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NONLINEAR ADAPTIVE PREDICTIVE CONTROL STRATEGY APPLIED TO AN INTERCONNECTED TANK SYSTEM

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Abstract: This paper presents the application of a nonlinear controller, using a predictive control strategy to an interconnected tank system. The design procedure of the controller uses the mathematical input-output model of the plant based on the basic physical relations to find a controller output. The applied search strategy minimizes the cost function for a given prediction horizon.
Since the nonlinear model that has been developed is linear in its parameters, it is possible to use the *Least Squares* method to estimate them. The recursive version is applied in order to model time-varying effects and to track disturbances. Thereby the presented controller forms an adaptive control system. A similar control strategy is possible also for related problems since level control is a frequently occuring problem.

Keywords: Adaptive control, predictive control, level control, nonlinear systems.

1. INTRODUCTION

This paper deals with an interconnected tank system for that exact level control is required and that is exposed to different disturbances. It is an laboratory scaled plant with two inputs and three outputs that is used for the education of students as well as for testing control algorithms. By different valves it is possible to configure the plant in a very variable way, so that it is possible to create different disturbances and to change significantly the process characteristics.

The basic physical relations of the plant lead to a nonlinear mathematical model. The advantage of such a model compared to a linearized one is that it should be valid in the complete range of the plant operation. However, this requires also a nonlinear controller. The parameters of that model should be constant for a certain state of the plant and alter only if there are variations at the plant (e.g., a change of the valve positions between the tanks). To be able to track also such plant modifications, the chosen model was made *adaptive* by estimating its parameters *on-line*. Since the nonlinear model is linear in its parameters, it is possible to carry out this task using well-known identification procedures, in this case the *recursive least squares* algorithm.

The design method for the indirect adaptive controller applied to the tank system is a modification and extension of the one presented in (Pickhardt and Unbehauen, 1994; Pickhardt and Silva, 1998). It consists of a predictive search strategy based on a mathematical model of the process that for a **s**ingle-**i**nput **s**ingle-**o**utput (SISO-)plant minimizes the cost function

$$J = \sum_{i=1}^{N_2}[w((k+i)T_S) - \hat{y}((k+i)T_S)]^2 + \rho_{\mathrm{u}}\sum_{i=1}^{N_2}[\Delta u((k+i-1)T_S)]^2, \quad (1)$$

where w is the reference, $\hat{y}$ are the future predictions based on the plant-model of the controlled signal, u is the manipulated signal, N_2

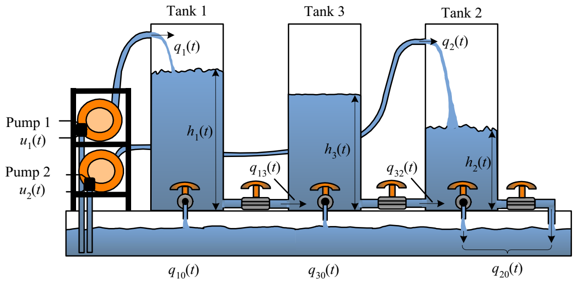

Fig. 1. Interconnected tank system

is the prediction horizon, T_S the sampling time and ρ_{u} is the weighting factor for the manipulated signal. Within the predictive controller, the manipulated signal is modified only for $N_{\mathrm{u}} \leq N_2$ samples, and for values i exceeding N_{u} the manipulated signal remains constant, i.e. $u((k+i)T_S) = u((k+N_{\mathrm{u}}-1)T_S)$ for the future values $N_{\mathrm{u}} \leq i < N_2$.

