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ACOUSTIC TRACKING OF AUTONOMOUS UNDERWATER VEHICLES BY A SET-MEMBERSHIP APPROACH

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Abstract: An algorithm is presented for tracking of Autonomous Underwater Vehicles (AUVs) from acoustic time-of-flight measurements received by a field of surface floating buoys. The algorithm assumes that measurements and AUV dynamics uncertainties are unknown but bounded, with known bounds, and produces as output the set of admissible AUV positions. The algorithm has been validated by simulation in which uncertainty models have been obtained from field data at sea.

Keywords: Autonomous Vehicles, Tracking.

1. INTRODUCTION

Recent years have witnessed an impressive growth in the technology of robotics for undersea exploration. Remotely Operated Vehicles (ROVs) linked with a thether cable to the mother ship are today a well-established technology routinely used in the off-shore industry. Autonomous Underwater Vehicles (AUVs) are still more research topics than commercial products; however, they held the promise of being the next significative step in ocean exploration and exploitation, cutting costs and allowing operations that are presently not possible from surface ships or by ROVs. One of the problems that prevents commercial applications of AUVs, or at least reduce their efficiency, is vehicle localization. The availability of (Differential) GPS systems on board of surface platforms has increased the demands on AUVs navigation accuracy. Inertial Navigation Systems (INS) cannot maintain the requested accuracy over the interval of operation of the system, and are highly expensive. It has to be remarked that the general problem of localization of autonomous vehicles has received much attention in the robotic literature (to name a few, Levitt and Lawton, 90; Leonard and Durrant-Whyte, 91; Sutherland and Thompson, 94; Borenstein et al., 95; Garulli and Vicino, 2001); however, the peculiarities and constraints of the underwater environment and of usual AUV missions prevent the simple transposition of available techniques for land or aerial vehicles, and require careful study of the implications of each chosen methodology for the underwater system performance (Tuhoy et al., 96; Caiti, 98). There are several navigation systems currently employed by AUVs researchers. The main non-acoustic approach consist in installing on the AUV a GPS receiver and an INS; the vehicle navigates with the INS, but periodically comes to surface to receive the GPS signal and to recalibrate the INS (Yun et al., 99). The acoustic approaches can be subdivided in the so-called Long Base-Line (LBL) and Short Base-Line (SBL) systems. In both cases the vehicle's position is determined on the basis of the acoustic returns detected by a set of receivers. In the LBL case, a set of acoustic transponders is deployed on the seafloor around the perimeter of the area of operation. The vehicle is able to locate itself with respect to the transponders with the required accuracy (Collin et al., 2000). In SBL systems, a ship follows the AUV at short range with a high-frequency directional emitter able to accurately determine the AUV position with respect to the mother ship; the same system allows for bidirectional communication among the AUV and the ship, so that the AUV navigation system is aware of its current absolute position (Størkersen et al., 98). All these methods have their merits and drawbacks. Augmented INS requires the use of sophisticated inertial sensors, and are vehicle-specific (i.e., the same system cannot be employed on more than one vehicle). LBL systems requires long time (with associated costs) for deployement and calibration. SBL systems need a ship to follow the vehicle, greatly reducing the cost-effectiveness of an AUV systems. A simpler alternative to LBL systems has been recently proposed. It consists in installing acoustic receivers/emitters on surface freely floating buoys having on board GPS receivers and radio interconnection. The vehicle is located through time-of-flight measurements of acoustic signals ("pings") from each buoy. The system has the ambition of becoming a true underwater GPS system, affordable, easy to deploy and recover, and autonomous during its time of operation. Localization and tracking performances have been recently investigated by several authors (Collin et al., 2000; Bechaz and Thomas, 2000; Mozzone et al. 2000); in all these cases, the algorithms analyzed have considered measurements affected by Gaussian-distributed noise, and have determined the resulting uncertainties through Monte Carlo analysis. In this paper, a different approach is proposed, based on the assumption that measurements and modeling errors are unknown but bounded, with known bounds. Tools from set-membership estimation theory (Milanese and Vicino, 91) are then employed to determine the admissible region in space where the vehicle is located. In particular, in this paper the performance of a set-membership based tracking algorithm is investigated, and compared with that of the Extended Kalman Filter (EKF). The tracking algorithm relies on a set-membership localization algorithm to obtain set-valued estimated positions, and on a simple and general vehicle kinematic model. The details of the localization algorithm are given in (Caiti et al., 2001). The performance of the tracking algorithm has been tested through simulations, considering buoys dislocation similar to those employed in (Collin et al., 2000; Mozzone et al., 2000), and error measurement characteristics taken from field experiments at sea (Mozzone et al., 2000). The results obtained show that when realistic disturbances are considered, as currents of unknown but bounded magnitude, the proposed algorithm has perfomance and robustness clearly superior to those of the EKF. The paper is organized as follows: in the next section the problem is formally stated, and the methodological set-membership approach is introduced; in section 3 the tracking algorithm is described; in section 4 simulation results are presented; finally conclusions are given.

