15th Triennial World Congress, Barcelona, Spain

ON MONOTONIC CONVERGENCE OF HIGH ORDER ITERATIVE LEARNING UPDATE LAWS [1]

Kevin L. Moore * **and YangQuan Chen** *

* *Center for Self-Organizing and Intelligent Systems (CSOIS)*
Dept. of Electrical and Computer Engineering
UMC 4160, College of Engineering, 4160 Old Main Hill
Utah State University, Logan, UT 84322-4160, USA

Abstract: High-order iterative learning control (ILC) in both iteration domain and time domain is investigated in this paper. We are interested in whether a high order iterative learning updating law is helpful in achieving a monotonic convergence in a suitable norm topology other than the exponentially weighted sup-norm. Discrete-time linear time invariant system is considered. With simulation illustrations, it is shown that a high-order scheme in both time domain and iteration domain is helpful. A new design framework for high order ILC is proposed.

Keywords: iterative learning control; high order updating law; time domain; iteration domain; small gain theorem; internal model principle.

1. INTRODUCTION

Iterative learning control (ILC) is proposed as a *value-added block* to enhance the feedback control performance by utilizing the fact that the system is operated repeatedly for the same task (Arimoto *et al.*, 1984; Moore, 1993). While the formal mathematically rigorous analysis is initially due to (Arimoto *et al.*, 1984), the basic idea can be traced back to (Uchiyama, 1978) and even to (Garden, 1967) which is commented in (Chen and Moore, 2000). Detailed literature reviews and recent developments on ILC research can be found in (Moore, 1999; Bien and Xu, 1998; Chen and Wen, 1999).

It is observed in (Lee and Bien, 1997) that although the λ-norm of tracking error from iteration to iteration can be proved to decay monotonically, the ∞-norm or sup-norm may increase to a huge value before it converges to the desired level. This transient behavior, which is a serious concern in the practical application of ILC schemes, can be improved by using an exponentially decay learning gain as discussed in (Lee and Bien, 1997). One may argue that to make the convergence monotonic in sup-norm or 2-norm, one can use a high-gain feedback (Owens, 1992)However, this is not practical because the high-gain feedback may saturate the actuators. The fact that in some ILC schemes the error can grow quite large before converging has also been qualitatively discussed in (Jang and Longman, 1994)from a frequency domain perspective. The effect can be explained as a result of the propagation of high-frequency components of the error by the ILC algorithm. Recently, in time domain, a condition for monotonic convergence of 1-norm of tracking errors is established in (Moore, 2001). There are some analysis results for monotonic convergence of ILC schemes via using approximate impulse response (Ishihara *et al.*, 1992), reduced sampling rate (Hillenbrand and Pandit, 2000) and for sampled data nonlin-

[1] This work is supported in part by U.S. Army Automotive and Armaments Command (TACOM) Intelligent Mobility Program (Agreement no. DAAE07-95-3-0023). Corresponding author: Dr YangQuan Chen. E-mail: yqchen@ieee.org; Tel. 01-435-7970148; Fax: 01-435-7972003. URL: http://www.crosswinds.net/~yqchen.

ear systems (Tayebi and Zaremba, 1999). How to achieve the monotonic convergence for discrete time systems via a proper ILC updating law design is addressed in a recent work (Moore *et al.*, 2001) where a time varying learning gain is used to achieve monotonic learning convergence.

On the other hand, high-order ILC was proposed to improve the ILC convergence (Bien and Huh, 1989; Chen *et al.*, 1998*b*,*a*). No convincing quantitative discussion can be found on why, how and in what sense high-order ILC outperforms the first order ILC scheme (Norrlöf and Gunnarsson, 2000). In this paper, the meaning of "*high-order*" is extended to the *iteration domain* and the *time domain*. We are interested in whether a high order iterative learning updating law is helpful in achieving a monotonic convergence in a suitable norm topology other than the exponentially weighted sup-norm, i.e., λ-norm. Discrete-time linear time invariant system is considered. With simulation illustrations, it is shown that a high-order scheme in both time domain and iteration domain is helpful. The major contribution of this paper is to point out two important facts **(1)** *the high-order in time-axis is to condition the system dynamics so that a monotonic convergence can be achieved* and **(2)** *the high-order in iteration-axis is to reject the iteration-dependent disturbance by virtue of the internal model principle (IMP)*. A new design framework for high order ILC is proposed.