Since the tank system is a **m**ulti-**i**nput **m**ulti-**o**utput (MIMO-)plant with three outputs (tank levels) and two inputs (voltages for the pumps), the cost function that is minimized has to be changed to

$$J = \sum_{j=1}^{3} \gamma_j \left\{ \sum_{i=1}^{N_2} [w_j(k+i) - \hat{y}_j(k+i)]^2 \right\} + \rho_{\mathrm{u}} \sum_{j=1}^{2} \sum_{i=1}^{N_2} [\Delta u_j(k+i-1)]^2. \quad (2)$$

The paper is organized as follows: Section 2 gives a short description of the interconnected tank system. Section 3 explains the nonlinear model of the plant and the way how its parameters are estimated. In section 4 the applied nonlinear predictive controller is described. Section 5 portrays the results obtained with this adaptive control scheme at the plant. The paper is completed with a conclusion (section 6).

2. DESCRIPTION OF THE PLANT

Fig. 1 shows the functional characteristics of the interconnected tank system. It consists of three cylindric tanks that are connected by valves with each other and that are equipped with outlet tabs (tank 1 and 3 have one tab, tank 2 has two tabs). Both pumps convey an amount of water $q_1(t)$ and $q_2(t)$ from the bottom vessel into tank 1 and 2 that is proportional to the applied voltages $u_1(t)$ and $u_2(t)$, respectively. The controlled variables are the fluid levels of the three tanks that are measured using difference pressure sensors. Sensors and pumps are connected with an electric supply unit that transforms the input voltages within the range of $\pm 10V$ to the pumps and also adapts the signals from the sensors, so that they also lie in the range of $\pm 10V$.

The normalised range of each manipulated signal of the controller is $[0, 2]$: $u_i = 0$ results in a voltage of $-10V$ that is equivalent to the minimum flow $q_{i\min} = 0$ and $u_i = 2$ yields $+10V$ and the maximum possible flow $q_{i\max}$. The output voltages of the process are transformed from $\pm 10V$ into the range $[0.0, 0.6]$ which is equivalent to the fluid level $h_i = y_i$ of each tank in meters.

3. NONLINEAR MODEL OF THE PLANT

The level for each tank depends on the sum of the water flowing into and flowing off the tank that can be adjusted by the voltages of the pumps and, respectively, by the position of the valves between the tanks and the outlet tabs. Depending on the cross-cut of the opening that can be considered by a constant p, the amount of water flowing off by an outlet tab according to Torricelli's law is

$$q_{i0}(t) = p_{i1}\sqrt{h_i(t)}. \quad (3)$$

Accordingly, if the level in tank 3 is lower than in tank 1 and 2, one gets for the amount of water flowing into tank 3 by the valves between tank 3 and the tanks 1 and 2

$$q_{i3}(t) = p_{i2}\sqrt{h_i(t) - h_3(t)}, i\epsilon\{1, 2\}. \quad (4)$$

The amount of water $q_i(t)$ flowing into tank 1 and tank 2 can be described by

$$q_i(t) = p_{i0}u_i(t). \quad (5)$$

Based on these physical relations one gets immediately the equations to describe the nonlinear dynamic behaviour of the plant:

$$\frac{d}{dt}h_1(t) = p_{10}u_1(t) + p_{11}\sqrt{h_1(t)} \begin{cases} +p_{12}\sqrt{h_1(t)-h_3(t)}, & h_1(t) \geq h_3(t) \\ -p_{12}\sqrt{h_3(t)-h_1(t)}, & h_1(t) < h_3(t) \end{cases} \quad (6a)$$

$$\frac{d}{dt}h_2(t) = p_{20}u_2(t) + p_{21}\sqrt{h_2(t)} \begin{cases} +p_{22}\sqrt{h_2(t)-h_3(t)}, & h_2(t) \geq h_3(t) \\ -p_{22}\sqrt{h_3(t)-h_2(t)}, & h_2(t) < h_3(t) \end{cases} \quad (6b)$$

$$\frac{d}{dt}h_3(t) = p_{31}\sqrt{h_3(t)} \begin{cases} +p_{32}\sqrt{h_1(t)-h_3(t)}, & h_1(t) \geq h_3(t) \\ -p_{32}\sqrt{h_3(t)-h_2(t)}, & h_1(t) < h_3(t) \end{cases} \begin{cases} +p_{33}\sqrt{h_2(t)-h_3(t)}, & h_2(t) \geq h_3(t) \\ -p_{33}\sqrt{h_3(t)-h_2(t)}, & h_2(t) < h_3(t) \end{cases}. \quad (6c)$$

