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GLOBAL TRACKING CONTROL OF UNDERACTUATED SURFACE SHIPS IN BODY FRAME

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Abstract: A controller is developed for underactuated surface ships with only surge force and yaw moment available to globally track a reference trajectory generated by a suitable virtual ship in a frame attached to the ship body. The control development is based on Lyapunov's direct method and backstepping technique, and utilizes several properties of ship dynamics and their interconnected structure. Numerical simulations validate the proposed controller. *Copyright @ 2002 IFAC*

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1. INTRODUCTION

This paper concentrates on the global tracking control of surface ships with only sway force and yaw moment available. The study is interested in designing a controller such that it makes the position (sway and surge) and orientation (yaw angle) of surface ships track the reference position and orientation generated by a virtual reference ship. Since the interested surface ships have fewer numbers of actuators than degrees of freedom to be controlled and the constraint on the acceleration (Reyhanoglu 1997, Pettersen 1996 and Sordalen and Engeland 1995) is nonintegrable, they are a class of underactuated systems with nonintegrable dynamics. Godhavn (1996) used a continuous time invariant state feedback controller to achieve global exponential position tracking under an assumption that the reference surge velocity is always positive. Unfortunately, the orientation of the ship was not controlled. Pettersen and Nijmeijer (1998) provided a high gain based semiglobal tracking result. Behal et al. (2000) designed a global tracker based on a transformation of the ship tracking system into the so-called convenient form (Dixon et al. (2000)). The dynamics of closed loop system is increased.

This paper proposes a constructive procedure to develop a controller to make design an underactuated surface ship with only surge force and yaw moment available track a reference trajectory generated by a virtual ship. The control development at the velocity level was based on the Lyapunov's direct method and utilized several nature properties of the underactuated ship dynamics. Based the backstepping technique (Krstic et al. 1995), the controls at the force and moment level ware designed. The proposed controller guarantees the global asymptotic and local exponential convergence of the tracking error to the origin. In addition, the reference surge and sway velocities are not required to be generated by the virtual ship. Simulations on a monohull ship with the length of 32 m and mass of 118×10^3 kg illustrate the effectiveness of the proposed controller.

2. PROBLEM FORMULATION

The underactuated ship moving in surge, sway and yaw can be described as

$$
\begin{aligned}
&\dot{x}=u\cos(\psi)-v\sin(\psi),\dot{y}=u\sin(\psi)+v\cos(\psi),\ \dot{\psi}=r,\\
&\dot{u}=\frac{m_{22}}{m_{11}}vr-\frac{d_{11}}{m_{11}}u+\frac{1}{m_{11}}\tau_u,\\
&\dot{v}=-\frac{m_{11}}{m_{22}}ur-\frac{d_{22}}{m_{22}}v,\\
&\dot{r}=\frac{(m_{11}-m_{22})}{m_{33}}uv-\frac{d_{33}}{m_{33}}r+\frac{1}{m_{33}}\tau_r,
\end{aligned}
\tag{1}
$$

where x, y and ψ are the surge displacement, sway displacement and yaw angle in the earth fixed frame, u, v and r denote surge velocity, sway velocity, yaw velocity. The positive constant terms d_{jj} and m_{jj}, $j = 1 \div 3$ denote the hydrodynamic damping and ship inertia including added mass in surge, sway and yaw. The available controls are the surge force τ_u and the yaw moment τ_r. The reference trajectory is generated by

$$
\begin{aligned}
&\dot{x}_d=u_d\cos(\psi_d)-v_d\sin(\psi_d),\dot{y}_d=u_d\sin(\psi_d)+v_d\cos(\psi_d),\dot{\psi}_d=r_d,\\
&\dot{u}_d=\frac{m_{22}}{m_{11}}v_dr_d-\frac{d_{11}}{m_{11}}u_d+\frac{1}{m_{11}}\tau_{ud},\\
&\dot{v}_d=-\frac{m_{11}}{m_{22}}u_dr_d-\frac{d_{22}}{m_{22}}v_d,\\
&\dot{r}_d=\frac{(m_{11}-m_{22})}{m_{33}}u_dv_d-\frac{d_{33}}{m_{33}}r_d+\frac{1}{m_{33}}\tau_{rd},
\end{aligned}
\tag{2}
$$