2. NOTATIONS AND PRELIMINARIES

Let an operation, or trial, of the system to be controlled be denoted by subscript "k" and let time during a given trial be denoted by "t," where $t \in [0, N]$. Each time the system operates the input to the system, $u_k(t)$, is stored, along with the resulting system error, $e_k(t) = y_d(t) - y_k(t)$, where $y_d(t)$ is the desired output. The plant to be controlled is a discrete-time, linear, time-invariant system of the form using $\mathcal{Z}$-transfer function:

$$\begin{aligned} Y(z) &= H(z)U(z) \\ &= (h_d z^{-d} + h_{d+1} z^{-(d+1)} + \cdots)U(z), \end{aligned} \quad (1)$$

where d is the relative degree of the system, z^{-1} is the standard delay operator in time, and the parameters h_i are the standard Markov parameters of the system $H(z)$. We will assume from here forward that $d = 1$. We will also assume the standard ILC reset condition: $y_k(0) = y_d(0) = y_0$ for all k. If we define the "supervectors" (Moore, 1998) $U_k = [u_k(0), u_k(1), \cdots, u_k(N-1)]^T$, $Y_k = [y_k(1), y_k(2), \cdots, y_k(N)]^T$, $Y_d = [y_d(1), y_d(2), \cdots, y_d(N)]^T$ and $E_k = [e_k(1), e_k(2), \cdots, e_k(N)]^T$, then the system can be written as

$$Y_k = H_p U_k, \quad (2)$$

where H_p is the matrix of Markov parameters of the plant, given by

$$H_p = \begin{bmatrix} h_1 & 0 & 0 & \dots & 0 \\ h_2 & h_1 & 0 & \dots & 0 \\ h_3 & h_2 & h_1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_N & h_{N-1} & h_{N-2} & \dots & h_1 \end{bmatrix}. \quad (3)$$

For this system, the learning controller's goal is to derive an optimal input $u^*(t)$, for $t \in [0, N-1]$ by evaluating the error $e_k(t) = y_d(t) - y_k(t)$ on the interval $t \in [1, N]$. This is accomplished by adjusting the input from the current trial (u_k) to a new input (u_{k+1}) for the next trial. This adjustment is done according to an appropriate algorithm. The so-called Arimoto-type discrete-time ILC algorithm is given by

$$u_{k+1}(t) = u_k(t) + \gamma e_k(t+1) \quad (4)$$

where γ is the constant learning gain.

Now following the notations of (Moore, 2000),introduce a new shift variable, w, with the property that

$$w^{-1} u_k(t) = u_{k-1}(t).$$

This is just the standard z-transform, re-named to reflect the fact that it is operating from trial-to-trial, with t fixed, as opposed to the standard z-transform operator which operates from time step-to-time step, with k fixed. Using the supervector, we can see that the 2D SISO dynamic system described by (1) and (4) can be transformed into 1D MIMO static plant (2). In w-domain, this 1D equivalent system including an iterative learning updating law, possibly high-order both in time domain and iteration domain, can be well described in general by Fig. 1 where L is the learning matrix and $C(w)$ is a scalar w-transfer function.