In these equations the constants p_{i0} are positive, p_{i1}, p_{12} and p_{22} are negative and due to the coupling of the tanks holds

$$p_{12} = -p_{32} \tag{7a}$$
$$p_{22} = -p_{33}. \tag{7b}$$

Although the equations (6) and (7) describe a *nonlinear* dynamic MIMO-system it is possible to determine the parameters p_{ij} with *linear* estimation algorithms: In discrete-time description all equations (6) can be represented as

$$h_i(k+1) = h_i(k) + \boldsymbol{m}_i^{\mathrm{T}}(k)\boldsymbol{p}_i, \quad i \in \{1,2,3\}. \tag{8}$$

The actual measurement vectors $\boldsymbol{m}_i(k)$ consists of the manipulated variables (voltages given to the pumps), the square roots of the levels $h_i(k)$ and the square roots of the difference of the levels. At each sampling instant it is necessary to make the distinctions according to eqs. (6) to obtain a positive argument of the square root.

If the parameters of the system are determined by a weighted estimation and its control is based on the nonlinear model thus obtained, this is an adaptive control concept. Since changes of the plant can be tracked by this identification, *all* possible disturbances with this plant that can be caused by changing valve positions and opening or closing of outlet tabs can be recognized and considered.

Since the levels of all the tanks are connected by the valves in between, with the two manipulated signals available it is only possible to control the levels of two tanks. The level of the third tank is determined automatically. Therefore, two of the three weighting factors $\gamma_1, \ldots, \gamma_3$ in equation (2), which have to be located in the interval $\gamma_i \in [0,1]$ are selected as 1 and the remaining factor is 0.

4. THE APPLIED CONTROL ALGORITHM

The control strategy developed for SISO-systems that has been presented in detail in Pickhardt & Unbehauen, (1994), and in Pickhardt & Silva, (1998) has been extended and adapted for MIMO-systems. Its aim is to find a trajectory for u that minimizes eq. (2). The following procedure is applied:

1. For all future values of u, N_{u} values $\frac{\Delta J}{\Delta u(k+i-1)} \approx \frac{\partial J}{\partial u(k+i-1)}$, $1 \leq i \leq N_{\mathrm{u}}$ are determined approximately by increasing and, respectively, decreasing each $u(k+i-1)$ with a small amount Δu. In order not to violate given limits for u a further increase and decrease of a certain $u(k+i)$ may not be possible if already $u(k+i-1) = u_{\max}$ or $u(k+i-1) = u_{\min}$ is valid. If it is possible to reduce J this reduction is saved in a vector $\boldsymbol{u_\delta}$ containing N_{u} elements. If $\boldsymbol{u_\delta}$ has been determined, all elements are divided by that element with the maximum amount. This normalizes $\boldsymbol{u_\delta}$ to the range $[-1, 0]$.
2. A step size k_{u} is selected that starts with the maximum value $k_{\mathrm{u_{max}}}$. The vector $k_{\mathrm{u}}\boldsymbol{u_\delta}$ is added to the vector with the future values of u and the corresponding J is computed. Here, likewise, given limits $u_{\min} \leq u(k+i-1) + k_{\mathrm{u}} u_\delta(k+i-1) \leq u_{\max}$ must not be violated.
3. Step by step k_{u} is decreased until either the optimal step size $k_{\mathrm{u_{opt}}}$ has been found that yields the maximum decrease of J or until the minimum step size $k_{\mathrm{u_{min}}}$ given by the user has been reached. $k_{\mathrm{u_{min}}}$ must not be smaller than the value Δu applied in the first step to determine the elements of the vector $\boldsymbol{u_\delta}$.
4. If it is possible to decrease J, the procedure continues with step 1. If, on the other hand, the minimum step size was reached, $k_{\mathrm{u_{min}}}$ it is obviously not possible to reduce the cost function by a simultaneous change of *several* future values of u. In this case it is aimed to change only that value of $u(k+i-1)$ for which $u_\delta(k+i-1) = -1.0$ is valid. If thereby it is possible to reduce J, again the algorithm continues with step 1, otherwise the optimal trajectory for u has been found.