where all the variables have similar meaning as in system (1) for the virtual reference ship. The global transformation of coordinates (Pettersen 1996)

$$
\begin{bmatrix} z_1\\ z_2\\ z_3\end{bmatrix}=\begin{bmatrix}\cos(\psi) & \sin(\psi) & 0\\ -\sin(\psi) & \cos(\psi) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix} x\\ y\\ \psi\end{bmatrix}
\tag{3}
$$

and a similar transformation for the reference virtual ship were used to transform system (1) into

$$
\begin{aligned}
&\dot{z}_1=u+z_2r,\ \dot{z}_2=v-z_1r,\ \dot{z}_3=r,\\
&\dot{u}=\frac{m_{22}}{m_{11}}vr-\frac{d_{11}}{m_{11}}u+\frac{1}{m_{11}}\tau_u,\\
&\dot{v}=-\frac{m_{11}}{m_{22}}ur-\frac{d_{22}}{m_{22}}v,\\
&\dot{r}=\frac{(m_{11}-m_{22})}{m_{33}}uv-\frac{d_{33}}{m_{33}}r+\frac{1}{m_{33}}\tau_r,
\end{aligned}
\tag{4}
$$

and a similar set of differential equations for the virtual ship. By introducing the auxiliary tracking variables $z_{ie} = z_i - z_{id}$, $1 \le i \le 3, u_e = u - u_d, v_e = v - v_d$ and $r_e = r - r_d$, the auxiliary tracking error dynamics are written as

$$
\begin{aligned}
&\dot{z}_{1e}=u_e+z_{2e}r_d+z_2r_e,\dot{z}_{2e}=v_e-z_{1e}r_d-z_1r_e,\ \dot{z}_{3e}=r_e,\\
&\dot{u}_e=\frac{m_{22}}{m_{11}}(vr-v_dr_d)-\frac{d_{11}}{m_{11}}u_e+\frac{1}{m_{11}}(\tau_u-\tau_{ud}),\\
&\dot{v}_e=-\frac{d_{22}}{m_{22}}v_e-\frac{m_{11}}{m_{22}}(ur-u_dr_d),\\
&\dot{r}_e=\frac{(m_{11}-m_{22})}{m_{33}}(uv-u_dv_d)-\frac{d_{33}}{m_{33}}r_e+\frac{1}{m_{33}}(\tau_r-\tau_{rd}).
\end{aligned}
\tag{5}
$$

Then the problem of tracking control was converted into that of stabilizing (5) at the origin. The first complete state tracking controller was developed by Pettersen and Nijmeijer (1998) and yielded the global practical stability. Based on the work in Jiang and Nijmeijer (1999), Pettersen and Nijmeijer (2000) proposed a controller that semi-globally asymptotically stabilized (5). A global tracking result based on a cascaded approach was proposed in (Lefeber (2000)). The stability analysis was based on theory of the linear time varying systems. A recent result on global tracking control of the underactuated surface ship based on Lyapunov's direct method and passivity approach was proposed by Jiang (2001). However, the quadratic function to design the control at the velocity level was motivated from that for the standard chain form system (Jiang and Nijmeijer (1999. It is difficult to determine the control gains.

This paper proposes a method to design a controller such that it makes the position and orientation x, y and ψ of the ship model (1) globally exponentially track a reference trajectory generated a suitable virtual ship as

$$
\begin{aligned}
&\dot{x}_d=u_d\cos(\psi_d)-v_d\sin(\psi_d),\dot{y}_d=u_d\sin(\psi_d)+v_d\cos(\psi_d),\\
&\dot{\psi}_d=r_d,\ \dot{v}_d=-\frac{m_{11}}{m_{22}}u_dr_d-\frac{d_{22}}{m_{22}}v_d.
\end{aligned}
\tag{6}
$$

We assume that the reference trajectory, x_d, y_d and ψ_d, and reference velocity, u_d and r_d, are bounded and differentiable once and r_d satisfies:

Assumption 1. There exists a constant σ such that, for any pair of $(t_0, t), 0 \le t_0 \le t < \infty$,

$$
\int_{t_0}^{t} r_d^2(\tau)d\tau \ge \sigma(t-t_0).
\tag{7}
$$

We introduce the position errors $x - x_d$ and $y - y_d$ in a frame attached to the ship body. This results in the error coordinates as

$$
\begin{bmatrix} x_e\\ y_e\\ \psi_e\end{bmatrix}=\begin{bmatrix}\cos(\psi) & \sin(\psi) & 0\\ -\sin(\psi) & \cos(\psi) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix} x-x_d\\ y-y_d\\ \psi-\psi_d\end{bmatrix}.
\tag{8}
$$

We also define the velocity tracking errors as $u_e = u - u_d, v_e = v - v_d$ and $r_e = r - r_d$. Taking the first time derivative both sides of (8) along the solution of (1) and (6) yield

$$
\begin{aligned}
\dot{x}_e &= u_e - u_d(\cos(\psi_e) - 1) - v_d \sin(\psi_e) + r_e y_e + r_e y_e, \\
\dot{y}_e &= v_e - v_d(\cos(\psi_e) - 1) + u_d \sin(\psi_e) - r_e x_e - r_d x_e, \\
\dot{\psi}_e &= r_e, \\
\dot{u}_e &= \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{1}{m_{11}} \tau_u - \dot{u}_d, \\
\dot{v}_e &= -\frac{m_{11}}{m_{22}} u_e r_d - \frac{m_{11}}{m_{22}} (u_e + u_d) r_e - \frac{d_{22}}{m_{22}} v_e, \\
\dot{r}_e &= \frac{(m_{11} - m_{22})}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_r - \dot{r}_d.
\end{aligned}
\tag{9}
$$

Hence in the sequel, stabilization of the system (9) is addressed.

3. CONTROL DESIGN

*Step 1.*For convenience, we write the error dynamics in ψ_e and r_e separately as

$$
\begin{aligned}
\dot{\psi}_e &= r_e, \\
\dot{r}_e &= \frac{(m_{11} - m_{22})}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_r - \dot{r}_d.
\end{aligned}
\tag{10}
$$

It is straightforward to design a control τ_{r} without canceling the useful damping term as

$$
\tau_r = m_{33}\left(-(k_1 + k_2)\tilde{r}_e + \dot{r}_d - \frac{m_{11} - m_{22}}{m_{33}} + \frac{d_{33}}{m_{33}}(r_d - k_1 \psi_e) \right)
\tag{11}
$$

where k_1 and k_2 are positive constants, and $\tilde{r}_e = r_e + k_1 \psi_e$. Take the Lyapunov function

$$
V_1 = \frac{k_1^2}{2} \psi_e^2 + \frac{1}{2} \tilde{r}_e^2
\tag{12}
$$

whose the first time derivative along the solution of (10) and (11) satisfies

$$
\dot{V}_1 = -k_1^3 \psi_e^2 - \left(k_2 + \frac{d_{33}}{m_{33}} \right) \tilde{r}_e^2 \leq -\rho_1 V_1
\tag{13}
$$

where $\rho_1 = \min\left(2k_1, 2\left(k_2 + \frac{d_{33}}{m_{33}} \right) \right)$. From (13), it is clear that the system (10) is globally exponentially stable at the origin, i.e., for any pair of initial conditions $(\psi_e(t_0), r_e(t_0))$ and any initial time instant $t_0 > 0$, the solution $(\psi_e(t), r_e(t))$ exists for each $t \geq t_0$ and satisfies

$$
\left\| \begin{bmatrix} \psi_e(t) \\ r_e(t) \end{bmatrix} \right\| \leq \zeta_1 e^{-\zeta_2 (t - t_0)} \left\| \begin{bmatrix} \psi_e(t_0) \\ r_e(t_0) \end{bmatrix} \right\|
\tag{14}
$$

for some constants $\zeta_1 > 0$ and $\zeta_2 > 0$.

