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ELECTROMECHANICAL OSCILLATIONS IN POWER SYSTEMS

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Abstract: The problem of electromechanical oscillations in power systems is revisited under a modern control theory perspective which corroborates, further clarifies and extends the classical view of the problem.

Keywords: Power system stability, power system stabilizers

1. INTRODUCTION

The problem of electromechanical oscillations in power systems has been studied for a long time. This problem arose in the early 60's, a time when modern control was still in its infancy. The solutions then developed for this problem made use of the analytical tools available at the time, that is, classical control methods. The basic stability concepts and design methods were developed from frequency response analysis and a lot of physical reasoning based on the system's operation experience (deMello and Concordia, 1969). This was quite a successful development, as the control methods thus derived have been and still are widely applied.

Today, several decades later, the application of modern multivariable control methods has become commonplace in the analysis of these phenomena and controller design aiming at their damping in multi-machine power systems (Kundur, 1994; Martins *et al.*, 1994; Costa *et al.*, 1997; Bazanella *et al.*, 1995). Yet, the basic stability concepts developed in the 60's still provide the guidelines and physical insights, and these concepts are still presented and understood from a classical control perspective (Kundur, 1994; Anderson and Fouad, 1995).

A formal analysis of the problem of electromechanical oscillations in power systems under the light of modern control in provided in this paper. The classical concepts in the stability of single machine systems are explained in terms of eigenanalysis, controllability and observability. Generic properties are formally derived from an algebraic-differential model for the power system. In so doing, these concepts get an elegant presentation in a rigorous mathematical framework, which is of particular interest for the control expert interested in power system stability. This rigorous mathematical derivation brings several implications and reveals some new aspects which will be discussed along the paper.

In section 2 the modeling of the power system is presented. Linearized models for multi-machine systems and for the single machine system are derived from a generic nonlinear model. The single machine case is studied in section 3, where the concepts and results in (deMello and Concordia, 1969) are derived from this model. The formal nature of this derivation provides formal proofs of those results and reveals some new properties. The multi-machine case is studied in section 4. The concepts developed for the single machine case are extrapolated based on physical insight and knowledge of typical system characteristics. A benchmark is used to illustrate these concepts. Some concluding remarks are given in Section 5.

2. POWER SYSTEM MODELING

The essential features of the synchronous machine regarding the power system stability problem are captured by the E_q' model, which for a round rotor machine is given by (Anderson and Fouad, 1995; Kundur, 1994):

$$\dot{\delta} = w \tag{1}$$

$$\frac{2H}{w_s}\dot{w} = P_m - E_q' I_q - Dw \tag{2}$$

$$T_{d0}'\dot{E}_q' = -E_q' + (X_d - X_d')I_d + E_f \tag{3}$$

where δ, w and E_q' are the state variables representing load angle, shaft speed deviation from synchronous speed and internal voltage respectively, P_m represents the mechanical power supplied by the turbine (which is assumed constant), E_f represents the field voltage, I_q and I_d stand for the quadrature and direct axis components of the output current, and the other symbols are physical parameters in standard notation (Arrillaga *et al.*, 1983).

The terminal voltage in the $d-q$ reference frame is given by

$$\begin{bmatrix} V_q \\ V_d \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} E_q' + \begin{bmatrix} 0 & X_d' \\ -X_q & 0 \end{bmatrix} \begin{bmatrix} I_q \\ I_d \end{bmatrix} \tag{4}$$

More complete models are known which describe the synchronous machine behavior more accurately. However, from a qualitative point of view - which is the scope of this paper - they yield the same results as the E_q' model.