The difference to the original version is that now in each step generally not only one, but all future values of u are changed, which increases the speed to find the optimal value for J. The applied method is based on a gradient procedure to optimize J. The gradient approximately is determined by $\frac{\partial J}{\partial \boldsymbol{u}}$. The factor k_{u} is determined in such a way that the modified vector containing the future values of u $\boldsymbol{u} + k_{\mathrm{u}} \frac{\partial J}{\partial \boldsymbol{u}}$ causes the maximum decrease of the cost. This method – determination of the gradient and optimization of J by adapting the future values of u, where given limits for u are not violated, is repeated until a further readuction of J is no more possible. One of the nice features of the described method is that at any time (e.g., if the sampling time is passed and the calculation of the optimal trajectory has not yet been finished) it is possible simply to stop the algorithm and to transfer the actual value $u(k)$ determined so far to the plant.

The user has to choose

(a) the prediction horizons N_{u} and N_2;

(b) the weighting factor ρ_{u};

(c) the start and the stop values $k_{\mathrm{u_{max}}}$ and $k_{\mathrm{u_{min}}}$ for the factor k_{u} that is used to change u.

5. ACHIEVED RESULTS

The conditions of the experiment displayed in figures 2 and 3 were as following: The sampling time was 2.5 seconds and the weighting factor was $\rho_u = 5 \times 10^{-5}$. The prediction horizon was $N_2 = 10$ steps for the controlled variables and $N_u = 3$ steps for the manipulated variables. The valve between tank 1 and 3 is open and the one between tank 2 and 3 is approximately half open, the tab of tank 3 is closed. The constellation of the valves and the tabs is selected in a way that allows all combinations of levels in the tanks. The case $h_3 > h_1 \wedge h_3 > h_2$, however, can be achieved only during a transition phase, where as well h_1 and h_2 are reduced rapidly. For different levels h_1 and h_2 in the end h_3 will be in between h_1 and h_2, but will be nearer to h_1 since the connection beween the tanks 1 and 3 is fully opened.

The main target of the experiment was to examine the effects of a disturbance. First, the outlet tab of tank 1 was half opened and that of tank 2 fully opened. Right at the beginning, when the estimated parameters have converged and are constant, the position of these tabs was exchanged.

To identify the paramters of the plant the recursive LS-estimation algorithm with a weighting near one ($\rho_{LS} = 0.995$) was used. For each of the three tanks an independent estimator was used, i.e. according to eqs. (6) and eqs. (7) for each tank three parameters have to be determined. With this method, the condition according to eqs. (7) is not fulfilled automatically. To secure this one could either

a) utilize the parameters p_{12} and p_{22} with tank 3 and estimate p_{31} only, or
b) use p_{32} with tank 1 and p_{33} with tank 2 and estimate p_{i0} and p_{i1} only.

However, this would have the disadvantage that the estimates are dependent on each other. For example, if b) is realized, the estimation of the parameters p_{i0} and p_{i1} for tanks 1 and 2 can be correct only if the estimation for tank 3 determines the true values for p_{32} and p_{33}.