Step 2. The control τ_u is designed to globally exponentially stabilize (9) with the control τ_{r} given in (11) in two sub-steps. For convenience, we write the system (9) without (10), which is already designed to be globally exponentially stable at the origin as

$$
\begin{aligned}
\dot{x}_e &= u_e - u_d(\cos(\psi_e) - 1) - v_d \sin(\psi_e) + r_e y_e + r_e y_e, \\
\dot{y}_e &= v_e - v_d(\cos(\psi_e) - 1) + u_d \sin(\psi_e) - r_e x_e - r_d x_e, \\
\dot{v}_e &= -\frac{m_{11}}{m_{22}} u_e r_d - \frac{m_{11}}{m_{22}} (u_e + u_d) r_e - \frac{d_{22}}{m_{22}} v_e, \\
\dot{u}_e &= \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{1}{m_{11}} \tau_u - \dot{u}_d.
\end{aligned}
\tag{15}
$$

Sub-step 2.1. We start by defining the virtual control error as $\tilde{u}_e = u_e - u_e^d$. Then (15) can be written as

$$
\begin{aligned}
\dot{x}_e &= u_e^d + \tilde{u}_e - u_d(\cos(\psi_e) - 1) - v_d \sin(\psi_e) + r_e y_e + r_e y_e, \\
\dot{y}_e &= v_e - v_d(\cos(\psi_e) - 1) + u_d \sin(\psi_e) - r_e x_e - r_d x_e, \\
\dot{v}_e &= -\frac{m_{11}}{m_{22}} (u_e^d + \tilde{u}_e) r_d - \frac{m_{11}}{m_{22}} (u_e^d + \tilde{u}_e + u_d) r_e - \frac{d_{22}}{m_{22}} v_e, \\
\dot{u}_e &= \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} (u_e^d + \tilde{u}_e + u_d) + \frac{1}{m_{11}} \tau_u - \dot{u}_d.
\end{aligned}
\tag{16}
$$

To design u_e^d that stabilizes the first three equations of (16), we consider the following quadratic function

$$
V_{21} = \frac{1}{2} x_e^2 + \frac{1}{2} y_e^2 + \frac{k}{2} (v_e + k_3 y_e)^2
\tag{17}
$$

where k and k_3 are positive constants to be chosen later. Taking the first time derivative of (17) along the solution of (16), after some manipulation, yields

$$
\begin{aligned}
\dot{V}_{21} = {} & x_e u_e^d + k(v_e + k_3 y_e)\left(-\left(\frac{d_{22}}{m_{22}} - k_3 \right) v_e - \frac{m_{11}}{m_{22}} u_e^d r_d \right. \\
& \left. - k_3 r_d x_e \right) + y_e v_e + \left(x_e - k \frac{m_{11}}{m_{22}} r_d (v_e + k_3 y_e) \right) \tilde{u}_e - \\
& x_e \left(u_d(\cos(\psi_e) - 1) + v_d \sin(\psi_e) \right) - \\
& \left(k k_3 (v_e + k_3 y_e) + y_e \right)\left(v_d(\cos(\psi_e) - 1) - u_d \sin(\psi_e) \right) - \\
& k(v_e + k_3 y_e) \left(\frac{m_{11}}{m_{22}} \left(\tilde{u}_e + u_e^d + u_d \right) + k_3 x_e \right) r_e.
\end{aligned}
\tag{18}
$$

We now design the virtual velocity control u_e^d as

$$
u_e^d = -k_4 x_e + k_5 r_d (v_e + k_3 y_e),
\tag{19}
$$

where k_4 and k_5 are positive constants to be selected later. Substituting (19) into (18) results in

$$\dot{V}_{21}=-k_4x_e^2+k_5r_d(v_e+k_3y_e)x_e-k\left(\frac{d_{22}}{m_{22}}-k_3\right)v_e^2-$$
$$\left(kk_3\left(\frac{d_{22}}{m_{22}}-k_3\right)-1\right)y_ev_e-kk_5r_d^2\frac{m_{11}}{m_{22}}(v_e+k_3y_e)^2+$$
$$k(v_e+k_3y_e)\left(k_4\frac{m_{11}}{m_{22}}-k_3\right)r_dx_e+\Omega+ \tag{20}$$
$$\left(x_e-k\frac{m_{11}}{m_{22}}r_d(v_e+k_3y_e)-\frac{m_{11}}{m_{22}}k(v_e+k_3y_e)r_e\right)\tilde{u}_e,$$

$$\Omega=-x_e(u_d(\cos(\psi_e)-1)+v_d\sin(\psi_e))-$$
$$(kk_3(v_e+k_3y_e)+y_e)(v_d(\cos(\psi_e)-1)-u_d\sin(\psi_e))- \tag{21}$$
$$k\frac{m_{11}}{m_{22}}(v_e+k_3y_e)(k_5r_d(v_e+k_3y_e)+u_d)r_e.$$