In order to write down a model for a power system consisting of N machines connected through a transmission network, each current and voltage must be put into a common reference frame, which is achieved through rotations:

$$\begin{bmatrix} V_{ri} \\ V_{mi} \end{bmatrix} = \begin{bmatrix} \cos\delta_i & -\sin\delta_i \\ \sin\delta_i & \cos\delta_i \end{bmatrix} \begin{bmatrix} V_{qi} \\ V_{di} \end{bmatrix} \tag{5}$$

with similar equations for the currents, where the subscripts r and m mean real and imaginary parts respectively and the subscript i means that the variable is related to the *ith* machine. In this common reference frame the currents and voltages are related through the transmission network equation:

$$\begin{bmatrix} I_{r1} \\ I_{m1} \\ \vdots \\ I_{rN} \\ I_{mN} \end{bmatrix} = [Y] \begin{bmatrix} V_{r1} \\ V_{m1} \\ \vdots \\ V_{rN} \\ V_{mN} \end{bmatrix} \tag{6}$$

where Y is the admittance matrix of the network.

Excitation control is usually applied in order to regulate the magnitude of the terminal voltage, which is given by

$$V_t = \sqrt{(V_q^2 + V_d^2)} \tag{7}$$

It is common practice to model the excitation control as a purely proportional action. This model captures the net effect of the controller, although actual implementations are far more complex. As in the case of the synchronous machine model, although more complete models give more accurate results in the analysis, they are equivalent from a qualitative standpoint.

3. THE SINGLE MACHINE CASE

3.1 *The model*

In the case of a single machine against an infinite bus the algebraic variables I_q and I_d can be eliminated from the model (1)-(3) to obtain a standard state space description of the system:

$$\dot{\delta} = \omega \tag{8}$$

$$\dot{\omega} = -b_1 E_q' \sin\delta - D\omega + \frac{P_m w_s}{2H} \tag{9}$$

$$\dot{E}_q' = b_3\cos\delta - b_4 E_q' + \frac{E_f}{T_{d0}'} \tag{10}$$

$$V_t = \sqrt{(b_5\cos\delta + b_6 E_q')^2 + (b_5\sin\delta)^2} \tag{11}$$

where the b parameters are all positive. These quantities are related to the physical parameters of the system and have been introduced in order to simplify the notation.

Let us initially study the case of constant field voltage, which represents the machine operated without AVR or with AVR after ceiling voltage has been reached. In this case, linearizing the system (8)-(10) around an arbitrary operating point yields

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ -b_1 E_{qo}'\cos\delta_o & 0 & -b_1\sin\delta_o \\ -b_3\sin\delta_o & 0 & -b_4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_{d0}'} \end{bmatrix} u \tag{12}$$

where

$$x \triangleq \begin{bmatrix} \delta - \delta_o \\ \omega \\ E_q' - E_{qo}' \end{bmatrix} \qquad u \triangleq E_f - E_{fo} \tag{13}$$

and the subscript o means the value of the variable at the operating point; the natural damping D has been neglected, which makes the analysis much simpler and has no qualitative effect on the results, since D is usually very small.

The eigenvalues of the dynamic system (12) are given by the roots of the characteristic polynomial of its dynamic matrix:

$$p_o(\lambda) = \lambda^3 + b_4\lambda^2 + b_1 E'_{qo} \cos\delta_o \lambda + \\ + b_1 b_4 E'_{qo} \cos\delta_o - b_1 b_3 \sin^2\delta_o \quad (14)$$

Applying the Routh-Hurwitz criterion to this polynomial shows that it has all its roots in the left half of the complex plane if and only if

$$b_1 b_4 E'_{qo} \cos\delta_o - b_1 b_3 \sin^2\delta_o > 0 \quad (15)$$

which is satisfied whenever an equilibrium exists. The point at which this inequality ceases to be satisfied represents a saddle-node bifurcation associated to the reaching of the maximum power transfer capability of the system (Bazanella *et al.*, 1998). The classical concept of *synchronizing torques* may suggest that the system's equilibrium becomes unstable, which is not the case: instability is observed only when the equilibrium ceases to exist.

By adding an AVR to the machine the loss of equilibrium point through a saddle-node bifurcation is avoided as long as ceiling voltage is not reached (when the system behaves again as if no AVR were present). Then small-signal monotonic instability is ruled out by the AVR. On the other hand, it can cause the electromechanical oscillations to become unstable, as can be seen in the analysis of its effect on the eigenvalues of the dynamic matrix.