If the model is appropriate to describe the plant the following relations will hold regarding the estimated parameters:

$p_{10} \approx p_{20} > 0$, since both pumps are of the same type (9a)

$0 > p_{21} > p_{11}$ (after the change), due to the position of the outlet tabs (9b)

$0 > p_{12} \approx -p_{32}, 0 > p_{22} \approx -p_{33}$ and $|p_{12}| > |p_{22}|$ due to the position of the valves between the tanks and (9c)

$p_{31} \approx 0$, since the outlet tab of tank 3 is closed. (9d)

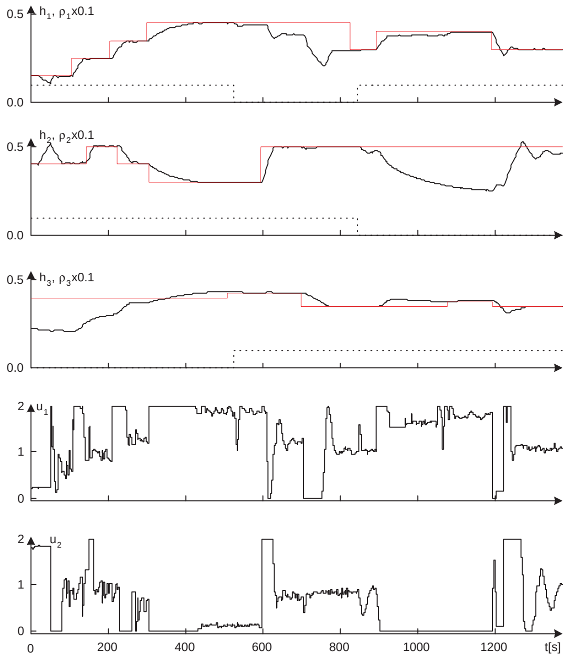

Fig. 2. Control of the interconnected tank system, $N_2 = 10$, $N_u = 3$

Determing p_{12}, p_{22}, p_{32} and p_{33} is anyhow difficult and dependent on changing tank levels. At equal levels $h_1 - h_3$ as well as $h_2 - h_3$ are 0 and for the correct value of p_{31} eq. (6c) is fulfilled for any p_{32} and p_{33}. If $h_1 - h_3 = h_3 - h_2$, again eq. (6c) is automatically fulfilled for all parameters p_{32} and p_{33} with $p_{32} = p_{33}$.

The result of the disturbance applied just after the start of the experiment is an increase of h_2 above and a decrease of h_1 below its references, whereas h_3 does not change much. After 60 seconds, the disturbance is almost compensated, however, since the parameters are not correctly estimated, the manipulated variables are still unsettled. Only after 430 seconds, wenn the controller outputs leave the saturation ($u_1 = 2.0$, $u_2 = 0.0$), they have again the same shape as before the disturbance.

During the experiment, the levels of different tanks have been controlled. First the levels of the tanks 1 and 2 are controlled, and the desired references can be reached exactly throughout the complete range of operation. When the reference is changed, the controller frequently operates at the given limits. However, to reach the reference faster after it has been increased would be possible only with more powerful pumps.

After 520 seconds the weightings γ_1 and γ_3 are exchanged, i.e. from this moment onwards the levels of tank 2 and 3 are controlled. Without giving a numerical analysis it is obvious that the controllability of the level of tank 3 is less than of tank 1: The direct impact of a manipulated variable is missing since it is only possible to control this level indirectly using the neighbouring tanks. Moreover, a change of the level of tank 1 due to the factor

