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# SLIDING MODE APPLICATION IN SPEED SENSORLESS TORQUE CONTROL OF AN INDUCTION MOTOR

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Abstract: Induction motor speed sensorless torque control, which allows operation at low and zero speed, optimizing both torque response and efficiency, is proposed. The control is quite different than the conventional field-oriented or direct torque control. The produced torque is explicitly continuous output variable of control. A new stator and rotor flux controller/observer based on continuous sliding mode and Lyapunov theory are developed. A smooth transition into the field weakening region and the full utilization of the inverter current and voltage capability are possible. The reference tracking performance of torque and rotor flux is demonstrated in terms of transient characteristics by experimental results.

Keywords: Induction motors, control system, torque control, sliding mode control, observers.

## 1. INTRODUCTION

Sensorless control of induction motor (IM) drives is receiving wide attention (Vas, 1998). The main reason is that the use of speed sensor spoils the ruggedness and simplicity of IM. Even more, in a hostile environment speed sensors cannot be mounted. However, due to the high order and nonlinearity of the dynamics of an IM, estimation of the angle speed and rotor flux without the measurement of mechanical variables becomes a challenging problem. To overcome this, various speed sensorless control algorithms have been presented (Holtz, 1996; Bose and Simoes, 1995; Kubota and Nakano, 1993). Due to its order reduction, disturbance rejection, strong robustness, and simple implementation by the means of power converter and digital signal processors, sliding mode control (SMC) theory is one of the prospective control methodologies for electric machines (Utkin, 1993).

In this paper, an SMC based rotor flux observer is designed and a sliding mode approach is applied to the torque tracking control of the IM. In principle, the proposed method is based on driving the stator flux towards the reference stator flux vector defined by the input commands, the reference torque and the reference rotor flux. This action is carried out by the robust sliding mode flux controller applying a suitable stator voltage vector to the machine in order to compensate the stator flux vector error.

## 2. MACHINE DYNAMICS AND PROBLEM STATEMENT

The state equation of the IM viewed from the stator frame (a, b frame) driven by voltage source inverter is given by

$$\frac{d\boldsymbol{\Psi}_s^s}{dt} = -\frac{R_s}{\sigma L_s}\boldsymbol{\Psi}_s^s + \frac{R_s L_m}{\sigma L_s L_r}\boldsymbol{\Psi}_r^s + \boldsymbol{u}_s^s , \quad (1)$$

$$\frac{d\boldsymbol{\Psi}_r^s}{dt} = \frac{L_m}{L_s}\frac{R_r}{\sigma L_r}\boldsymbol{\Psi}_s^s - \frac{R_r}{\sigma L_r}\boldsymbol{\Psi}_r^s + jp\omega_r\boldsymbol{\Psi}_r^s , \quad (2)$$

$$\boldsymbol{\Psi}_s^s = \sigma L_s \boldsymbol{i}_s^s + \frac{L_m}{L_r}\boldsymbol{\Psi}_r^s , \quad (3)$$

$$T_e = \frac{2}{3} p \frac{L_m}{\sigma L_s L_r}\left(\psi_{ra}^s \psi_{sb}^s - \psi_{rb}^s \psi_{sa}^s\right), \quad (4)$$

$$J\frac{d\omega_r}{dt} = T_e - T_L , \quad (5)$$

where two dimensional vectors $\boldsymbol{\Psi}_s^s = \left[\psi_{sa}^s, \psi_{sb}^s\right]^T$,

$\boldsymbol{\Psi}_r^s = \left[\psi_{ra}^s, \psi_{rb}^s\right]^T$, $\boldsymbol{u}_s^s = \left[u_{sa}^s, u_{sb}^s\right]^T$, $\boldsymbol{i}_s^s = \left[i_{sa}^s, i_{sb}^s\right]^T$ are stator and rotor flux, stator voltage and current, respectively. $T_e$ is motor torque and $T_L$ is load torque. $J$ is inertia of the rotor and $p$ is the number of pole pairs, and $\omega_{\mathrm{r}}$ is the mechanical rotor angle velocity. $R_{\mathrm{s}}$ and $R_{\mathrm{r}}$ are the stator and rotor resistance, whereas $L_{\mathrm{s}}$ and $L_{\mathrm{r}}$ are the stator and rotor self-inductance. $L_{\mathrm{m}}$ is the mutual inductance and $\sigma = 1 - L_m^2 / \left(L_r L_s\right)$.