2. PROBLEM STATEMENT

Let us consider the situation in which n buoys are placed in arbitrary positions on the sea surface over an area of interest. An absolute earth reference system (x, y, z) is assumed, with z=0 on the water surface, and the z-axis pointing upward from the sea surface. Each buoy position $(x_i(t), y_i(t), 0)$ is assumed known. In practice, any buoy position will be known at D-GPS accuracy; however, the uncertainty in the position can be treated as an additional uncertainty in the measurement. The buoys are allowed to move freely; however, since their movement will be due to waves and current, with a time scale much larger than that of the travelling acoustic signals, it is assumed that the buoys do not change position between transmission and reception of each ping. Without loss of generality, it is assumed that each buoy transmits at regular pre-specified time intervals an acoustic signal encoding its current GPS position. Symmetric situations, in which the AUV acts as acoustic source, or in which the acoustic signal are reflected from the AUV and received at the buoys, can be dealt in a similar fashion. The low-level signal processing needed to discriminate among the various buoys is not considered here. It is assumed that the received signals are suitably processed so that the AUV has available from the i-th buoy, at time t_k, the measurement $s_i(t_k)$ of the travel time of the emitted ping to the AUV. Each measurement is affected by an unknown but bounded uncertainty $e_i(t_k)$, i.e.:

$$\begin{aligned} &s_i(t_k) = \tilde{s}_i(t_k) + e_i(t_k), \\ &\left| e_i(t_k) \right| \le E_i \quad \forall k, \quad i = 1, \cdots, n \end{aligned} \tag{1}$$

being $\tilde{s}_i$ the measurement when no uncertainties are present, and E_i known bounds on the error, in which all the uncertainties can be concentrated. The sound speed $c(x, y, z)$ in the area of interest is assumed known. Acoustic propagation is modeled with ray path theory, including multipath effects, and considering lossless reflection at the water surface and seafloor boundaries. At each time t_k, the measurement from the i-th buoy, once converted from time to distance taking into account the sound speed profile, and considering the worst case uncertainty (equation (1)), defines a region $S_i(t_k)$, of the admissible space B_0, for the vehicle position. The space B_0 is bounded by the sea surface and sea bottom, and by the defined extension of the area of interest; the buoys are located inside B_0. Merging the information from all the available buoys, the region $V(t_k)$ in space which bounds the true vehicle position is given by:

$$V(t_k) = B_0 \cap \left(\bigcap_{i=1}^{n} S_i(t_k) \right) \tag{2}$$

The region $V(t_k)$ may have a complex geometrical shape, making it unfeasible its exact computation. The localization algorithm described in (Caiti et al.,

2001) determines an approximation of $V(t_k)$ in terms of the orthotope (a parallelepiped with orthogonal edges) $B(t_k) \supseteq V(t_k)$ of minimal volume. The tracking algorithm relies on these orthotope approximations as output measurements, and on a dynamic model of the AUV. Referring to Figure (1), the following simple uncertain kinematic model has been considered:

$$
\begin{aligned}
\dot{x} &= u \cdot \cos(\boldsymbol{j}) - v \cdot sin(\boldsymbol{j}) + w_x \\
\dot{y} &= u \cdot sin(\boldsymbol{j}) + v \cdot \cos(\boldsymbol{j}) + w_y \\
\dot{z} &= 0 \\
\dot{\boldsymbol{f}} &= r + w_{\boldsymbol{j}}
\end{aligned}
\tag{3}
$$