By choosing

$$kk_3\left(\frac{d_{22}}{m_{22}}-k_3\right)-1=0,\frac{m_{11}}{m_{22}}k_4=k_3 \tag{22}$$

and noting that

$$|k_5r_d(v_e+k_3y_e)x_e|\le\frac{k_5^2r_d^2}{4\varepsilon}(v_e+k_3y_e)^2+\varepsilon x_e^2,\ \forall\varepsilon>0 \tag{23}$$

(20) can be written as

$$\dot{V}_{21}=-(k_4-\varepsilon)x_e^2-k\left(\frac{d_{22}}{m_{22}}-k_3\right)v_e^2-\left(kk_5\frac{m_{11}}{m_{22}}-\frac{k_5^2}{4\varepsilon}\right)r_d^2(v_e+k_3y_e)^2+$$
$$\left(x_e-k\frac{m_{11}}{m_{22}}r_d(v_e+k_3y_e)-\frac{m_{11}}{m_{22}}k(v_e+k_3y_e)r_e\right)\tilde{u}_e+\Omega \tag{24}$$

Sub-step 2.2. We take the quadratic function

$$V_{22}=V_{21}+\frac{1}{2}\tilde{u}_e^2 \tag{25}$$

whose derivative satisfies

$$\dot{V}_{22}\le-(k_4-\varepsilon)x_e^2-k\left(\frac{d_{22}}{m_{22}}-k_3\right)v_e^2-$$
$$\left(kk_5\frac{m_{11}}{m_{22}}-\frac{k_5^2}{4\varepsilon}\right)r_d^2(v_e+k_3y_e)^2+\Omega+$$
$$\tilde{u}_e\left(\left(x_e-k\frac{m_{11}}{m_{22}}r_d(v_e+k_3y_e)-\frac{m_{11}}{m_{22}}k(v_e+k_3y_e)r_e\right)+\right.$$
$$\frac{m_{22}}{m_{11}}vr-\frac{d_{11}}{m_{11}}(\tilde{u}_e+u_e^d+u_d)+\frac{1}{m_{11}}\tau_u-\dot{u}_d-\frac{\partial u_e^d}{\partial r_d}\dot{r}_d-$$
$$\frac{\partial u_e^d}{\partial x_e}(u_e-u_d(\cos(\psi_e)-1)-v_d\sin(\psi_e)+r_ey_e+r_dy_e)-$$
$$\frac{\partial u_e^d}{\partial y_e}(v_e-v_d(\cos(\psi_e)-1)+v_d\sin(\psi_e)-r_ex_e-r_dx_e)-$$
$$\left.\frac{\partial u_e^d}{\partial v_e}\left(-\frac{d_{22}}{m_{22}}v_e-\frac{m_{11}}{m_{22}}u_er_d-\frac{m_{11}}{m_{22}}(u_e+u_d)r_e\right)\right) \tag{26}$$

It is now straightforward to design the control τ_u without canceling the damping term as

$$\tau_u=m_{11}\left(-k_6\tilde{u}_e-\left(x_e-k\frac{m_{11}}{m_{22}}r_d(v_e+k_3y_e)-\right.\right.$$
$$\left.\frac{m_{11}}{m_{22}}k(v_e+k_3y_e)r_e\right)-$$
$$\frac{m_{22}}{m_{11}}vr+\frac{d_{11}}{m_{11}}(u_e^d+u_d)+\dot{u}_d+\frac{\partial u_e^d}{\partial r_d}\dot{r}_d+\frac{\partial u_e^d}{\partial x_e}$$
$$(u_e-u_d(\cos(\psi_e)-1)-v_d\sin(\psi_e)+r_ey_e+r_dy_e)+$$
$$\frac{\partial u_e^d}{\partial y_e}(v_e-v_d(\cos(\psi_e)-1)+v_d\sin(\psi_e)-r_ex_e-r_dx_e)+$$
$$\left.\frac{\partial u_e^d}{\partial v_e}\left(-\frac{d_{22}}{m_{22}}v_e-\frac{m_{11}}{m_{22}}u_er_d-\frac{m_{11}}{m_{22}}(u_e+u_d)r_e\right)\right) \tag{27}$$

where k_6 is a positive constant. Substituting (27) into (26) yields

$$\dot{V}_{22}\le-(k_4-\varepsilon)x_e^2-k\left(\frac{d_{22}}{m_{22}}-k_3\right)v_e^2-$$
$$\left(kk_5\frac{m_{11}}{m_{22}}-\frac{k_5^2}{4\varepsilon}\right)r_d^2(v_e+k_3y_e)^2-\left(\frac{d_{11}}{m_{11}}+k_6\right)\tilde{u}_e^2+\Omega. \tag{28}$$