In order to illustrate the nonlinear behavior of induction motor control, a theoretical deduction of torque derivatives (4) was carried out. This yields:

$$\frac{dT_e}{dt} = \frac{2p}{3}\frac{L_m}{\sigma L_s L_r}\left(\boldsymbol{\Psi}_r^s \times \boldsymbol{u}_s^s - p\omega \boldsymbol{\Psi}_s^s \bullet \boldsymbol{\Psi}_r^s\right) - \left(\frac{R_s}{\sigma L_s} + \frac{R_r}{\sigma L_r}\right) T_e \quad , \qquad (6)$$

where $\times$ indicates dot product (active term) and $\cdot$ indicates scalar product (residual term). Equation (6) shows that the torque derivative is composed of two terms: an active term which is dependent on the voltage space vector and a residual term which is a linear function of delivered torque and rotor speed. The active term will be influenced by the stator flux controller. The residual term depends only on the operation point on the torque/speed plane, and is then supported to vary in a very wide range, depending on the demands of the mechanical process.

The conventional field oriented approach and direct torque control don't take full advantage and capability of servo drive with induction motor, because they both don't consider the torque derivative (6) in the control scheme. The proposed sliding mode torque control scheme also includes the derivative (6) thus slightly improving the control of induction motor.

It is known that the value of rotor flux is needed for the implementation of torque or speed control. Unfortunately, it cannot be measured directly. If the rotor speed is available, the rotor flux can be estimated with a simple observer exploiting the well-known current model. However, if no information about the mechanical variables is acquired, the design of the observer is not a trivial problem. Our objective in this paper is to design a torque tracking control for the IM given by (1)-(5) without the measurement of mechanical variables. Therefore, we have the following problem statement: design a flux/speed observer to estimate the flux and speed simultaneously based on the measurement of the stator currents and voltages and then design a corresponding controller to guarantee that the real torque tracks the desired one.

## 3. PROPOSED CONTROL SYSTEM

The control objective in this section is to design a torque tracking controller for the electromechanical system given by (1)-(6). Specifically, based on the rotor flux and speed estimation strategy described in this section, we design a corresponding sliding mode torque controller to guarantee the asymptotic stability of the sliding mode observer and torque tracking controller. The additional goal of the control dictated by technological requirements is to make the flux track the command flux input.

### *3.1 Torque control*

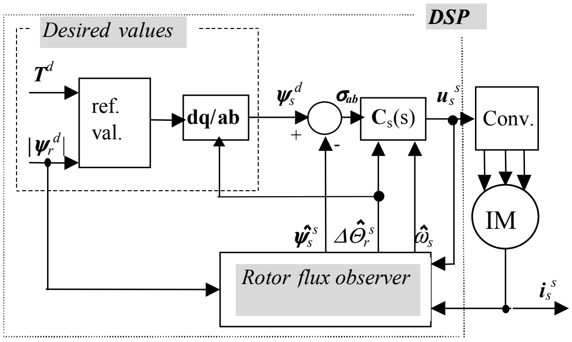


Fig.1. Proposed control scheme with the included closed-loop observer

The proposed control system is summarized in the scheme on Fig. 1. The desired stator flux components in a rotor flux reference frame $\boldsymbol{\Psi}_s^d = \left[\psi_{sd}^d, \psi_{sq}^d\right]^T$ are calculated from the torque and rotor flux commands

$$\psi_{sd}^d = \frac{L_s}{L_m}\left(\psi_r^d + \sigma \frac{L_r}{R_r}\frac{d\psi_r^d}{dt}\right), \qquad (7)$$

$$\psi_{sq}^d = \frac{3}{2}\frac{\sigma L_s L_r}{pL_m}\frac{T_e^d}{\psi_r^d}, \qquad (8)$$

and afterwards transformed into a stator reference frame. The transformation angle $\theta_r^s$ is the phase angle of the estimated rotor flux vector in a stator reference frame and can be defined as

$$\hat{\theta}_r^s = \arctan\left(\hat{\psi}_{rb}^s / \hat{\psi}_{ra}^s\right). \qquad (9)$$

By comparing the reference value of the stator flux space vector with the estimated one, the error in the stator flux space vector ($\boldsymbol{\sigma}_s = \boldsymbol{\Psi}_s^d - \hat{\boldsymbol{\Psi}}_s^s$) is readily obtained. The knowledge of this error allows the determination of the appropriate voltage space vector $\boldsymbol{u}_s^d$, which the PWM inverter has to apply to IM during next sampling period. The desired voltage space vector of PWM inverter is calculated with the use of continuous sliding mode control law in discrete form (Sabanovic, *et al.,* 1994; Rodic, *et al.,* 2000) and will be explained in next section.