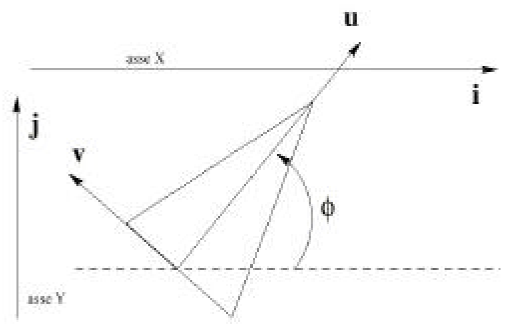

Fig. 1: coordinate system for AUV kinematic model

In equation (3), u, v and r are the surge, sway and yaw velocities of the AUV, that will be considered as known input. The terms w_h are the model uncertainties (including input uncertainties). The model is planar, since AUV depth is easily and most efficiently measured by pressure gauges on board the vehicle. The set-membership tracking algorithm will rely on a discretization in time of the system in equation (3). The general approach is here briefly described. Let $\mathbf{x}(k)$ be the state of the dynamic system of interest, and $\mathbf{y}(k)$ the vector of available measurements. Let the dynamic equations be expressed in compact form as:

$$
\begin{aligned}
\mathbf{x}(k+1) &= \mathbf{f}(\mathbf{x}(k), k) + \mathbf{w}(k) \\
\mathbf{y}(k) &= \mathbf{h}(\mathbf{x}(k), k) + \mathbf{m}(k)
\end{aligned}
\tag{4}
$$

where the uncertainty vectors $\mathbf{w}$ and $\mathbf{m}$ are unknown but bounded, with worst case bounds known, i.e. $\|\mathbf{w}\|_\infty \le E_w, \|\mathbf{m}\|_\infty \le E_m$; the admissible set to which the vector $\mathbf{x}$ belongs is indicated in the following as *X*. Let the initial conditions be specified as: $\mathbf{x}(1) \in X(1/0)$, being $X(1/0)$ a known set. The objective of the tracking algorithm is to recursively compute the set of states $X(k/k)$ (state estimate) and $X(k+1/k)$ (state prediction) compatible with the information available at time k. Let $C_\infty(k)$ the set of system states compatible with the measurement at time k:

$$
C_\infty(k) = \{\mathbf{x} : \|\mathbf{y}(k) - \mathbf{h}(\mathbf{x}(k), k)\|_\infty \le \mathrm{E}_m\} \tag{5}
$$

The sets $X(k/k)$ and $X(k+1/k)$ are then determined by the following recursive relations:

$$
\begin{aligned}
X(k/k) &= X(k/k-1) \cap C_\infty(k) \\
X(k+1/k) &= \mathbf{f}(X(k/k), k) + B_\infty E_w
\end{aligned}
\tag{6}
$$

where B_∞ is the unit box. Since the geometrical shape of the sets $X(k/k)$ and $X(k+1/k)$ may be fairly complex, they will be approximated by bounding orthotopes of minimial volume $B(k/k)$, $B(k+1/k)$ such that:

$$
\begin{aligned}
B(1/0) &\supseteq X(1|0) \\
B(k/k) &\supseteq B(k/k-1) \cap C_\infty(k) \\
B(k+1/k) &\supseteq \mathbf{f}(B(k/k), k) + B_w
\end{aligned}
\tag{7}
$$