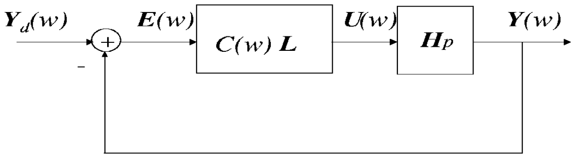


Fig. 1. w-plane closed loop control system interpretation of ILC

Referring to (4), this particular case corresponds to

$$C(w) = \frac{w^{-1}}{1 - w^{-1}}, \quad L = \gamma I.$$

The convergence properties of the Arimoto-type ILC algorithm have been well-established in the literature. Using a contraction mapping approach it is easy to see that the ILC scheme converges if the induced operator norm satisfies

$$\|I - \gamma H_p\|_i < 1. \quad (5)$$

Note that this sufficient condition ensures monotone convergence in the sense of the relevant norm topology. It is also possible to give the following necessary and sufficient condition for convergence (Moore, 1998):

$$|1 - \gamma h_1| < 1. \quad (6)$$

Unfortunately, this second condition does not guarantee monotone convergence as shown in (Moore, 2001). In addition to the necessary and sufficient condition for convergence (6), the other conditions to guarantee the monotone convergence can be found in a theorem given in (Moore, 2001), i.e.,

$$|h_1| > \sum_{j=2}^{N} |h_j|. \quad (7)$$

3. HIGH-ORDER IN TIME DOMAIN

Throughout this paper, we shall use second order IIR models for simulation. All initial conditions are set to 0. Further, h_1 is 1. Therefore, we fix γ=0.9, if not otherwise specified, such that $|1 - \gamma h_1| < 1$. In all simulation experiments, we fix N=60 and maximum number of iterations 60. The desired trajectory, if not otherwise specified, is a triangle with a maximum height 1 which is given by

$$y_d(t) = \begin{cases} 2t/N & , \ i = 1, \cdots, N/2 \\ 2(N-t)/N & , \ i = N/2+1, \cdots, N. \end{cases} \quad (8)$$

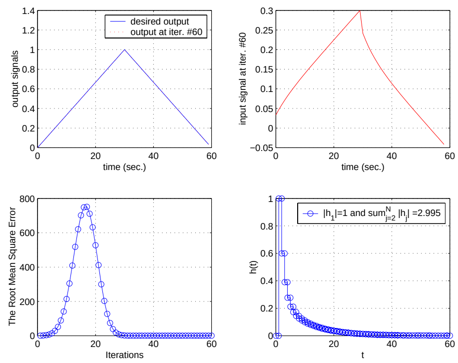
***Case 1.* Stable lightly damped system.** The $\mathcal{Z}$-transfer function for simulation is

$$H_1(z) = \frac{z - 0.8}{(z-0.5)(z-0.9)}. \quad (9)$$

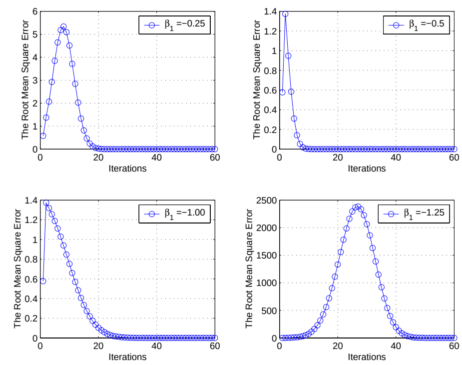
Fig. 2(a) shows the ILC result using the constant learning gain $\gamma = 0.9$ and updating law (4). The top left subplot of Fig. 2(a) shows the desired trajectory and the output at the 60-th ILC iteration where we can observe the good tracking result. The desired input signal at the 60-th ILC iteration is shown in the top right subplot of Fig. 2(a). The impulse response of $H_1(z)$ is shown in the bottom right subplot of Fig. 2(a) where we can read that h_1=1 and $\sum_{j=2}^{60} = 2.995$. Clearly, the monotonic condition (7) is not satisfied. Therefore, in the bottom left subplot of Fig. 2(a), a quite big peak transient can be observed for the root mean squares (RMS) of the tracking error with respect to iteration number. Now let's apply the high-order ILC scheme in time-domain

$$u_{k+1}(t) = u_k(t) + \gamma(e_k(t+1) - \beta_1 e_k(t)). (10)$$

For some different values $\beta_1 > 0$, for the same learning gain $\gamma = 0.9$, the resulted plots for tracking error RMS vs. iteration number are shown in Fig. 2(b). Clearly, tuning β_1 can make the ILC convergence monotonic.