### *3.2 Chattering free sliding mode control*

The VSS theory (Sabanovic, 1993; Sabanovic, *et al.,* 1994; Utkin, 1981) has been applied to nonlinear processes. One of the main features of this approach is that one only needs to drive the error to a "switching surface", after which the system is in "sliding mode" and will not be affected by any modeling uncertainties and/or disturbances. Let us consider the plant

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t) + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}, \qquad (10)$$

with $\mathrm{rank}(\boldsymbol{B}) = m$, $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{u} \in \mathbb{R}^m$. In VSS control, the goal is to keep the system motion on the manifold $\boldsymbol{S}$, which is defined as

$$\boldsymbol{S} = \{\boldsymbol{x} : \boldsymbol{\sigma}(\boldsymbol{x},t) = \boldsymbol{0}\}. \tag{11}$$

The solution to achieve this goal can be calculated from the requirement that $\boldsymbol{\sigma}(\boldsymbol{x},t) = \boldsymbol{0}$ is stable. The control should be chosen such that the candidate Lyapunov function satisfies the Lyapunov stability criteria. The aim is to force the system states to the sliding surface defined by

$$\boldsymbol{\sigma} = \boldsymbol{G}(\boldsymbol{x}^d - \boldsymbol{x}). \tag{12}$$

Firstly, a candidate Lyapunov function is selected as

$$v = \frac{\boldsymbol{\sigma}^T \boldsymbol{\sigma}}{2} > 0 \text{ and } \dot{v} = \boldsymbol{\sigma}^T \dot{\boldsymbol{\sigma}} < 0. \tag{13}$$

It is aimed that the derivative of the Lyapunov function is negative definite. This can be assured if we can somehow make sure that

$$\dot{v} = -\boldsymbol{\sigma}^T \boldsymbol{D}\boldsymbol{\sigma} < 0, \tag{14}$$

$\boldsymbol{D}$ is always positive definite.

From (13) and (14)

$$\dot{\boldsymbol{\sigma}} = -\boldsymbol{D}\boldsymbol{\sigma} \tag{15}$$

can be written. Equalizing (15) to zero results in what is known as "equivalent control". In other words, the control that makes the derivative of the sliding function equal to zero is called equivalent control. Derivative of (12) is

$$\boldsymbol{G}\dot{\boldsymbol{x}}^d - \boldsymbol{G}(f(\boldsymbol{x},t) + \boldsymbol{B}\boldsymbol{u}_{eq}) = \boldsymbol{0}. \tag{16}$$

As a result, the equivalent control can be written in the following form:

$$\boldsymbol{u}_{eq} = (\boldsymbol{G}\boldsymbol{B})^{-1} \boldsymbol{G}(f(\boldsymbol{x},t) - \dot{\boldsymbol{x}}^d). \tag{17}$$

From derivative of (12) and using (17)

$$\frac{d\boldsymbol{\sigma}}{dt} = (\boldsymbol{G}\boldsymbol{B})(\boldsymbol{u}_{eq} - \boldsymbol{u}) \tag{18}$$

is obtained. Then, another equation for equivalent control can be written as given below.

$$\boldsymbol{u}_{eq}(t) = \boldsymbol{u}(t) + (\boldsymbol{G}\boldsymbol{B})^{-1} \frac{d\boldsymbol{\sigma}}{dt} \tag{19}$$

By using the definition given by (10) and (12) in (15)

$$\boldsymbol{G}(\dot{\boldsymbol{x}}^d - \dot{\boldsymbol{x}}) = \boldsymbol{G}(\dot{\boldsymbol{x}}^d - f(\boldsymbol{x},t) - \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}) = -\boldsymbol{D}\boldsymbol{\sigma} \tag{20}$$

the control is obtained as:

$$\boldsymbol{u} = (\boldsymbol{G}\boldsymbol{B}(\boldsymbol{x}))^{-1} \left(\boldsymbol{G}\left(\dot{\boldsymbol{x}}^d - f(\boldsymbol{x},t)\right) + \boldsymbol{D}\boldsymbol{\sigma}\right). \tag{21}$$

Using (11) for the equivalent control can be written as:

$$\boldsymbol{u}(t) = \boldsymbol{u}_{eq}(t) + (\boldsymbol{G}\boldsymbol{B})^{-1} \boldsymbol{D}\boldsymbol{\sigma}. \tag{22}$$