3. THE TRACKING ALGORITHM

The set-membership tracking algorithm for the AUV kinematic model of equation (3) is now described. The system state is composed by the variables x, y and $\boldsymbol{j}$, i.e. planar dynamics are considered, with no AUV movement along the depth axis *z*. It is assumed that the system inputs u, v and r are exactly known; the model and input uncertainties $w_x, w_y, w_{\boldsymbol{j}}$ are unknown but with known bounds $W_x, W_y, W_{\boldsymbol{j}}$. Available measurements are the (x, y) position of the AUV, in terms of a bounding orthotope $B(t_k) \supseteq V(t_k)$, and the measurement of $\boldsymbol{j}$, obtained from gyro on board the AUV, affected by an unknown but bounded uncertainty $\boldsymbol{e}_{\boldsymbol{j}}$, $|e_{\boldsymbol{j}}| \le E_{\boldsymbol{j}}$. Let us set Δt as sampling interval for the acquisition of new measurements. As described through equations (4-7), the set-membership tracking algorithm is based on the intersection of the sets generated by the system predictions and by the available measurements; the sets are computed on the basis of the worst case modeling and measurement errors. In order to compute predictions for the system described in equation (3), let us consider first the dynamic equation of the state variable $\boldsymbol{j}$, which is independent from the other states. Let us suppose that at the $(k-1)$-th sampling interval it is available the information on the compact set $I(\Phi_{1,k-1}, \Phi_{2,k-1})$ to which $\boldsymbol{j}(k-1)$ must belong: $\Phi_{1,k-1} \le \boldsymbol{j}(k-1) \le \Phi_{2,k-1}$. Then the state $j(k)$ must belong to the set:

$$
\begin{aligned}
&I(\Phi_{1,k}, \Phi_{2,k}) = \\
&= I(\Phi_{1,k-1} - E_{\boldsymbol{f}}\Delta t, \Phi_{2,k-1} + E_{\boldsymbol{f}}\Delta t) + \\
&+ \int_{(k-1)\Delta t}^{k\Delta t} r \, dt
\end{aligned}
\tag{8}
$$

and, at any time $t, (k-1)\Delta t \le t < k\Delta t$, $\boldsymbol{j}(t)$ must belong to the set:

$$
\begin{aligned}
&I(\Phi_{1,k}, \Phi_{2,k}) = \\
&= I(\Phi_{1,k-1} - E_{\boldsymbol{f}}\Delta t, \Phi_{2,k-1} + E_{\boldsymbol{f}}\Delta t) + \\
&+ \int_{(k-1)\Delta t}^{t} r \, dt
\end{aligned}
\tag{9}
$$

Equation (9) is needed because the goal of the analysis is now to bound the admissible evolution of the states x and y within the sampling interval. In order to reach the goal, a bound on the maximum and minimum time derivative of the states must be determined. Consider the following quantities related to the variable x:

$$d_1(t) = u(t)\cdot\cos(\Phi_{1,t}) - v(t)\cdot sin(\Phi_{1,t}) + W_x$$

$$d_2(t) = u(t)\cdot\cos(\Phi_{2,t}) - v(t)\cdot sin(\Phi_{2,t}) + W_x$$

$$\begin{aligned} d_3(t) = u(t)\cdot\cos\left(-\operatorname{arctg}\left(\frac{v(t)}{u(t)}\right)\right) + \\ -v(t)\cdot sin\left(-\operatorname{arctg}\left(\frac{v(t)}{u(t)}\right)\right) + E_x \end{aligned} \quad (10)$$

which are the derivatives of x computed at the boundaries of the admissible interval of $\boldsymbol{j}$, and at the value of $\boldsymbol{j}$ that maximizes the x derivative, i.e., such that $\partial\dot{x}/\partial\boldsymbol{j} = -usin(\boldsymbol{j}) - v\cos(\boldsymbol{j}) = 0$. At any t the maximum derivative of x is given by:

$$\dot{x}_{\max}(t) = \max(d_1(t), d_2(t), d_3(t)) \quad (11)$$

subject to the constraint:

$$-\operatorname{arctg}\left(\frac{v(t)}{u(t)}\right) + n\cdot\boldsymbol{p} \in I(\Phi_{1,t}, \Phi_{2,t}) \quad (12)$$