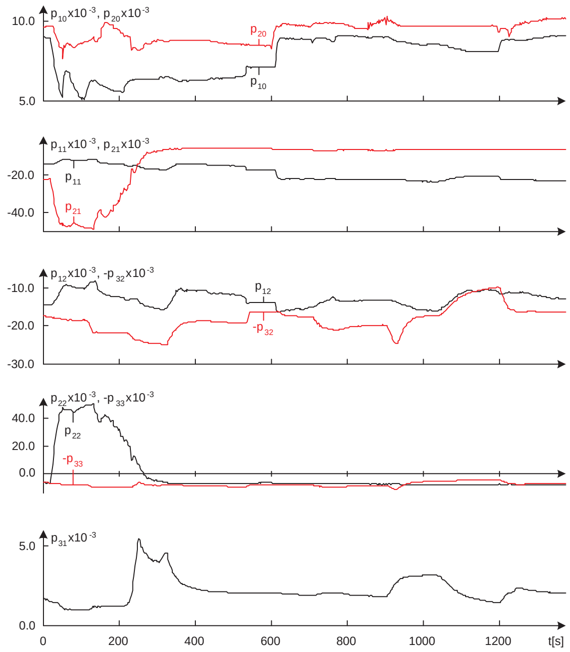

Fig. 3. Estimated parameters for the interconnected tank system, $N_2 = 10$, $N_u = 3$

$\sqrt{|h_1 - h_3|}$ causes a much smaller change of the level of tank 3 only. Nevertheless, the control behaviour is almost similar as before. An important presupposition for a good control performance regarding the level of tank 3 obviously is that the parameters of eq. (6c) have been determined precisely.

To control tank 3 with the pump of tank 2 (after 850 seconds) is even more difficult than with the pump of tank 1, since the valve between these two tanks is only half opened: At a constant level of tank 1 differences of several centimeters in tank 2 have only a very small and slow impact on the level of tank 3. The result is quite satisfactory, even if a slowly decreasing oscillation of u_2 appears at the end of the experiment. It has, however, almost no influence on h_3, which confirms the statement made regarding the controllability.

Interesting is here the period after 900 seconds. The required reference w_3 cannot be reached, although u_2 is permanently zero and h_2 is decreasing. The reference $w_1 = 0.4$ would be possible, however, since the cost function of this MIMO-plant equally evaluates the level of both tanks, h_1 is kept below its reference. Otherwise, h_3 would even more deviate from $w_3 = 0.35$ and the value of the cost to be minimized would increase. After about 1080 seconds w_3 is increased to 0.375, and thereafter $h_1 = w_1$ as well as $h_3 = w_3$ is reached.

The trend diagram of the estimated parameters in fig. 3 confirms the relations of eqs. (9). This result therefore means that eqs. (6) indeed are appropriate to describe the dynamic behaviour of the pilot plant.

p_{10} and p_{20} are almost identical. They should not be affected by the change of the outlet tabs, but after a disturbance always *all* parameters of the concerned tanks change, and only after 650 seconds p_{10} and p_{20} have again the same value as before this disturbance.

However, the applied disturbance touches only tank 1 and tank 2, and their parameters change. The parameters for tank 3 remain constant. This is one of the advantages of the applied method to use an independent estimator for each tank and not to couple them. Otherwise, by mistake also the estimated parameters of tank 3 would have changed due to the influence of the disturbance.

At the beginning, p_{11} has approximately half the value as p_{21}. At the end of the experiment it is just reverse. This is expected because the position of the outlet tabs of tank 1 and tank 2 was exchanged. The parameters p_{12} and $-p_{32}$ as well as p_{22} and $-p_{33}$ do not agree exactly, but the order of magnitude is the same. At the end of the test, when all estimated parameters have been converged towards their true values, the eqs. (7) are almost fulfilled. Due to the different position of the valves between the tanks $|p_{12}| > |p_{22}|$ should be valid which is confirmed by the trend curves in fig. (3). Finally it has to be portrayed that, as expected, p_{31} is almost zero.

6. CONCLUSIONS

In this paper, the application of a nonlinear predictive controller to an interconnected tank system using a nonlinear model is presented. It shows good results and exhibits a very smooth controller output. Since the parameters of the nonlinear model are estimated *on-line*, it is an adaptive control scheme that is able to track changes of the plant.

The applied control scheme seems to be especially appropriate if a suitable nonlinear model of the plant to be controlled can be found, as it is the case here based on the physical equations describing the plant dynamics.

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