By looking at (19) estimation for $\boldsymbol{u}_{eq}$ can be made using the property that $\boldsymbol{u}(t)$ is continuous and can not change too much in a short time as given below:

$$\hat{\boldsymbol{u}}_{eq}(t) = \hat{\boldsymbol{u}}(t - \delta t) + (\boldsymbol{G}\boldsymbol{B})^{-1} \frac{d\boldsymbol{\sigma}}{dt}, \tag{23}$$

where $\delta t$ is a short delay time. This estimation is also consistent with the logic that $\boldsymbol{u}_{eq}$ is selected as the average of $\boldsymbol{u}$. By putting (23) into (22), we get the last form for the controller

$$\boldsymbol{u}(t) = \boldsymbol{u}(t - \delta t) + (\boldsymbol{G}\boldsymbol{B})^{-1}(\boldsymbol{D}\boldsymbol{\sigma} + \dot{\boldsymbol{\sigma}}). \tag{24}$$

By using Euler interpolation

$$\boldsymbol{u}(t) = \boldsymbol{u}(t - \delta t) + \frac{(\boldsymbol{G}\boldsymbol{B})^{-1}}{\delta t}\left((\boldsymbol{D}\delta t + 1)\boldsymbol{\sigma}(t) - \boldsymbol{\sigma}(t - \delta t)\right). \tag{25}$$

The next step in controller design is the discretization of (25) into the state equation on the assumption that the voltage vectors $\boldsymbol{u}_s(k)$ and $\boldsymbol{u}_s(k-1)$ are constant within small *kT*-th and *(k+1)T*-th sampling intervals. The stator flux control problem can be formulated as follows: let us define the input voltage vector in (1) so that each error of the vector $\boldsymbol{\sigma}_s = \boldsymbol{\Psi}_s^d - \hat{\boldsymbol{\Psi}}_s^s$ asymptotically approaches zero as quickly as possible at varying load parameters and operating conditions. For the relatively small sampling time $T$ achieved in advanced PWM controlled inverters, the magnitude of supply can be assumed to remain constant, so the new value of the voltage vector can be predicted from the previous value by rotation in the plane:

$$\boldsymbol{u}_s^d(k) = \boldsymbol{C}(\Delta\theta_r)\boldsymbol{u}_s^d(k-1);$$
$$\boldsymbol{C}(\Delta\theta_r) = \begin{bmatrix} \cos\Delta\theta_r & -\sin\Delta\theta_r \\ \sin\Delta\theta_r & \cos\Delta\theta_r \end{bmatrix}. \tag{26}$$

Desired voltage $\boldsymbol{u}_s^d$ is calculated as in discrete time expression:

$$\begin{aligned} \boldsymbol{u}_s^d(k+1) &= \boldsymbol{C}(\Delta\theta_r)\boldsymbol{u}_s^d(k) + (\boldsymbol{G}\boldsymbol{B}T)^{-1} \cdot \\ &((\boldsymbol{I} + T\boldsymbol{D})\boldsymbol{C}(\Delta\theta_r)\boldsymbol{\sigma}_s(k) - \boldsymbol{C}(2\Delta\theta_r)\boldsymbol{\sigma}_s(k-1)) \end{aligned}. \tag{27}$$

Our system will reach the sliding mode manifold $\boldsymbol{S}$ in infinite time, but we can define the ε–vicinity of the manifold, which is reached in finite time.

The matrices $\boldsymbol{C}(\Delta\theta_r), \boldsymbol{C}(2\Delta\theta_r)$ are used in algorithm for compensation of computational delay, namely the stator flux error $\boldsymbol{\sigma}_s(k)$, $\boldsymbol{\sigma}_s(k-1)$ are measured and computed from actual and reference stator flux in previous sampling time one step ($\Delta\theta_r$) and two steps ($2\Delta\theta_r$) backwards.

The stability condition of algorithm is fulfilled if $\boldsymbol{D}$ is selected so that all eigenvalues of ($\boldsymbol{I}$-$T\boldsymbol{D}$) are within the unity circle of z-plane. Since in addition to the stator flux control error derivative is further employed to estimate the changing tendency of flux error, the switching frequency of PWM inverter is reduced.