(a) First order ILC



(b) Second order ILC in time-domain

Fig. 2. Time-domain high-order ILC for stable lightly damped system $H_1(z)$

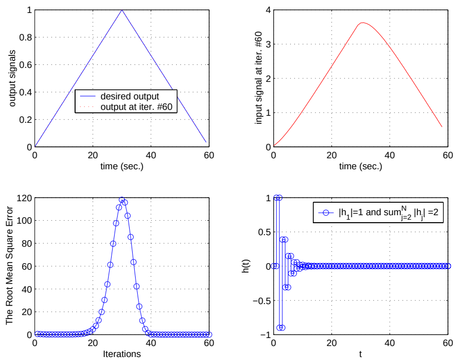
***Case 2.* Stable oscillatory system.** The $\mathcal{Z}$-transfer function for simulation in this case is

$$H_2(z) = \frac{z - 0.8}{(z-0.5)(z+0.6)}. \quad (11)$$

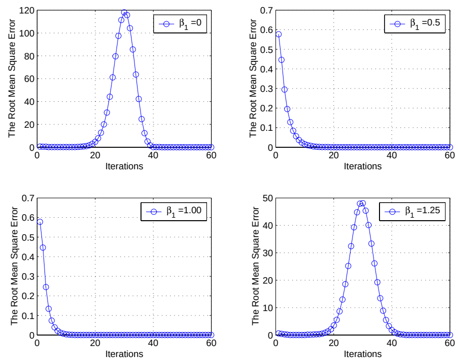
Fig. 3(a) shows the ILC result using the constant learning gain $\gamma = 0.9$. The descriptions about the subplots are similar to Fig. 2(a). For $H_2(z)$, here h_1=1 and $\sum_{j=2}^{60} = 2$. Clearly, the monotonic condition (7) is not satisfied, too. Therefore, in the bottom left subplot of Fig. 3(a), a big peak transient can again be observed. Now let's apply the high-order ILC scheme in time-domain

$$u_{k+1}(t) = u_k(t) + \gamma(e_k(t+1) + \beta_1 e_k(t)). (12)$$

For some different values $\beta_1 > 0$ and for the same learning gain $\gamma = 0.9$, the resulted plots for tracking error RMS vs. iteration number are shown in Fig. 3(b). Clearly, tuning β_1 can make the ILC convergence monotonic. However, note here that there is a difference in sign before β_1.



(a) First order ILC



(b) Second order ILC in time-domain

Fig. 3. Time-domain high-order ILC for stable lightly damped system $H_2(z)$

4. HIGH-ORDER IN ITERATION DOMAIN

In ILC, it is assumed that desired trajectory $y_d(t)$ and external disturbance are invariant with respect to iterations. We will show that when this assumption is invalid, the conventional integral type ILC (4) will no longer work well. Instead, we need to use high-order ILC scheme along iteration direction.

***Case a.* Ramp type disturbance in iteration domain.** Consider a stable plant

$$H_a(z) = \frac{z - 0.8}{(z - 0.55)(z - 0.75)}. \qquad (13)$$

Assume that $y_d(t)$ does not vary w.r.t. iterations. However, we add disturbance $d(k,t)$ at the output $y_k(t)$. In iteration k, the disturbance is a constant w.r.t. time but its value is proportional to k. Therefore, we can write $d(k,t) = c_0 k$. In the simulation, we set $c_0 = 0.01$. Using the conventional ILC scheme (4), the results are summarized in Fig. 4(a). We can clearly see the steady state error and the tracking performance in the final iteration is not satisfactory.