Expressions similar to those of equations (10-12) can be derived for the minimum time derivative of x, and for the state y. Consider now the compact set $I(X_{1,k-1}, X_{2,k-1})$ to which $x(k-1)$ must belong, $X_{1,k-1} \le x(k-1) \le X_{2,k-1}$, and similarly for $y(k-1)$. Then $x(k)$ and $y(k)$ will be bounded by the following expressions:

$$\begin{aligned} X_{1,k} = X_{1,k-1} + \int_{(k-1)\Delta t}^{k\Delta t}(\dot{x}_{\min}(t) - W_x)dt \le \\ \le x(k) \le \\ \le X_{2,k-1} + \int_{(k-1)\Delta t}^{k\Delta t}(\dot{x}_{\max}(t) + W_x)dt = X_{2,k} \end{aligned} \quad (13)$$

$$\begin{aligned} Y_{1,k} = Y_{1,k-1} + \int_{(k-1)\Delta t}^{k\Delta t}(\dot{y}_{\min}(t) - W_y)dt \le \\ \le y(k) \le \\ \le Y_{2,k-1} + \int_{(k-1)\Delta t}^{k\Delta t}(\dot{y}_{\max}(t) + W_y)dt = Y_{2,k} \end{aligned} \quad (14)$$

Equations (8), (13) and (14) allows to iteratively generate predictions of the admissible region of the state space. Since the orientation is independent from the (x, y) position, let us consider the set $B(k/k-1)$ as the predicted orthotope in the (x, y) space bounding the admissibile system positions. Let $B(k)$ be the bounding orthotope on the system position obtained from the acoustic measurements at time k with the localization algorithm. The estimated system state will be bounded by the orthotope $B(k/k)$ of minimal volume such that:

$$B(k/k) \supseteq B(k) \cap B(k/k-1) \quad (15)$$

A similar set-membership estimate can be easily derived for $\boldsymbol{j}$, taking into account that in this case the orthotopes are segments of minimal length. The computation of minimal volume bounding orthotopes has been implemented with linear programming methods, since all the constraints can be directly described as intersections of planes.

4. PERFORMANCE ANALYSIS

The set-membership tracking algorithm described in the previous section has been compared with the classic Extended Kalman Filter (EKF) tracking. Two cases are presented in the following, all of them assuming constant sound speed in water ("winter conditions"). In the first case, the process errors and the measurement errors (i.e., the uncertainties in the measured ranges from each buoy and the uncertainty in the gyro reading of $\boldsymbol{j}$) are generated by uniform distributions with zero mean. The set-membership algorithm has knowledge of the bounds on the distribution intervals, and the EKF has been initialized with a diagonal covariance matrix, where the elements of the diagonal have been taken so that the resulting ellipsoid covers the 95% of the volume of the uncertainty orthotopes. The second case is similar to the first, but for the presence of a constant bias in some of the process uncertainties, constant bias which is unknown to both the set-membership and the EKF algorithms. An admissible region of 30×30 Km in the (x, y) plane is considered, with water depth of 150 m. Three buoys are considered, placed as vertex of an equilateral triangle of 16 Km side. The process errors have the following bounds: $W_x = W_y = 0.2$ m/s, $W_{\boldsymbol{j}} = 0.02$ rad/s. The acoustic range measurements are independent and identically distributed (i.i.d.) with measurement errors $F_i \le 100$ m for every buoy i and at any sample instant (as taken from Mozzone et al., 2000). The gyro measurement errors are i.i.d., with uniform probability distribution and error bound $E_{\boldsymbol{j}} \le 0.0175$ rad. The sampling interval Δt has been taken as 10 s. Vehicle depth has been held constant at 75 m. In the simulations to be presented in the following, the complete system dynamics have been taken into account, in particular the following planar dynamic model has been implemented (Indiveri 98):

$$\begin{aligned} m_{11}\dot{v} &= -m_{22}\cdot u\cdot r - k_v\cdot v - k_{v|v|}\cdot v\cdot|v| \\ m_{33}\dot{r} &= (m_{11} - m_{22})\cdot u\cdot v - k_v\cdot r + \\ &\quad - k_{r|r|}\cdot r\cdot|r| + \boldsymbol{t} \\ \boldsymbol{j} &= r \\ \dot{x} &= u\cdot\cos(\boldsymbol{j}) - v\cdot sin(\boldsymbol{j}) \\ \dot{y} &= u\cdot sin(\boldsymbol{j}) + v\cdot\cos(\boldsymbol{j}) \end{aligned} \quad (16)$$