From the above control design procedure, we require

$$k>0,\ k_1>0,k_2>0,\ k_4\frac{m_{11}}{m_{22}}-k_3=0,$$
$$-kk_3\left(\frac{d_{22}}{m_{22}}-k_3\right)+1=0,\frac{d_{22}}{m_{22}}-k_3>0, \tag{29}$$
$$\varepsilon>0,\ kk_5\frac{m_{11}}{m_{22}}-\frac{k_5^2}{4\varepsilon}>0,\ k_6>0.$$

All the constants $k_i, 1\le i\le 6$ in (29) are computed in order as follows

$$k_1>0,k_2>0,k_6>0,\ k>4\left(\frac{m_{22}}{d_{22}}\right)^2,$$
$$k_3=\frac{1}{2}\left(\frac{d_{22}}{m_{22}}+\sqrt{\left(\frac{d_{22}}{m_{22}}\right)^2-\frac{4}{k}}\right), \tag{30}$$
$$k_4=k_3\frac{m_{22}}{m_{11}},\ 0<\varepsilon<k_4,0<k_5<4\varepsilon k\frac{m_{11}}{m_{22}}.$$

*Theorem 1***.** Under the assumptions that the reference trajectory (x_d,y_d) is bounded, the reference velocities u_d and r_d and their first time derivatives are bounded, and r_d satisfies assumption 1, the tracking control problem posed in section 2 is solved by the controls (11) and (27). In particular, letting $X_e=[x_e,y_e,\psi_e,u_e,v_e,r_e]^T$, there exists a *K*-function γ and a constant $\sigma>0$ such that for any

$t_0 \geq 0$ and any $X_e(t_0) \in R^6$, the solution $X_e(t)$ exists for each $t \geq t_0$ and satisfies

$$\left|X_e(t)\right| \leq \gamma\left(\left|X_e(t_0)\right|\right)e^{-\sigma(t-t_0)}. \quad (31)$$

Proof. We first show that there exists a positive time varying coefficient $c(t)$ such that

$$\dot{V}_{22} \leq -c(t)V_{22} + \Omega. \quad (32)$$

To simplify representation in the proof of (32), by defining

$$c_1 = (k_4 - \varepsilon), c_2 = k\left(\frac{d_{22}}{m_{22}} - k_3\right),$$
$$c_3 = \left(kk_5\frac{m_{11}}{m_{22}} - \frac{k_5^2}{4\varepsilon}\right)r_d^2, c_4 = \frac{d_{11}}{m_{11}} + k_6 \quad (33)$$

then (28) can be written as

$$\dot{V}_{22} \leq -c_1 x_e^2 - (c_3 - \varepsilon_1)(v_e + k_3 y_e)^2 - \varepsilon_2 y_e^2 - \left(c_2 v_e^2 + \varepsilon_1 (v_e + k_3 y_e)^2 - \varepsilon_2 y_e^2\right) - c_4 \tilde{u}_e^2 + \Omega \quad (34)$$

where ε_1 and ε_2 are some positive constants. We now show that there exist $0 < \varepsilon_1 < c_3$ and $\varepsilon_2 > 0$ such that

$$M := c_2 v_e^2 + \varepsilon_1 (v_e + k_3 y_e)^2 - \varepsilon_2 y_e^2 \geq 0 \; \forall x_e \in R, y_e \in R. \quad (35)$$

By opening the square bracket, after some manipulation, (35) can be written as

$$M := \left(c_2 + \varepsilon_1 - \frac{\varepsilon_1^2 k_3^2}{\varepsilon_1 k_3^2 - \varepsilon_2}\right)v_e^2 + \left(\varepsilon_1 k_3^2 - \varepsilon_2\right)\left(y_e + \frac{\varepsilon_1 k_3}{\varepsilon_1 k_3^2 - \varepsilon_2}v_e\right)^2. \quad (36)$$