Now let's apply the high-order ILC scheme in iteration-domain

$$u_{k+1}(t) = 2u_k(t) - u_{k-1}(t) + \gamma(2e_k(t+1) - e_{k-1}(t+1)). \qquad (14)$$

The results are summarized in Fig. 4(b) where we can clearly observe that the steady state error is eliminated via the high-order scheme (14) and the tracking performance in the final iteration is now satisfactory. Note that, looking at the subplots regarding the 2-norm of control signal in Fig. 4, no much difference can be observed. Therefore, although the control efforts are similar, the results are totally different due the "*order*" of ILC updating law. It is interesting to note that, for this case, almost the same results are obtained if we use the ILC scheme

$$u_{k+1}(t) = 2u_k(t) - u_{k-1}(t) + \gamma(3e_k(t+1) - 2e_{k-1}(t+1)). \qquad (15)$$

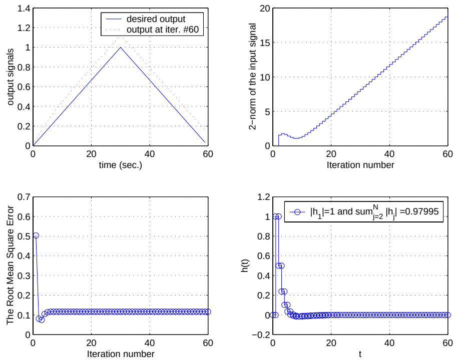
***Case b.* Alternating type disturbance in iteration domain.** Similar to Case a, now we change the disturbance to $d(k,t) = c_0(-1)^{k-1}$. In the simulation, we set $c_0 = 0.01$ as in Case b. This is an alternating disturbance. If the iteration number k is odd, the disturbance is positive constant in iteration k while when k is even, the disturbance jumps to a negative constant. In this example, we set $\gamma = 0.9$. Using the conventional ILC scheme (4), the results are summarized in Fig. 5(a). We can clearly see the steady state error and the tracking performance in the final iteration is not satisfactory. Now let's apply the high-order ILC scheme in the iteration-domain

$$u_{k+1}(t) = u_{k-1}(t) + \gamma e_{k-1}(t+1). \qquad (16)$$

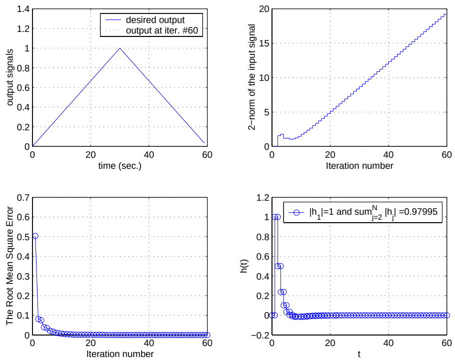
The results are summarized in Fig. 5(b) where we can clearly observe that the steady state error is eliminated via the high-order scheme (16) and the tracking performance in the final iteration is now satisfactory.

5. HIGH-ORDER ILC: DESIGN FRAMEWORK

In the previous two sections, we have shown via examples that the high-order in time-axis is to condition the system dynamics so that a monotonic convergence can be achieved and

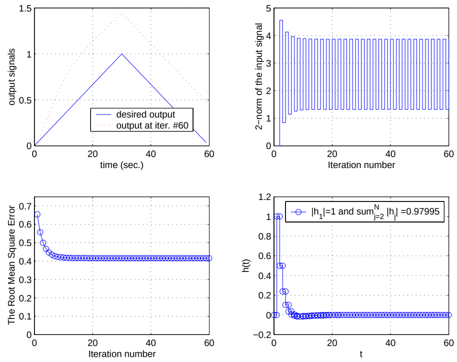


(a) First order ILC

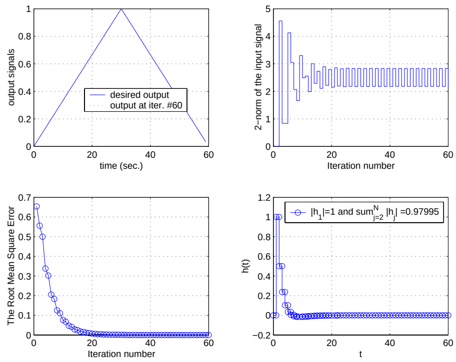


(b) Second order ILC in iteration domain

Fig. 4. ILC for stable system $H_a(z)$ with ramp type disturbance in iteration domain