The output equation linearized yields

$$y = \begin{bmatrix} \dfrac{-b_5 b_6 E'_{qo} \sin\delta_o}{\sqrt{2b_5 b_6 E'_{qo}\cos\delta_o + b_6^2 E'^2_{qo} + b_5^2}} \\ 0 \\ \dfrac{-b_5 b_6 \cos\delta_o}{\sqrt{2b_5 b_6 E'_{qo}\cos\delta_o + b_6^2 E'^2_{qo} + b_5^2}} \end{bmatrix}^T x \quad (16)$$

where $y \triangleq V_t - V_{to}$. From this state space model the transfer function from field voltage to terminal voltage is calculated as

$$\frac{y(s)}{u(s)} = \frac{b_6}{\sqrt{2b_5 b_6 \cos\delta_o E'_{qo} + b_6^2 E'^2_{qo} + b_5^2}} \frac{n(s)}{p_o(s)} \quad (17)$$

where

$$n(s) \triangleq (b_5 \cos\delta_o + b_6 E'_{qo}) s^2 + \\ + (b_1 b_6 \cos\delta_o E'^2_{qo} + b_1 b_5 E'_{qo}) \quad (18)$$

and $p_o(\cdot)$ is the characteristic polynomial given in (14).

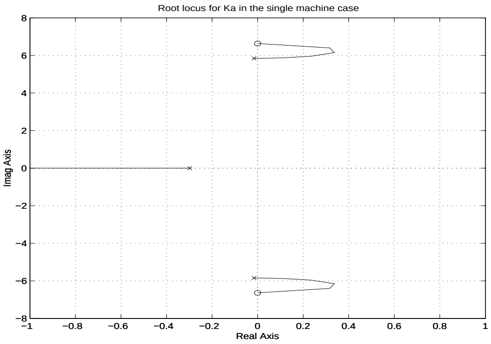

Fig. 1. Typical root locus for the AVR gain in a single machine system.

For typical operating conditions there is a pair of complex poles with low damping and a real pole. The zeros of the transfer function (17) are purely imaginary and given by

$$z_{1,2} = \pm \jmath b_1 E'_{qo} \cos\delta_o \alpha \quad (19)$$

with

$$\alpha \triangleq \frac{1}{1 - \frac{b_5 \sin^2\delta_o}{b_5 + b_6 E'_{qo}\cos\delta_o}} > 1 \quad (20)$$

If the proportional controller is applied around the transfer function (17) then the closed-loop poles move towards the imaginary zeros, eventually crossing the imaginary axis. Figure 1 shows a typical case (data for the example are given in the appendix).

The particular configuration of the root locus depends on the system parameters and the operating condition, but two important characteristics are always present: i) there always exists a pair of imaginary zeros; ii) the poles always cross the imaginary axis. The first of these characteristics has been shown above; the second can be proven as follows. The characteristic equation for the closed-loop system is

$$p_c(s) = X_o s^3 + (b_4 X_o + K_a b_6 (b_5 \cos\delta_o + b_6 E'_{qo})) s^2 \\ + X_o (b_1 E'_{qo} \cos\delta_o) s + X_o (b_1 b_3 \sin^2\delta_o + b_1 b_4 \cos\delta_o) \\ + K_a b_6 (b_1 b_5 E'_{qo} + b_1 b_6 \cos\delta_0 E'^2_{qo} \quad (21)$$

where $X_o \triangleq \sqrt{2b_5 b_6 E'_{qo}\delta_o + b_6^2 E'^2_{qo} + b_5^2}$. A condition for existence of purely imaginary poles is obtained by substituting $s = \jmath\Omega$ into (21), which after some manipulation yields

$$K_a = \frac{b_3}{b_5 E'_{qo}} \quad (22)$$

$$\Omega = \sqrt{b_1 E'_{qo} \cos\delta_o} \quad (23)$$

meaning that for K_a given by (22) the closed-loop poles cross the imaginary axis at the frequency $\pm\jmath\Omega$ given by (23).