It is clear that $M \geq 0$ when

$$c_2 + \varepsilon_1 - \frac{\varepsilon_1^2 k_3^2}{\varepsilon_1 k_3^2 - \varepsilon_2} \geq 0, \; \varepsilon_1 k_3^2 - \varepsilon_2 > 0 \quad (37)$$

or equivalently

$$\varepsilon_2 < \min\left(\varepsilon_1 k_3^2, \frac{c_2 \varepsilon_1 k_3^2}{c_2 + \varepsilon_1}\right). \quad (38)$$

Therefore we can always pick $0 < \varepsilon_1 < c_3$ and ε_2 such that (38) is satisfied. With this choice of ε_1 and ε_2, $M \geq 0$, we can write (34) as

$$\dot{V}_{22} \leq -c_1 x_e^2 - (c_3 - \varepsilon_1)(v_e + k_3 y_e)^2 - \varepsilon_2 y_e^2 - c_4 \tilde{u}_e^2 + \Omega \quad (39)$$

which yields (32) with

$$c(t) = \min(2c_1, 2\varepsilon_2, 2(c_3 - \varepsilon_1)/k, 2c_4).$$

From assumption 1, there exists a positive constant σ_3 such that

$$\int_{t_0}^{t} c(\tau)d\tau \geq \sigma_3(t - t_0). \quad (40)$$

Next, we show that there exists a time varying vector valued signal $\xi \in R^2$ that exponentially converges to zero such that Ω defined in (21) satisfies

$$|\Omega| \leq V_{22} f_1(\xi(t)) + f_2(\xi(t)) \quad (41)$$

with

$$f_1(\xi(t)) \leq \gamma_1(\xi(t_0))e^{-\sigma_1(t-t_0)}$$
$$f_2(\xi(t)) \leq \gamma_2(\xi(t_0))e^{-\sigma_2(t-t_0)} \quad (42)$$

where γ_1 and γ_2 are class *K*-functions, and σ_1 and σ_2 are positive constants. By noting that

$$\left|(\cos(\psi_e) - 1)/\psi_e\right| \leq 1, \; \left|\sin(\psi_e)/\psi_e\right| \leq 1,$$
$$\lim_{\psi_e \to 0}\left((\cos(\psi_e) - 1)/\psi_e\right) = 1, \lim_{\psi_e \to 0}\left(\sin(\psi_e)/\psi_e\right) = 1 \quad (43)$$

it is straightforward to show that (21) satisfies

$$|\Omega| \leq \left(\frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 + \frac{k}{2}(v_e + k_3 y_e)^2\right)\left(|u_d| + |v_d|\right)|\psi_e| + \left(1 + 2kk_3^2\right)\left(|u_d| + |v_d|\right)|\psi_e| + \frac{k}{2}(v_e + k_3 y_e)^2 \times \left(2k_5|r_d| + |u_d|\right)\frac{m_{11}}{m_{22}}|r_e| + |u_d|\frac{m_{11}}{m_{22}}\frac{k}{2}|r_e| \quad (44)$$

which is further satisfied

$$|\Omega| \leq V_{22}\left(\left(|u_d| + |v_d|\right)|\psi_e| + \left(2k_5|r_d| + |u_d|\right)\frac{m_{11}}{m_{22}}|r_e|\right) + \left(1 + 2kk_3^2\right)\left(|u_d| + |v_d|\right)|\psi_e| + |u_d|\frac{m_{11}}{m_{22}}\frac{k}{2}|r_e| \quad (45)$$

By noting from (14) that ψ_e and r_e exponentially converge to zero and that u_d, v_d and r_d are bounded by assumption, hence (41) follows from (45) readily. Substituting (41) into (32) yields

$$\dot{V}_{22} \leq -(c(t) - f_1(\xi(t)))V_{22} + f_2(\xi(t)) \quad (46)$$

We now consider the following differential equation

$$\dot{x} = -(c(t) - f_1(\xi(t)))x + f_2(\xi(t)) \quad (47)$$

whose solution $x(t)$ at $t > t_0$, for any $t_0 \geq 0$, satisfies

$$x(t) = x(t_0)e^{-\int_{t_0}^{t}(c(\tau) - f_1(\xi_1(\tau))d\tau} + \int_{t_0}^{t} f_2(\xi(\tau))e^{-\int_{\tau}^{t}(c(s) - f_1(\xi_1(s))ds}d\tau \quad (48)$$