(a) First order ILC



(b) Second order ILC in iteration domain

Fig. 5. ILC for stable system $H_a(z)$ with alternating type disturbance in iteration domain

the high-order in iteration-axis is to reject the iteration-dependent disturbance by virtue of the internal model principle (IMP). In this section, a new design framework for high order ILC is proposed.

Fig. 6 presents a general description of ILC scheme with uncertainties in both time and iteration axis with the use of the so-called w-transformation and supervector representation. In Fig. 6, Y_d, Y, U, E, H_p, $C(w)$ and L are the same as in Fig. 1. Here Y_d should be in the sense of $Y_d(w)$ which is generated from a constant or iteration-invariant vector signal R_d passed to the w transfer function matrix $F(w)$. Similarly, D is a constant vector signal and $G(w)$ is a w transfer function matrix. $G(w)D$ and $F(w)R_d$ describe, respectively, the disturbance and reference signals which may be variant w.r.t. to time and iteration. ΔH_p represents the uncertainty in plant model. When described by $\Delta(w)H_p$, the model uncertainty can be iteration-dependent. We can see that the description via Fig. 6 is fairly general to cover almost all ILC schemes such as high-order (in time and in iteration), iteration dependent uncertainties, disturbances and references.

Note that the general ILC block diagram in Fig. 6 is also valid for nonlinear systems since only finite time interval is concerned and a good linear approximation is always possible in practice. The price to pay here is the high dimension in the MIMO structure in Fig. 6. However, cost of memory is not critical nowadays and ILC computation can be done off-line. The more important is actually the robustness issue. Especially, the iteration-dependent disturbance or uncertainty cannot be suitably attacked in previous ILC framework. Therefore, the ILC analysis and design based on the general description in Fig. 6 are practically attractive.

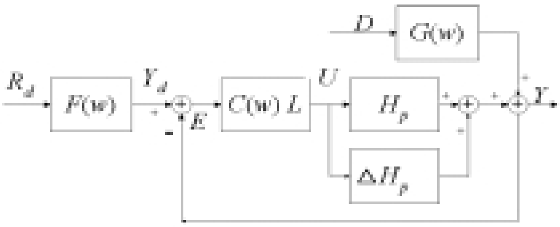


Fig. 6. General setting of the high-order ILC control design problem

Based on the framework shown in Fig. 6, we have developed a joint high-order ILC design method for $C(w)$ and L based on IMP and small gain theorem, which will be reported separately due to space limit.

6. CONCLUDING REMARKS

High-order iterative learning control (ILC) in both iteration domain and time domain is investigated in this paper. Via extensive simulation experiments, the major contribution of this paper is to point out two important facts **(1)** *the high-order in time-axis is to condition the system dynamics so that a monotonic convergence can be achieved* and **(2)** *the high-order in iteration-axis is to reject the iteration-dependent disturbance by virtue of the internal model principle (IMP).* A new design framework for high order ILC is outlined. Future research efforts would be in borrowing H_∞ notion to design $C(w)$. It is also valuable to investigate the application background which invalidates the conventional assumption that the system dynamics is invariant with respect to iteration.

References

Arimoto, S., S. Kawamura and F. Miyazaki (1984). 'Bettering operation of robots by learning'. *J. of Robotic Systems* **1**(2), 123–140.

Bien, Z. and Jian-Xin Xu (1998). *Iterative Learning Control - Analysis, Design, Integration and Applications.* Kluwer Academic Publishers.

Bien, Z. and K. M. Huh (1989). 'High-order iterative learning control algorithm'. *IEE Proc., Pt-D, Control Theory and Appl.* **136**(3), 105–112.