The ac phase voltage of PWM inverter is uniquely determined from $\boldsymbol{u}_s^d(k+1)$ using the simple coordinate transformation:

$$\boldsymbol{u}_{123} = \boldsymbol{K}_{ab}^{123}\boldsymbol{u}_s^d, \quad \boldsymbol{K}_{ab}^{123} = \begin{bmatrix} 1 & 0 \\ -1/2 & -\sqrt{3/2} \\ -1/2 & \sqrt{3/2} \end{bmatrix}. \tag{28}$$

## 4. SLIDING MODE FLUX AND SPEED OBSERVER

In the observer design suitable for the sensorless drive the observer control input should be a known function of the motor speed so that, after establishing sliding mode in torque tracking loop, the speed can be determined as unique solution. This leads to the following selection of the structure of the stator current observer with the combination of voltage and current model

$$\frac{d\hat{\boldsymbol{i}}_s^s}{dt} = \frac{1}{\sigma L_s}\left(\boldsymbol{u}_s^s - R_s\hat{\boldsymbol{i}}_s^s - \frac{L_m}{L_r}\left(-\hat{\boldsymbol{e}}_r^s + \frac{L_m}{L_r}\frac{R_r}{\sigma L_s}\hat{\boldsymbol{\Psi}}_s^s\right)\right) \quad (29)$$

with $\hat{\boldsymbol{e}}_r^s = \frac{\hat{R}_r}{\sigma L_r}\hat{\boldsymbol{\Psi}}_r^s - jp\omega_r\hat{\boldsymbol{\Psi}}_r^s$.

Estimation error is determined as

$$\frac{d\boldsymbol{\varepsilon}_i}{dt} = \frac{1}{\sigma L_s}\left(\frac{L_m}{L_r}\boldsymbol{\varepsilon}_e - R_s\boldsymbol{\varepsilon}_i + \left(\frac{L_m}{L_r}\right)^2\frac{R_r}{\sigma L_s}\left(\boldsymbol{\Psi}_s^s - \hat{\boldsymbol{\Psi}}_s^s\right)\right), \quad (30)$$

with $\boldsymbol{\varepsilon}_i = \boldsymbol{i}_s^s - \hat{\boldsymbol{i}}_s^s$, $\frac{d\boldsymbol{\varepsilon}_i}{dt} = \frac{d\left(\boldsymbol{i}_s^s - \hat{\boldsymbol{i}}_s^s\right)}{dt}$, and $\boldsymbol{\varepsilon}_e = \boldsymbol{e}_r^s - \hat{\boldsymbol{e}}_r^s$.

Sliding mode algorithm (25) could be used to calculate control, which has meaning of electromotive force

$$\hat{\boldsymbol{e}}_r^s(k) = \hat{\boldsymbol{e}}_r^s(k-1) + \frac{\sigma L_s}{T}\frac{L_m}{L_r}\left(\left(\boldsymbol{I} + T\boldsymbol{D}_i\right)\boldsymbol{\varepsilon}_i(k) - \boldsymbol{\varepsilon}_i(k-1)\right) \quad (31)$$

Rotor flux observer could be selected from (2) having the structure of flux model with additional convergence term $\Delta\boldsymbol{\Psi}_r^s$

$$\frac{d\hat{\boldsymbol{\Psi}}_r^s}{dt} = \frac{L_m}{L_r}\frac{R_r}{\sigma L_s}\hat{\boldsymbol{\Psi}}_s^s - \hat{\boldsymbol{e}}_r^s + \Delta\boldsymbol{\Psi}_r^s. \quad (32)$$

Now flux estimation error can be calculated as

$$\frac{d\boldsymbol{\varepsilon}_\psi}{dt} = \frac{d(\boldsymbol{\Psi}_r^s - \hat{\boldsymbol{\Psi}}_r^s)}{dt} = -\boldsymbol{\varepsilon}_e + R_r\frac{L_m}{L_r}\boldsymbol{\varepsilon}_i - \Delta\boldsymbol{\Psi}_r^s. \quad (33)$$

To ensure convergence $\Delta\boldsymbol{\Psi}_r^s$ could be selected in the form

$$\Delta\boldsymbol{\Psi}_r^s = K_\psi\boldsymbol{\varepsilon}_\psi. \quad (34)$$

Design parameter $K_\psi$ could be selected from (34) so that estimated rotor flux tends to its real value.