3.2 *Controllability and observability*

When the problem of electromechanical oscillations in power systems first appeared, for some time it was possible to design AVR's as lead and/or lag controllers to guarantee closed-loop stability (deMello and Concordia, 1969). However, as the systems became more heavily loaded, more machines were equipped with AVR's and the performance requirements became more strict, the difficulty in designing such controllers was growingly overwhelming, eventually reaching the point where no satisfactory solution could be found.

It is easy to see why this is so by looking at the root locus given in Figure 1. However complex transfer function one may think of using as an AVR, it will be very hard to move the poles away from the imaginary zeros, since they are so close to begin with. This almost cancellation of the poles by the zeros in the transfer function can be due to poor controllability or observability of these modes. The controllability matrix is given by

$$U = \begin{bmatrix} 0 & 0 & -b_1 \sin \delta_o \\ 0 & -b_1 \sin \delta_o & b_1 b_4 \sin \delta_o \\ 1 & -b_4 & b_4^2 \end{bmatrix} \qquad (24)$$

and its determinant is $|U| = -b_1^2 \sin^2 \delta_o$, which is zero only at zero load and grows as the load grows. Hence controllability is not the issue and observability must be.

3.3 *The Power System Stabilizer*

The terminal voltage feedback is necessary to regulate the output voltage but, as shown above, it is impractical to design an AVR with a transfer function to make the system stable in closed-loop. Hence, the use of an additional controller is necessary to provide the system stable, while the AVR still performs its function of regulating the terminal voltage in steady state. This additional controller can use the same input, since the modes are controllable by this input, but must measure another output from which the modes are strongly observable.

Practical measurable quantities are rotor speed, electric power, accelerating power and bus frequency. From a theoretical point of view bus frequency and rotor speed are the same signal, differences arising exclusively from practical considerations regarding instrumentation issues. Accelerating power is also equivalent to electrical power from a theoretical point of view, since they are different only by a constant (P). Moreover, electrical power and rotor speed are directly related by equation (2), where we can see that the transfer function from w and P_e is just a zero at the origin, which does not influence controllability or observability of the complex eigenvalues, since they are far from the origin. Different controllers must be designed for either one of these signals, but none can be said to be more efficient from a theoretical point of view. For this reason only rotor speed feedback is considered in the following analysis.

The transfer function from the new input V_r to the output w is given by

$$\frac{w(s)}{V_r(s)} = -b_1 \sin(x_1) \frac{s}{p_c(s)} \qquad (25)$$

which does not present zeros close to any of the eigenvalues. Therefore the system is observable from this output. This can also be seen calculating the observability matrix, whose determinant is

$$\begin{aligned} det(O) &= b_1^2 \sin(x_1)[b_4 x_3 \cos(x_1) + b_3 \sin^2(x_1)] \\ &= b_1 b_4 P \cos(x_1) + b_1^2 b_3 \sin^3(x_1) \qquad (26) \end{aligned}$$

which is nonzero for any valid operating point.

4. THE MULTI-MACHINE CASE

In the multi-machine case it is not possible to eliminate the algebraic variables in the nonlinear model to obtain a generic state space model as in the single machine case. Therefore one can not perform an analysis similar to what was done for the single machine, obtaining exact results of generic nature. Instead, one can take those results as an initial approximation for the multi-machine system's performance and build upon them to determine the general characteristics to be expected in a multi-machine system.

The single machine model is a simplified model for a multi-machine system in which all the machines in the system but one are modeled as a static voltage source. A Thevenin equivalent is then derived for this static network, resulting in the model (8)-(10). This model is valid only for phenomena more closely related to the variables of that machine whose dynamics has been kept. The oscillation modes in this model, identified by the complex eigenvalues of the linearized model (12), are the *local modes* of this machine. As a first approximation to the system stability studies one can take one single machine model for each machine in the system.

