{ij0} + \hat{\gamma}_{ij1}\|\mathbf{x}_i\| \\ &+ \sum_{k=2,k\neq i+1}^{N+1}\hat{\gamma}_{ijk}x_{k-1,sup}\Bigg) sat\left(\frac{\sigma_{ij}}{\beta_{ij}}\right),\end{aligned}$$

$j = 1, \cdots, m_i$, where β_{ij} is a small positive constant. The adaptive gains are given by

$$\dot{\hat{\gamma}}_{ij0} = \frac{1}{\alpha_{i0}}(-\eta_{ij0}\gamma_{ij0} + |\sigma_{ij}|), \tag{10}$$

$$\dot{\hat{\gamma}}_{ij1} = \frac{1}{\alpha_{i1}}(-\eta_{ij1}\gamma_{ij1} + |\sigma_{ij}|\|\mathbf{x}_i\|), \tag{11}$$

$$\begin{aligned}\dot{\hat{\gamma}}_{ijk} &= \frac{1}{\alpha_{i2}}(-\eta_{ijk}\gamma_{ijk} + |\sigma_{ij}|x_{k-1,sup}), \\ &\quad k = 2, \cdots, N+1;\ k \neq i+1,\end{aligned} \tag{12}$$

with zero initial conditions, where α_{i0}, α_{i1}, α_{i2}, η_{ij0}, η_{ij1} and η_{ijk} are all positive constants specified by the designer.

Step 3: Estimation of $\Delta\mathbf{P}_i$
First define a nominal sliding function vector as

$$\begin{aligned}\boldsymbol{\sigma}_{i,n}(t) \triangleq &\int_0^t(-\mathbf{K}_{si}\boldsymbol{\sigma}_{i,n} - \Delta\mathbf{P}_{i,est} - \mathbf{u}_{i,adp})d\tau \\ &+\boldsymbol{\sigma}_{i,n}(0),\end{aligned} \tag{13}$$

where $\boldsymbol{\sigma}_{i,n}(t) = \left[\sigma_{i1,n}(t) \ \cdots \ \sigma_{im_i,n}(t)\right]^T$. From (7) and (9), one can obtain

$$\dot{\boldsymbol{\sigma}}_i = -\mathbf{K}_{si}\boldsymbol{\sigma}_i + \Delta\mathbf{P}_i - \Delta\mathbf{P}_{i,est} - \mathbf{u}_{i,adp}, \tag{14}$$

Let $\Delta\mathbf{I}_i(t) \triangleq \boldsymbol{\sigma}_i(t) - \boldsymbol{\sigma}_{i,n}(t)$. From (13) and (14), one can obtain

$$\Delta\mathbf{P}_i(t) = \left(\frac{d}{dt} + \mathbf{K}_{si}\right)\Delta\mathbf{I}_i(t). \tag{15}$$

(15) implies that one can estimate $\Delta\mathbf{P}_i$ by utilizing a filter with transfer function $\frac{s}{1+\epsilon_i s}$ (Cheng *et al.*, 2001; Chen, 1993), which means that

$$\Delta\mathbf{P}_{i,est}(s) = \left(\frac{s}{1+\epsilon_i s} + \mathbf{K}_{si}\right)\Delta\mathbf{I}_i(s), \tag{16}$$

where ϵ_i is a small positive constant.

4. ROBUSTNESS OF SYSTEM'S STABILITY

The robustness of the proposed control system's stability is stated in the following theorem.

Theorem: Consider the large-scale system (1) with the sliding function (5) for each subsystem and the proposed controller (9). If all the aforementioned assumptions are valid, and the perturbation estimation error satisfies

$$\begin{aligned}|\Delta p_{ij} - \Delta p_{ij,est}| \leq &\ \gamma_{ij0} + \gamma_{ij1}\|\mathbf{x}_i\| \\ &+ \sum_{k=2,k\neq i+1}^{N+1}\gamma_{ijk}x_{k-1,sup},\end{aligned} \tag{17}$$

$i = 1, \cdots, N$, $j = 1, \cdots, m_i$, where γ_{ijl}, $l \in \{0, \cdots, N+1\}$, $l \neq i+1$, are unknown positive constants, then

(a) each sliding function σ_{ij} of the i-th subsystem is globally uniformly ultimately bounded;
(b) each tracking error $\mathbf{e}_i$ of the i-th subsystem will be bounded.