To analyze convergence of the estimates to real values for the proposed observer structure, we first need to analyze the rotor flux property. The rotor flux error yields from (33)

$$\begin{bmatrix}\varepsilon_{\psi a}\\ \varepsilon_{\psi b}\end{bmatrix} = -\begin{bmatrix}\hat{\psi}_{ra} & -\hat{\psi}_{rb}\\ \hat{\psi}_{rb} & \hat{\psi}_{ra}\end{bmatrix}\begin{bmatrix}\mu\\ \eta\end{bmatrix}; \quad \mu = \frac{\Delta\hat{x}_r\hat{x}_r + \Delta\hat{\omega}_r\hat{\omega}_r}{\hat{\omega}_r^2 + \hat{x}_r^2}; \quad \eta = \frac{\Delta\hat{x}_r\hat{\omega}_r - \Delta\hat{\omega}_r\hat{x}_r}{\hat{\omega}_r^2 + \hat{x}_r^2}, \quad (35)$$

and

$$\begin{bmatrix}\hat{\omega}_r\\ \hat{x}_r\end{bmatrix} = \frac{1}{\|\hat{\psi}_r\|^2}\begin{bmatrix}\hat{\psi}_{rb} & -\hat{\psi}_{ra}\\ \hat{\psi}_{ra} & \hat{\psi}_{rb}\end{bmatrix}\begin{bmatrix}\hat{e}_{ra}\\ \hat{e}_{rb}\end{bmatrix}, \quad (36)$$

$$\begin{bmatrix}\Delta\hat{\omega}_r\\ \Delta\hat{x}_r\end{bmatrix} = \frac{\sigma L_s L_r}{L_m T}\begin{bmatrix}\hat{\psi}_{ra} & -\hat{\psi}_{rb}\\ \hat{\psi}_{rb} & \hat{\psi}_{ra}\end{bmatrix}\left(\boldsymbol{D}_i\boldsymbol{\varepsilon}_i + \frac{d\boldsymbol{\varepsilon}_i}{dt}\right). \quad (37)$$

If sliding mode in current control (30) exists both $\Delta\hat{\omega}_r = 0$ and $\Delta\hat{x}_r = 0$, then $\mu = 0$ and $\eta = 0$ and consequently $\boldsymbol{\varepsilon}_\psi = 0$. $\hat{x}_r$ has a meaning of $\hat{x}_r = \hat{R}_r/(\sigma\hat{L}_r)$ and is estimated inverse value of leakage rotor time constant.

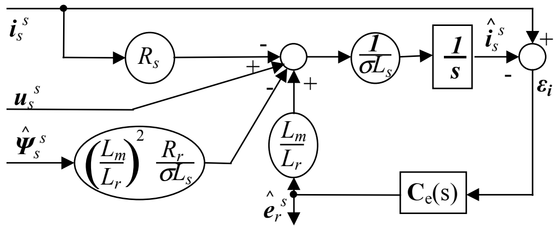


Fig. 2. Closed-loop EMF ($\hat{\boldsymbol{e}}_r^s$) observer.

In our proposed system shown in Fig. 2, the EMF is estimated by (31). The rotor flux is estimated by (32), where the estimated stator flux ($\hat{\boldsymbol{\Psi}}_s^s$) is replaced by the desired stator flux. The rotor speed and rotor leakage time constant are estimated by (36). The proposed sliding mode EMF observer shown in Fig. 2 is robust against motor parameter variation, such as stator resistance thermal variation and saturation of inductance parameters.

The synchronous speed can be estimated by the derivative of transformation angle

$$\hat{\omega}_s = \frac{d\hat{\theta}_r}{dt} = \frac{\dot{\hat{\psi}}_{rb}\hat{\psi}_{ra} - \dot{\hat{\psi}}_{ra}\hat{\psi}_{rb}}{\hat{\psi}_{ra}^2 + \hat{\psi}_{rb}^2}. \quad (38)$$

With the estimated rotor speed (36) the actual slip speed will be

$$\hat{\omega}_{sl} = \hat{\omega}_s - \hat{\omega}_r. \quad (39)$$

The stator and rotor resistance are also of great importance to system stability of servo drive. Therefore, an identification algorithm using the information on EMF (31) and estimated rotor flux with (36) can be used for rotor resistance estimation:

$$\hat{R}_r = \sigma L_r\frac{\hat{\psi}_{ra}\hat{e}_{ra} + \hat{\psi}_{rb}\hat{e}_{rb}}{\|\hat{\psi}_r\|^2}. \quad (40)$$

Stator and rotor resistance are explicitly expressed with (6). For interval of operation of induction motor with constant torque, the derivative of torque will be zero. From that the identification algorithm for resistance can be obtained:

$$\hat{R}_s = \sigma L_s\frac{\hat{\boldsymbol{\Psi}}_r^s \times \boldsymbol{u}_s^s - p\omega_r\hat{\boldsymbol{\Psi}}_r^s \bullet \hat{\boldsymbol{\Psi}}_s^s}{\hat{\boldsymbol{\Psi}}_r^s \times \hat{\boldsymbol{\Psi}}_s^s} - \frac{\hat{R}_r}{\sigma L_s}. \quad (41)$$

In the proposed sliding mode control scheme of the induction motor only the inductances $\sigma L_s$ and $L_m/L_r$ remain constant values.