In Figure 2 the AUV true path is illustrated. Figure 3 reports the errors between the estimated positions and the true ones along the path, for both the EKF and the set-membership algorithm. The orthotope center has been taken as estimated position of the set-membership algorithm. Figure 4 reports the 99% confidence interval of the EKF and the worst case bounds from the set-membership algorithm. From the figures, it is evident that the EKF tracking algorithm has a better performance with respect to the the set-membership tracking algorithm; moreover, even in this case in which statistical assumption are closer to the set-membership approach, not only the EKF has a good performance, but its estimation of the error (Figure 4) is consistent with the effective error, so that the EKF covariance matrix can be employed as a check of the tracking accuracy. The worst case error bounds as estimated by the set-membership tracking algorithm are also consistent with the effective errors.

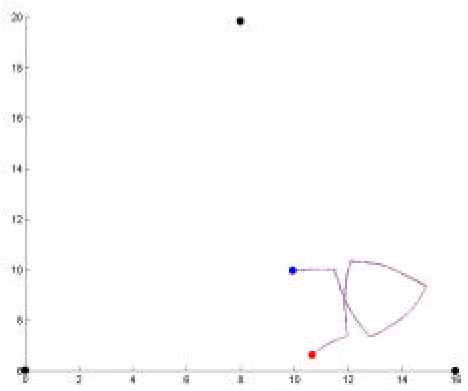

Fig. 2: the AUV simulated path with respect to the buoy configuration (black dots). Both axis in Km. The blue dot is the path starting point, the red dot is the path end point

The second case is similar to the first, but for the uncertainties w_x, w_y, which are generated independently from a uniform probability distribution with interval $[-0.12, 0.2]$ m/s. The average model uncertainties mimic the presence of a very modest constant current of 0.06 m/s over the area of operation. The set-membership algorithm still has the a priori knowledge of a worst case error of magnitude 0.2 m/s, and the EKF covariance matrix is initialized as in the previous case from the knowledge of the worst case bounds on the errors. In Figure 5 the tracking errors are reported for this case; in Figure 6 the 99% confidence interval of the EKF tracking and the worst case bounds on the set-membership tracking error are reported. It is well known that, in presence of unmodelled biases, the Kalman Filter performs poorly, and this case is no exception, as it can be seen in Figure 5. However, the important point here is that, as shown in Figure 6, the EKF has no knowledge of its poor performance: the estimated confidence intervals from the covariance matrix are very small, indicating a very precise tracking estimate (which is obviously not the case). The set-membership tracking algorithm, on the contrary, has by construction always control on the worst case error.

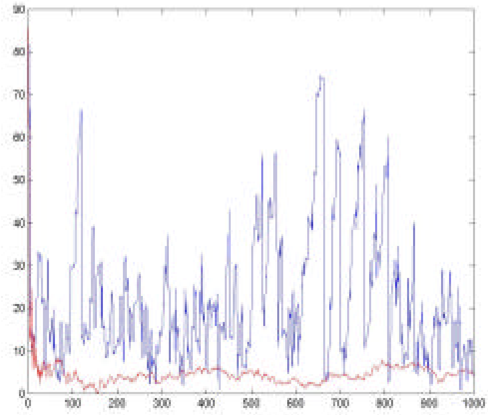

Fig. 3: Error as a function of sampling instant along the path. X-scale is in sampling intervals, Y-scale is in meters. The blue line is the error of the set-membership tracking algorithm, the red line is the error of the EKF tracking

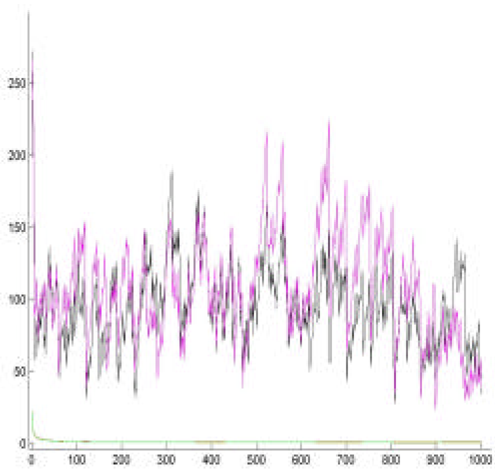

Fig. 4: worst case error bounds for the set-membership tracking (*x*-black; *y*-violet) and 99% confidence intervals estimated by the EKF through the covariance matrix (*x*-green; *y*-red - almost undistinguishable). X-axis in the figure is in sampling intervals, Y-axis is in meters