Furthermore, the stability of overall controlled system is guaranteed by the proposed control scheme. □

Proof: (a) For each subsystem, define a Lyapunov candidate function $\mathbf{V}_i = \left[V_{i1} \ \cdots \ V_{im_i}\right]^T$, where

$$V_{ij} = \frac{1}{2}\left(\sigma_{ij}^2 + \sum_{k=0,k\neq i+1}^{N+1}\alpha_{ijk}\tilde{\gamma}_{ijk}^2\right),$$

and $\tilde{\gamma}_{ijk} \triangleq \hat{\gamma}_{ijk} - \gamma_{ijk}$ are the errors of adaptive gains. Differentiating V_{ij} with respect to time and using (14) yields

$$
\begin{aligned}
\dot{V}_{ij} &= \sigma_{ij}(-k_{sij}\sigma_{ij} + \Delta p_{ij} - \Delta p_{ij,est} - u_{ij,adp}) \\
&\quad + \sum_{k=0,k\neq i+1}^{N+1} \alpha_{ijk}\tilde{\gamma}_{ijk}\dot{\tilde{\gamma}}_{ijk} \\
&= -k_{sij}\sigma_{ij}^2 + \sigma_{ij}(\Delta p_{ij} - \Delta p_{ij,est}) - \sigma_{ij} \times \\
&\quad sat\left(\frac{\sigma_{ij}}{\beta_{ij}}\right)\Bigg(\hat{\gamma}_{ij0} + \hat{\gamma}_{ij1}\|\mathbf{x}_i\| \\
&\quad + \sum_{k=2,k\neq i+1}^{N+1} \hat{\gamma}_{ijk}x_{k-1,sup}\Bigg) + \tilde{\gamma}_{ij0}(-\eta_{ij0} \times \\
&\quad \hat{\gamma}_{ij0} + |\sigma_{ij}|) + \tilde{\gamma}_{ij1}(-\eta_{ij1}\hat{\gamma}_{ij1} + |\sigma_{ij}|\|\mathbf{x}_i\|) \\
&\quad + \sum_{k=2,k\neq i+1}^{N+1} \tilde{\gamma}_{ijk}(-\eta_{ijk}\hat{\gamma}_{ijk} + |\sigma_{ij}|x_{k-1,sup}).
\end{aligned}
$$

If $|\sigma_{ij}| > \beta_{ij}$, then using (10), (11), (12) and (17), the preceding equation can be further derived as

$$
\begin{aligned}
\dot{V}_{ij} &= -k_{sij}\sigma_{ij}^2 + \sigma_{ij}(\Delta p_{ij} - \Delta p_{ij,est}) - |\sigma_{ij}| \times \\
&\quad \left(\hat{\gamma}_{ij0} + \hat{\gamma}_{ij1}\|\mathbf{x}_i\| + \sum_{k=2,k\neq i+1}^{N+1} \hat{\gamma}_{ijk}x_{k-1,sup}\right) \\
&\quad +\tilde{\gamma}_{ij0}(-\eta_{ij0}\hat{\gamma}_{ij0} + |\sigma_{ij}|) \\
&\quad +\tilde{\gamma}_{ij1}(-\eta_{ij1}\hat{\gamma}_{ij1} + |\sigma_{ij}|\|\mathbf{x}_i\|) \\
&\quad + \sum_{k=2,k\neq i+1}^{N+1} \tilde{\gamma}_{ijk}(-\eta_{ijk}\hat{\gamma}_{ijk} + |\sigma_{ij}|x_{k-1,sup}) \\
&\leq -k_{sij}\sigma_{ij}^2 + |\sigma_{ij}|\Bigg(\gamma_{ij0} + \gamma_{ij1}\|\mathbf{x}_i\| \\
&\quad + \sum_{k=2,k\neq i+1}^{N+1} \gamma_{ijk}x_{k-1,sup}\Bigg) - |\sigma_{ij}| \times \\
&\quad \left(\hat{\gamma}_{ij0} + \hat{\gamma}_{ij1}\|\mathbf{x}_i\| + \sum_{k=2,k\neq i+1}^{N+1} \hat{\gamma}_{ijk}x_{k-1,sup}\right) \\
&\quad +\tilde{\gamma}_{ij0}(-\eta_{ij0}\hat{\gamma}_{ij0} + |\sigma_{ij}|) \\
&\quad +\tilde{\gamma}_{ij1}(-\eta_{ij1}\hat{\gamma}_{ij1} + |\sigma_{ij}|\|\mathbf{x}_i\|) \\
&\quad + \sum_{k=2,k\neq i+1}^{N+1} \tilde{\gamma}_{ijk}(-\eta_{ijk}\hat{\gamma}_{ijk} + |\sigma_{ij}|x_{k-1,sup}) \\
&= -k_{sij}\sigma_{ij}^2 - \sum_{k=0,k\neq i+1}^{N+1} \eta_{ijk}\tilde{\gamma}_{ijk}\hat{\gamma}_{ijk} \\
&= -k_{sij}\sigma_{ij}^2 - \sum_{k=0,k\neq i+1}^{N+1} \eta_{ijk}\tilde{\gamma}_{ijk}^2 \\
&\quad - \sum_{k=0,k\neq i+1}^{N+1} \eta_{ijk}\hat{\gamma}_{ijk}\gamma_{ijk} + \sum_{k=0,k\neq i+1}^{N+1} \eta_{ijk}\gamma_{ijk}^2.
\end{aligned}
$$