## 5. FLUX CONTROL FOR EFFICIENCY IMPROVEMENT

Torque control is skillfully obtained by controlling the instantaneous slip frequency, whereas improvement of the efficiency can be achieved by controlling the amplitude of $\|\psi_r^d\|$ like a field-weakening operation. In the case of the frequent changes of the torque command, it is necessary to keep the flux amplitude at maximum. Under the field-weakening state, the sufficient torque or quick torque response would not be expected because of the slow response of the flux increase. On the other hand, in the steady-state operation, especially at small loads, the maximum efficiency can be obtained at a low flux level. Therefore, in order to obtain the maximum efficiency the flux level is adjusted in accordance with the torque command. The maximal allowed rotor flux would depend on actual stator frequency and dc-link voltage:

$$\psi_{r\max}^{d} = \frac{U_{dc}/\sqrt{3} - |\boldsymbol{i}_s^s|\,\hat{R}_s}{\hat{\omega}_s}, \tag{42}$$

and torque dependent efficiency optimal rotor flux will be (Stefanski and Karys, 1996):

$$\psi_{r\mathrm{opt}}^{d} = \frac{T_e^d}{p}\sqrt[4]{\frac{\hat{R}_s + \hat{R}_r}{\hat{R}_s + \dfrac{L_m^2}{\hat{R}_{gls} + \hat{R}_r}\hat{\omega}_s^2}}, \tag{43}$$

$\hat{R}_{gls}$ is the resistance of iron losses and should be determined by experimental identification. The actual rotor flux command will be used

$$\psi_{r\mathrm{opt}}^{d} \le \psi_{r\max}^{d}. \tag{44}$$

## 6. EXPERIMENTAL RESULTS

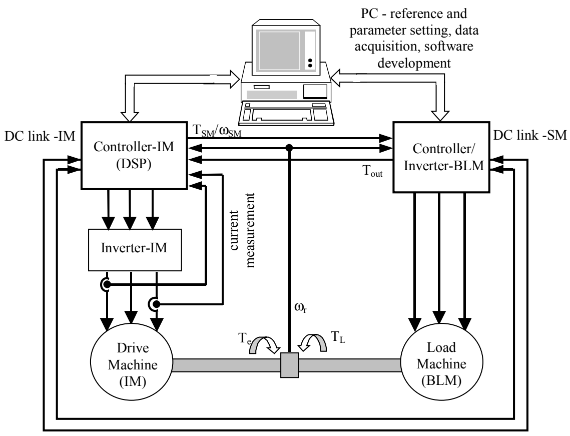


Fig.3. Experimental system.

Experiments were carried out to verify the feasibility of the developed control scheme using the 1.5 kW three-phase IM. Controller is based on Texas Instruments DSP TMS320C31 and the PWM inverter is built with IGBT module produced by Semicron. The controller has the diagnostic features that are necessary for drive installation test problem detection and elimination. The control algorithm is calculated every 160 μs. Brushless AC servomotor mechanical connected with the IM under test was used as the Load Machine. The control of both, speed and applied torque, is possible, thus hardware-in-the-loop operation can be performed (Fig. 3).

In the presented work the applied torque of the Load Machine is measured and thus the value of the torque applied by the Drive Machine is obtained. Desired, actual and observed values of torque are featured on Fig. 4, whereas Fig. 5 shows actual rotor speed and estimated stator frequency. From the figures the performance of the algorithm in the low speed area can be evaluated. Some problems occur during the transition through the stator frequency zero, however they are resolved by the presented control scheme, proving its convergence. Torque tracking performance of the system is satisfactory even at very low speeds including speed zero.