Chen, Y. and Changyun Wen (1999). *Iterative Learning Control: Convergence, Robustness and Applications.* vol. LNCIS-248. Springer-Verlag. London.

Chen, Y. and Kevin L. Moore (2000). Comments on US Patent #3555252: Learning control of actuators in control systems. In 'Proc. of The 6-th Int. Conf. on Control, Automation, Robotics and Vision'. Singapore.

Chen, Y., C. Wen, J.-X. Xu and M. Sun (1998*a*). 'High-order iterative learning identification of projectile's aerodynamic drag coefficient curve from radar measured velocity data'. *IEEE Trans. on Contr. Sys. Tech.* **6**(4), 563–570.

Chen, Y., Z. Gong and C. Wen (1998*b*). 'Analysis of a high order iterative learning control algorithm for uncertain nonlinear systems with state delay'. *Automatica* **34**(3), 345–353.

Garden, M. (1967). 'Learning control of actuators in control systems'. *United States Patent #3555252.*

Hillenbrand, S. and M. Pandit (2000). 'An iterative learning control with reduced sampling rate for plants with variations of initial states'. *Int. J. of Contr.* **73**(10), 882–889.

Ishihara, T., K. Abe and H. Takeda (1992). 'A discrete-time design of robust iterative learning controllers'. *IEEE Trans. on Systems, Man and Cybernetics* **22**(1), 74–84.

Jang, H. S. and R. W. Longman (1994). A new learning control law with monotonic decay of the tracking error norm. In 'Proceedings of the 15-th Annual Allerton Conf. on Communication, Control and Computing'. UIUC, USA. pp. 314–323.

Lee, H.-S. and Zeungnam Bien (1997). 'A note on convergence property of iterative learning controller with respect to sup norm'. *Automatica* **33**(8), 1591–1593.

Moore, K. L. (1993). *Iterative learning control for deterministic systems.* Advances in Industrial Control. Springer-Verlag.

Moore, K. L. (1998). Multi-loop control approach to designing iterative learning controllers. In 'Proceedings of the 37th IEEE Conference on Decision and Control'. Tampa, FL, USA. pp. 666–671.

Moore, K. L. (1999). 'Iterative learning control - an expository overview'. *Applied & Computational Controls, Signal Processing, and Circuits* **1**(1), 151–241.

Moore, K. L. (2000). A matrix fraction appraoch to higher-order iterative learning control: 2-D dynamics through repetition-domain filtering. In 'Proc. of the 2nd Int. Workshop on Multidimensional (ND) Systems'. Czocha Castle, Poland. pp. 99–104.

Moore, K. L. (2001). An observation about monotonic convergence in discrete-time, P-type iterative learning control. In 'Proc. of IEEE Int. Symp. on ISIC'. Mexico City, Mexico.

Moore, K. L., YangQuan Chen and Vikas Bahl (2001). 'Monotonically convergent iterative learning control for linear discrete-time systems'. *for submission to Automatica* pp. 1–16.

Norrlöf, M. and S. Gunnarsson (2000). Comparative study on first and second order ILC - frequency domain analysis and experiments. In 'Proc. of the 39th IEEE Conf. on Decision and Control'. Sydney, Australia.

Owens, D. H. (1992). Iterative learning control - convergence using high gain feedback. In 'Proc. of the 31st Conf. on Decision and Control'. Tucson, AZ, USA. pp. 2545–2546.

Tayebi, A. and M. B. Zaremba (1999). Exponential convergence of an iterative learning controller for time varying nonlinear systems. In 'Proc. of the 38th IEEE Conf. on Decision and Control'. Phoenix, AZ, USA. pp. 1593–1598.

Uchiyama, M. (1978). 'Formulation of high-speed motion pattern of a mechanical arm by trial'. *Trans. SICE (Soc. Instrum. Contr. Eng.)* **14**(6), 706–712(in Japanese).