Since $\hat{\gamma}_{ijk} \geq 0$,

$$
\begin{aligned}
\dot{V}_{ij} &\leq -k_{sij}\sigma_{ij}^2 - \sum_{k=0,k\neq i+1}^{N+1} \eta_{ijk}\tilde{\gamma}_{ijk}^2 + \sum_{k=0,k\neq i+1}^{N+1} \eta_{ijk}\gamma_{ijk}^2 \\
&\leq -\zeta_{ij}\left[\frac{1}{2}\left(\sigma_{ij}^2 + \sum_{k=0,k\neq i+1}^{N+1} \alpha_{ijk}\tilde{\gamma}_{ijk}^2\right)\right] \\
&\quad + \sum_{k=0,k\neq i+1}^{N+1} \eta_{ijk}\gamma_{ijk}^2 \\
&= -\zeta_{ij}V_{ij} + \sum_{k=0,k\neq i+1}^{N+1} \eta_{ijk}\gamma_{ijk}^2, \qquad (18)
\end{aligned}
$$

where $\frac{1}{2}\zeta_{ij} \triangleq \min\left(k_{sij}, \min_k\left(\frac{\eta_{ijk}}{\alpha_{ijk}}\right)\right) \geq 0$. Note that η_{ijk} and γ_{ijk} are bounded, it is easy to see that V_{ij} is globally uniformly ultimately bounded, which implies that σ_{ij} and $\tilde{\gamma}_{ijk}$ (and hence $\hat{\gamma}_{ijk}$) are also globally uniformly ultimately bounded.
(b)From (11), it can be shown that

$$
\begin{aligned}
\hat{\gamma}_{ij1}(t) &= \int_{t_0}^{t} e^{-\frac{\eta_{ij1}}{\alpha_{ij1}}(t-\tau)}\frac{1}{\alpha_{ij1}}|\sigma ij|\|\mathbf{x}_i\|d\tau \\
&\quad + e^{-\frac{\eta_{ij1}}{\alpha_{ij1}}(t-t_0)}\frac{1}{\alpha_{ij1}}\hat{\gamma}_{ij1}(t_0). \qquad (19)
\end{aligned}
$$

Since from (a) it has been derived that $\hat{\gamma}_{ij1}$ is bounded, and the integrand in (19) is positive, one can conclude that $\|\mathbf{x}_i\|$ is bounded, this also implies $x_{i,sup}$, $i = 1, \cdots, N$, are also bounded. From (17), one can see that $\Delta p_{ij} - \Delta p_{ij,est}$ is bounded. Therefore according to (8), (15), (16) and assumption A3 and A4, Δp_{ij} and $p_{ij,est}$ are bounded respectively. On the other hand, from (3) one can also obtain that $\mathbf{e}_i(t)$ is bounded since x_{mi} is bounded. Therefore, the stability of overall controlled system is guaranteed by the proposed control scheme. △

If there is no perturbation estimation error, i.e., $\Delta p_{ij} = \Delta p_{ij,est}$, then according to (17), $\gamma_{ijk} = 0$, $k = 0, \cdots, N$. From (18), one can see that $\dot{V}_{ij} \leq -\zeta_{ij}V_{ij}$, it implies $\boldsymbol{\sigma}_i \to 0$ as $t \to \infty$.