In the actual multi-machine system the oscillation modes present in the single machine models will still be observed. New modes of oscillation, involving the interaction between the dynamics of the different (groups of) machines - which are not modeled in the single machine models - will also be present. These are called *inter area modes*, as each

Table 1. Electromechanical modes of the New England system.

1	$-0.48 \pm \jmath 8.91$	2	$-0.51 \pm \jmath 8.63$
3	$-0.34 \pm \jmath 8.40$	4	$-0.25 \pm \jmath 7.64$
5	$-0.24 \pm \jmath 7.08$	6	$-0.34 \pm \jmath 6.84$
7	$-0.25 \pm \jmath 6.41$	8	$-0.29 \pm \jmath 6.06$
9	$-0.77 \pm \jmath 2.91$	10	$\pm \jmath 2.45$

machine or group of machines which oscillates coherently is called an area.

The typical characteristics of electromechanical oscillations in multi-machine systems will be illustrated with a benchmark: the well-known New England system. A highly stressed operating condition is studied. The one line diagram, system and bus data for the benchmark can be found in (Byerly *et al.*, 1978).

4.1 *Controllability and observability*

The multi-machine model as described in Section 1 can be put, after linearization and elimination of the algebraic variables z, in a standard linear state space form:

$$\dot{x} = Ax + Bu \tag{27}$$

$$y = Cx \tag{28}$$

The first step in analyzing the system's stability is the calculation of the eigenvalues of the dynamic matrix A in (27). If N is the number of machines in the model, $N+1$ modes of electromechanical oscillation can be expected: one local mode for each machine and an inter area mode.

The model used for the New England system is of order 75. The complex eigenvalues related to the electromechanical oscillations are identified among the 75 eigenvalues and listed in Table 1. The operating condition was selected at the stability limit, at which one of the modes (# 10) turns unstable by crossing the $\jmath\Omega$ axis.

The association of each local mode to its corresponding machine can be made by means of participation factors, which relate eigenvalues to state variables (Kundur, 1994). The larger is the participation factor p_{ij} the stronger will be the observability of the $j-th$ mode in the dynamics of the $i-th$ state variable. Verifying the largest participation factors for a mode determines whether it is local or inter area, and to which machine(s) it is associated. Table 2 presents the largest participation and to which variable it is associated for each one of the electromechanical modes.

From the complete participation matrix it follows that eigenvalue # 10 is an inter area mode, since it has large participation factors in the variables of all the machines.

Table 2. Largest participation factors for the New England system.

Mode	state variable	PF
1	w_{G36}	0.27
2	w_{G37}	0.45
3	w_{G33}	0.39
4	w_{G32}	0.26
5	w_{G30}	0.35
6	w_{G34}	0.13
7	w_{G31}	0.16
8	w_{G38}	0.27
9	E'_{qG38}	0.19
10	E'_{qG38}	0.23

4.2 *PSS design*

The participation factors have strong implications on the control design. The available signals are the reference voltages of each generator as control inputs and their speeds as measured signals (outputs), that is:

$$B = \begin{bmatrix} b_1 & \dots & b_N \end{bmatrix} \tag{29}$$

$$C = \begin{bmatrix} c_1 \\ \vdots \\ c_N \end{bmatrix} \tag{30}$$

with $b_i \in \Re^{N\times 1}$ and $c_i \in \Re^{1\times N}$ the input and output vectors corresponding to each each machine. Each b_i and c_i has only one nonzero element: in each b_i this nonzero element is at the line corresponding to the state variable E'_{qi} and in each c_i it is at the column corresponding to w_i.

In the example several electromechanical modes present poor damping and the inter area mode is not stable, so the corresponding eigenvalues should be moved to the left in the complex plane by means of supplementary control. This is possible through these inputs and outputs if and only if the poorly damped eigenvalues are not fixed modes of the control structure to be used, which is of decentralized feedback.