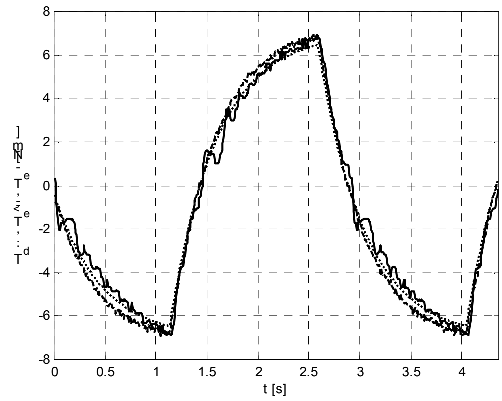


Fig. 4. Locked rotor - desired, estimated, and actual value of IM torque $T_e$.

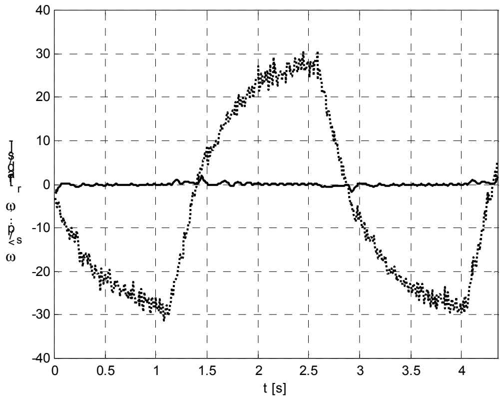


Fig. 5. Locked rotor - estimated value of stator frequency and actual value of speed

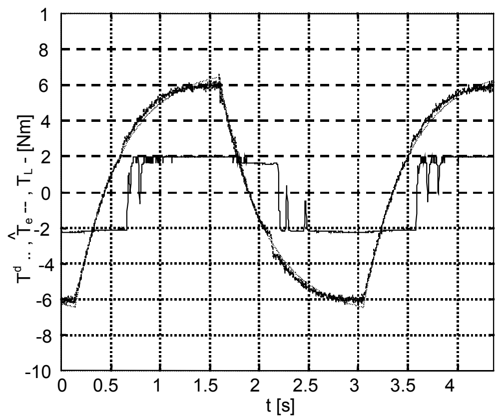


Fig. 6. Step load - desired and estimated value of IM torque $T_e$ and load torque $T_L$.

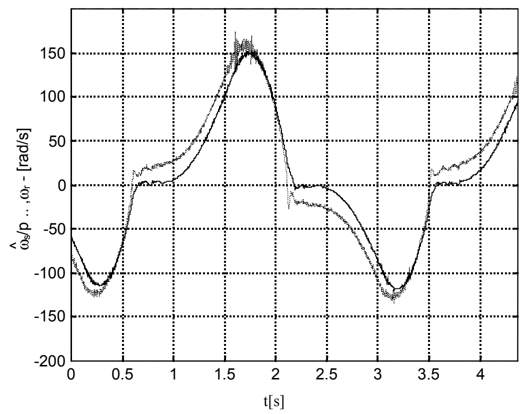


Fig. 7. Step load - estimated value of stator frequency and actual value of speed.

In the presented work the applied torque of the Load Machine is measured and thus the value of the torque applied by the Drive Machine is obtained. Desired, actual and observed values of torque are featured on Fig. 4, whereas Fig. 5 shows actual rotor speed and estimated stator frequency. From the figures the performance of the algorithm in the low speed area can be evaluated. Some problems occur during the transition through the stator frequency zero, however they are resolved by the presented control scheme, proving its convergence. Torque tracking performance of the system is satisfactory even at very low speeds including speed zero.

Second test investigates the performance at the load torque changing with nearly step function dependant on speed, as presented in (45):

$$T_L = c_3 \operatorname{sign}(\omega_r) . \tag{45}$$

The results are presented in Fig. 6 and Fig. 7. Fig. 6 features the desired and actual value of applied torque $T_e$ together with the load torque $T_L$. Again actual torque is not presented. The actual speed measured on the rotor axis together with the estimated value of stator frequency is presented in Fig. 7. By this experiment it is proven that the shape of load torque has no effect on the torque tracking performance of the presented control algorithm.

## 7. CONCLUSION

Sensorless torque and flux control of an IM is an emerging new technology, though in the early state of development. A sliding mode algorithm for torque and flux tracking control is presented in the paper. The algorithm is aimed to solve practical highly nonlinear problem of the operation of IM at high and low speed including zero speed. This algorithm is especially suitable for the applications where desired torque and rotor flux are varying during the operation. For example when efficiency of the IM operation is an important issue, like in electric vehicles. Simulations and experiments verified the performance of the proposed algorithm.

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