5. EXAMPLE

Consider a large-scale system with dynamic equation given by
subsystem I:

$$
\begin{aligned}
\dot{\mathbf{x}}_1 &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}\mathbf{x}_1 + \begin{bmatrix} 0 \\ 0 \\ 1 - 0.2\cos(t) \end{bmatrix}u_1 \\
&\quad + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_{21}(t - 0.3|\cos(t)|) \\ x_{22}(t - 0.1|\cos(2t)|) \end{bmatrix} + \begin{bmatrix} d_{11} \\ d_{12} \\ d_{13} \end{bmatrix}, \\
d_{11} &= 0.3\cos(2t)x_{11} + (0.2\sin(t) - 0.1)x_{12}, \\
d_{12} &= -0.2\cos(t)x_{13} - 0.2x_{11}x_{12}\cos(x_{13}), \\
d_{13} &= 0.1x_{13}u_1,
\end{aligned}
$$

subsystem II:

$$\dot{\mathbf{x}}_2 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \mathbf{x}_2 + \begin{bmatrix} 1+0.3\cos(2t) & 0 \\ 1 & 1+0.2\sin(t) \end{bmatrix} \mathbf{u}_2 + \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_{11}(t-0.1|\cos(t)|) \\ x_{12}(t-0.2|\sin(t)|) \\ x_{13}(t-0.3|\sin(3t)|) \end{bmatrix} + \begin{bmatrix} d_{21} \\ d_{22} \end{bmatrix},$$

$$d_{21} = -0.2\sin(t)x_{21} + (0.5\cos(t)+0.3\sin(x_{21}))x_{22},$$
$$d_{22} = 0.2\cos(t)x_{21} + (0.3x_{21} - 0.2\sin(t))x_{22},$$

where $\mathbf{x}_1 = \begin{bmatrix} x_{11} & x_{12} & x_{13} \end{bmatrix}^T$, $\mathbf{x}_2 = \begin{bmatrix} x_{21} & x_{22} \end{bmatrix}^T$. The desired reference models are chosen as

$$\dot{\mathbf{x}}_{m1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix} \mathbf{x}_{m1} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r_1,\ r_1 = \sin(t)$$
$$\dot{\mathbf{x}}_{m2} = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} \mathbf{x}_{m2} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{r}_2,\ \mathbf{r}_2 = \begin{bmatrix} 1 \\ \cos(t) \end{bmatrix}$$

where $\mathbf{x}_{m1} = \begin{bmatrix} x_{m11} & x_{m12} & x_{m13} \end{bmatrix}^T$, $\mathbf{x}_{m2} = [x_{m21}\ x_{m22}]^T$. Let $\mathbf{C}_1 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$, $\mathbf{C}_2 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $\mathbf{K}_1 = \begin{bmatrix} -4 & -8 & -4 \end{bmatrix}$, $\mathbf{K}_2 = \begin{bmatrix} 0 & 0 \\ -5 & -2 \end{bmatrix}$, $\epsilon_1 = \epsilon_{21} = \epsilon_{22} = \beta_1 = \beta_{21} = \beta_{22} = 0.001$, $(k_{s1}, k_{s21}, k_{s22}) = (2, 1, 1)$ and the design parameters of adaptive gains are chosen to be $\alpha_{10} = \alpha_{11} = \alpha_{12} = \alpha_{20} = \alpha_{21} = \alpha_{22} = 10$, $\eta_{10} = \eta_{11} = \eta_{12} = \eta_{210} = \eta_{211} = \eta_{212} = \eta_{220} = \eta_{221} = \eta_{222} = 0.01$. The initial conditions are set to be $\mathbf{x}_1(0) = \begin{bmatrix} 0.9 & -0.4 & 0.3 \end{bmatrix}^T$, $\mathbf{x}_2(0) = \begin{bmatrix} -0.8 & 0.1 \end{bmatrix}^T$, $\mathbf{x}_{m1}(0) = \mathbf{x}_{m2}(0) = 0$. The simulation results are shown from Fig. 1 to Fig. 4. From Fig. 1 and Fig. 2 one can see that the controller can achieve very good tracking accuracy. There is almost no chattering, as shown in Fig. 3. Fig. 4 shows that the adaptive gains $\hat{\gamma}_{110}$, $\hat{\gamma}_{111}$ and $\hat{\gamma}_{113}$ are all bounded. The other adaptive gains $\hat{\gamma}_{2jk}$, $j = 1, 2$, $k = 0, 1, 2$, whose figures are omitted in this paper, are all bounded too.