The controllability and observability can be assessed by looking at the transfer functions for each input-output pair separately. To this end, take

$$\dot{x} = Ax + b_i u \tag{31}$$

$$y = c_i x \tag{32}$$

with $i = 1, \dots, N$, whose transfer function is

$$\frac{w_i(s)}{V_{ri}(s)} = c_i(sI - A)^{-1} b_i \tag{33}$$

Then for each i one can analyze what can be done with each one of the local controllers by looking at the transfer functions $\frac{w_i(s)}{V_{ri}(s)}$. Figures 2 and 3 show a zoom at the pole-zero configuration of the transfer functions $G_{33}(s)$ and $G_{38}(s)$; only the singularities close to the $\jmath\Omega$ axis are shown.

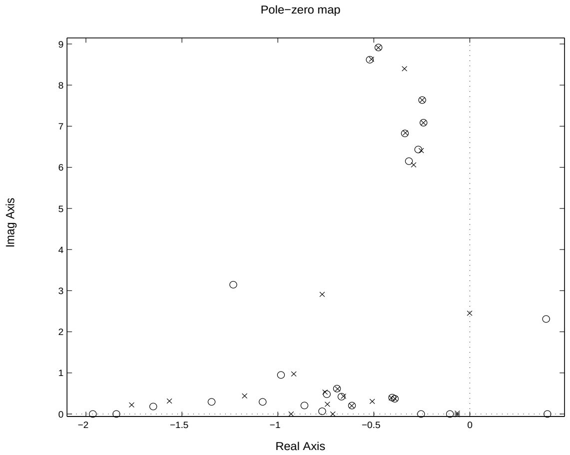

Fig. 2. Pole-zero configuration for the input-output pair $V_{rG33} - w_{G33}$.

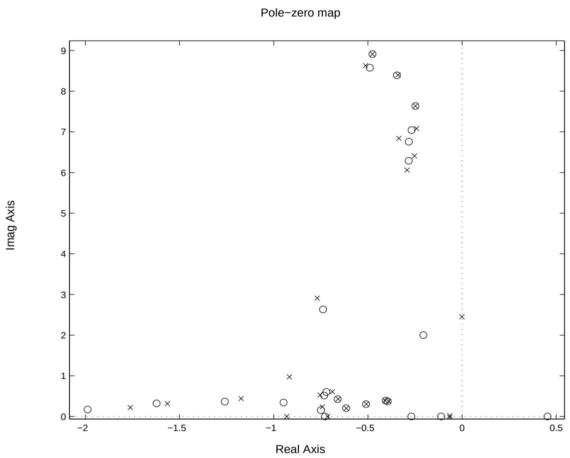

Fig. 3. Pole-zero configuration for the input-output pair $V_{rG38} - w_{G38}$.

For each machine only the eigenvalues related to its local oscillation mode(s) is (are) controllable and observable; the remaining eigenvalues appear canceled by zeros in the figures. A PSS installed in generator G_{33}, for instance, can shift significantly to the left only its local mode, which is # 3. This is the only eigenvalue which is not almost canceled by a very close zero in the figure, which is in accordance with the fact that w_{G33} is the variable most strongly associated to this eigenvalue, as shown in Table 2. The PSS in generator G_{33} could hardly move the inter area mode (seen sitting on the $\jmath\Omega$ axis in the figures) to the left either, since there is a right-half plane zero to attract it. On the other hand, a PSS placed at generator G_{38} could probably do it, as in Figure 3 this zero is on the other side of the $\jmath\Omega$ axis.

5. CONCLUDING REMARKS

The classical concepts in the analysis of electromechanical oscillations in power systems can be derived formally from a state-space modeling of the power system. This approach allows to cast these concepts into the modern control framework, which completes the physical reasoning behind these concepts with formal mathematical analysis, leading to shorter and more formal derivations of the results. It also sheds new light on the problem, revealing some additional details and providing further insight into the problem which is very useful in the design of PSS's.

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Appendix A. DATA FOR THE SINGLE MACHINE SYSTEM

b_1	34.290	b_3	0.149	b_4	0.330
b_5	0.504	b_6	0.496		