As a last consideration, it is worth to remark that constant bias of unknown magnitude can be included in the EKF setting, improving the tracking performance. On the other hand, time-varying non-zero mean disturbances will always give origin to EKF tracking errors as those reported here. Ocean currents and tides are both space and time varying, so the EKF is bound to see its performance severely degraded in realistic oceanic conditions. The example has been kept very simple, with constant current, in order to better focus on the algorithm.

5. CONCLUSIONS

A tracking algorithm for AUV with measurements from a sparse field of acoustic buoys has been presented. The algorithm is based on set-membership estimation theory, and produces as output the region in space to which the AUV must belong, on the basis of the worst case bounds on the measurement and process errors. No statistical assumptions on the disturbances are made. The set-membership tracking performance has been compared with that of the EKF. As long as no bias are present in the process or measurement uncertainties, the EKF tracking gives more accurate results, and is able to correctly estimate also its own cinfidence interval. However, when realistic oceanic conditions are considered, in particular the presence of space and time varying currents and tides, not only the EKF performance degrades, but also its confidence interval estimation falls apart. On the contrary, the proposed algorithm has no difference in performance from the no-bias case to the one in which biases are included, and it is always able, by construction, to indicate its range of accuracy. Based on the above consideration, it is believed that the algorithm proposed may represent a significative in-the-field alternative to the ones based on statistical error characterization, for those situations in which the disturbances cannot be well characterized or anticipated, and its imperative to exactly bound the region where the vehicle is located.

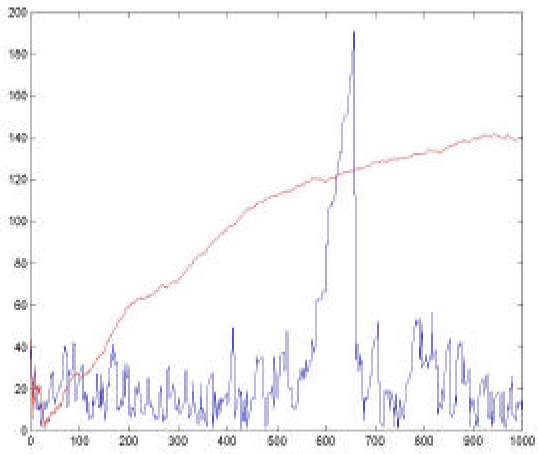

Fig. 5: error as a function of sampling instant along the path. X-scale is in sampling intervals, Y-scale is in meters. The blue line is the error of the set-membership tracking algorithm, the red line is the error of the EKF tracking.

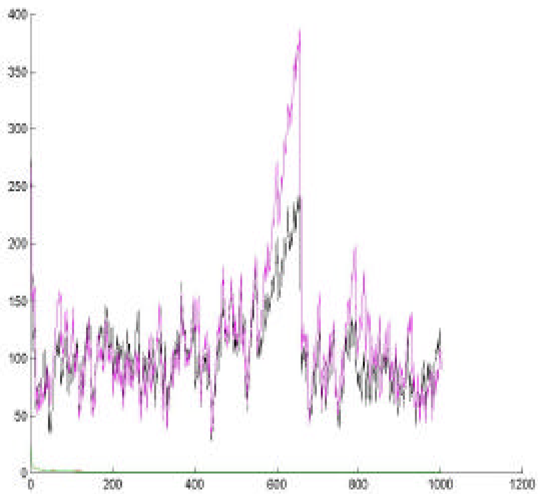

Fig. 6: worst case error bounds for the set-membership tracking (*x*-black; *y*-violet) and 99% confidence intervals estimated by the EKF through the covariance matrix (*x*-green; *y*-red - almost undistinguishable, and always in the interval 0-10m). X-axis in the figure is in sampling intervals, Y-axis is in meters

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