6. CONCLUSIONS

A decentralized model reference control scheme with perturbation estimation process and adaptive mechanism embedded is successfully proposed for a class of large-scale mismatched perturbed systems for solving robust tracking problems. Due to the adaptive mechanism, the knowledge of upper-bounds of perturbation as well as the knowledge of the exact function of time-delay are not required. Since the perturbation estimation process is embedded in the controller, the proposed adaptive gains need only to overcome the unknown upper-bounds of perturbation estimation errors, high control gain is avoided. Therefore the chattering phenomenon is further alleviated than those of traditional VSC in which boundary layer controllers are used.

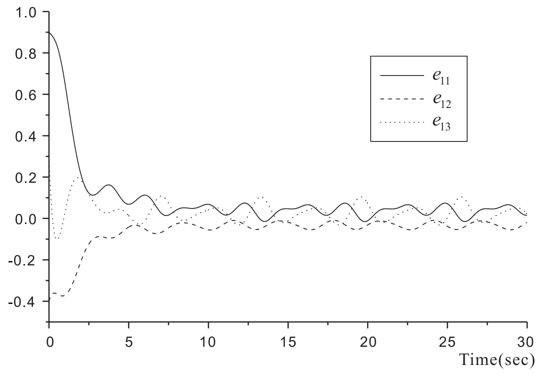

Fig. 1. Tracking error $\mathbf{e}_1$.

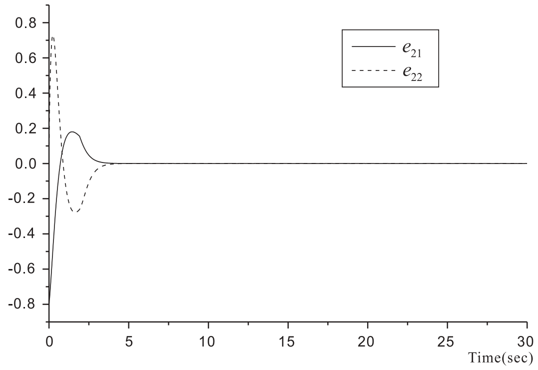

Fig. 2. Tracking error $\mathbf{e}_2$.

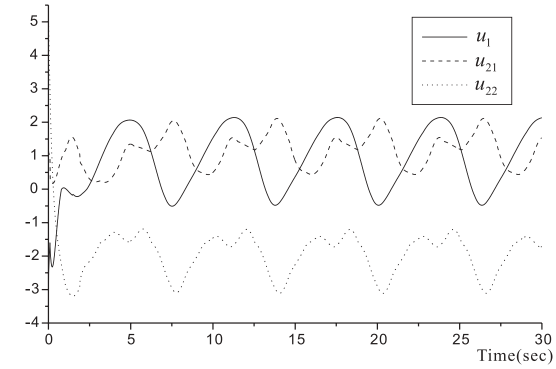

Fig. 3. Control effort u_1 and $\mathbf{u}_2$.

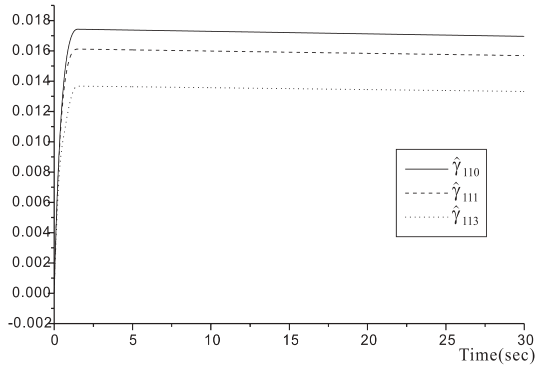

Fig. 4. Adaptive gains $\hat{\gamma}_{110}$, $\hat{\gamma}_{111}$ and $\hat{\gamma}_{113}$.

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