which, from (42), also satisfies

$$|x(t)| \leq |x(t_0)|e^{\gamma_1(|\xi(t_0)|)/\sigma_1}e^{-\sigma_3(t-t_0)} + \frac{1}{\sigma_1}\gamma_1(|\xi(t_0)|)\gamma_2(|\xi(t_0)|)e^{-\sigma_3 t + \sigma_2 t_0}\int_{t_0}^{t} e^{(\sigma_3 - \sigma_2)\tau}d\tau \quad (49)$$

It can be seen that there exists a positive constant $\sigma_4 < \min(\sigma_2, \sigma_3)$ and a positive constant σ_5 such that

$$e^{-\sigma_3 t + \sigma_2 t_0}\int_{t_0}^{t} e^{(\sigma_3 - \sigma_2)\tau}d\tau \leq \sigma_5 e^{-\sigma_4(t-t_0)} \quad (50)$$

From (50) and (49), there exist a class *K*-function γ and a positive constant σ_6 such that

$$|x(t)| \leq \gamma\left(\left|(x(t_0), \xi(t_0))\right|\right)e^{-\sigma_6(t-t_0)}. \quad (51)$$

From (51) and (46), by comparison principle (Khalil 1992), (31) follows readily.

4. SIMULATIONS

This section validates the control laws (26) and (27) by simulating them on a monohull ship with the length of 32 m, mass of 118×10^3 kg and other parameters calculated by using Marintek Ship Motion program version 3.18 as

$$m_{11}=120\times10^3\text{ kg}, m_{22}=177.9\times10^3\text{ kg}, m_{33}=636\times10^5\text{ kgm}^2,$$

$$d_{11}=215\times10^2\text{ kgs}^{-1}, d_{22}=497\times10^2\text{ kgs}^{-1}, d_{33}=802\times10^4\text{ kgm}^2s^{-1}.$$

In the simulation, based on (30), the control parameters are taken and computed as

$$\varepsilon=0.1, k_1=0.5, k_2=1,\ k=102.5, k_3=0.24,\ k_4=0.35,$$
$$k_5=20.7, k_6=6.$$

The initial conditions are chosen as

$$[x(0), y(0), \psi(0), u(0), v(0), r(0)]=$$
$$[-5\text{m}, -5\text{m}, 0.5\text{ rad}, 0\text{ ms}^{-1}, 0\text{ ms}^{-1}, 0\text{ rad s}^{-1}].$$

The reference trajectory is generated by a virtual ship with the initial conditions as

$$[x_d(0), y_d(0), \psi_d(0), v_d(0)]=[0\text{m}, -0\text{ m}, 0\text{ rad}, 0\text{ ms}^{-1}]$$

and the reference velocities as $u_d=1\text{ ms}^{-1}$ and $r_d=0.1\text{rads}^{-1}$. This choice means that the reference trajectory is a circle with a radius of 10 m.

The tracking trajectory in (*x*,*y*) plane is plotted in Figure 1. It can be seen that the tracking errors asymptotically converge to zero as proven in Theorem 1. Due to space limitation, comparison with other tracking controllers available for underactuated ships is omitted.

5. CONCLUSIONS

The constructive approach has been proposed in this paper to develop a controller to make design an underactuated surface ship with only surge force and yaw moment available track a reference trajectory generated by a virtual ship in the ship body frame. The proposed controller guarantees the global exponential and local exponential, in sense of Lyapunov, convergence of the tracking error to the origin. Simulation results validated the effectiveness of our proposed controller.

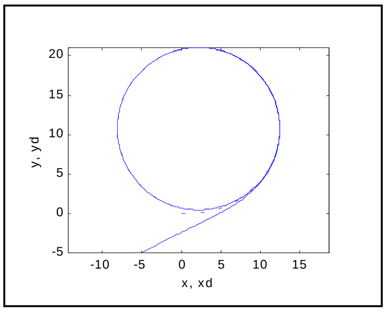

Figure 1. Tracking trajectory in (*x*,*y*) plane, (x, y): solid, (x_d, y_d): dash.

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