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SUPERSTABLE CONTROL SYSTEMS

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Abstract: In this paper we show how the newly introduced concept of superstable systems can be used to efficiently solve some traditionally hard design problems in control theory, such as stabilization with fixed–order controllers, rejection of bounded disturbances and robust controller design. Moreover, this approach also allows for formulating and treating some new problems such as the "linear-linear regulator" design. As we show in the paper, in the superstability context, these problems become convex, which allows for straightforward numerical solutions, e.g., using linear programming. *Copyright*© 2002 *IFAC*

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1. INTRODUCTION

The notion of stability is a cornerstone in control theory. Obviously, any control system must be stabilized prior to further satisfying additional performance specifications. However, the classical notion of stability is not always convenient for a number of reasons: For instance, since the the set of stable systems is nonconvex in the parameter space of the plant, analyzing its stability properties in the presence of parametric perturbations is difficult. Similarly, lack of convexity of the set of all stabilizing controllers of a given structure, renders fixed order controller design a difficult problem.

One of the possible ways to overcome such difficulties is to consider another (smaller) class of systems, which we call *superstable*. Since these systems are convex in parameter space, the problem of stabilization also becomes convex in the space of controller parameters and, hence, admits an easy solution using linear programming techniques. Moreover, in this context some classically difficult problems such as static output stabilization, simultaneous stabilization and robust stabilization under interval matrix uncertainty become tractable. Finally, this approach allows for formulating and solving some nonstandard optimal control problems such as the minimization of an integral functional involving absolute values of the phase variables rather than their squares.

On the other hand, the superstability-based approach has its own drawbacks. Firstly, since superstability is a stronger property than stability, not every controllable system can be rendered superstable. Additionally, for optimal control, these tools provide only upper bounds of the actual performance.

The main goal of the present paper is to apply the superstability concept to control system design and to provide a systematic analysis of the advantages and drawbacks of the approach.

2. PROPERTIES OF SUPERSTABLE SYSTEMS

In this section we precisely formulate the concept of superstability and present some properties of superstable systems. We use $\|\cdot\|$ to denote the ∞-norm for vectors: $\|a\| \doteq \max_i |a_i|$, $a = (a_1, \ldots, a_n)^{\mathrm{T}} \in \mathcal{R}^n$, and $\|\cdot\|_\infty$ stands for the ℓ^∞ ($\mathcal{L}^\infty$) norm of a sequence (function). Also, we use the 1-norm for matrices: $\|A\|_1 \doteq \max_i \sum_j |a_{ij}|$, $A = ((a_{ij}))$.

2.1 Preliminary Definitions

Definition 1. A matrix $A = ((a_{ij}))$ is said to be *d-superstable*, if it satisfies the following condition:

$$\mu_d(A) = \mu_d \doteq 1 - \|A\|_1 > 0. \tag{1}$$

Similarly, a matrix A is said to be *c-superstable* if it satisfies the condition

$$\mu_c(A) = \mu_c \doteq \min_i \Big(-a_{ii} - \sum_{j \neq i} |a_{ij}| \Big) > 0. \tag{2}$$

Clearly, a *d–superstable* matrix is Schur, i.e.

$$\rho \doteq \max_i |\lambda_i| < 1 \tag{3}$$

(where λ_i are the eigenvalues of A), but the converse does not hold. For instance, the matrix $A = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$ is stable (with $\rho = 0$), but not superstable. Similarly, if a matrix satisfies condition (2) then it is Hurwitz [1]. In the sequel, we refer to matrices satisfying either (1) or (2) simply as *superstable* and use μ to denote either μ_d or μ_c when the meaning is clear from the context.

Definition 2. A linear time invariant system

$$\sigma x(t) = Ax(t) + Bw(t), \quad x(0) = x_0, \tag{4}$$

where $x(t) \in \mathcal{R}^n$ and $w(t) \in \mathcal{R}^m$ denote the state and an exogenous disturbance, respectively, and where σ denotes either the shift (discrete time) or time-derivative (continuous time) operator, is said to be *superstable* if the matrix A is superstable (for discrete-time systems, formula (4) reads $x_{k+1} = Ax_k + Bw_k$, and $x(t)$ is understood as x_k).

[1] These matrices are often referred to as *negative diagonal dominant matrices*, and $-A$ is sometimes referred to as the *Hadamard matrix*.

2.2 Stability and Disturbance Rejection Properties of Superstable Systems

Fact 1. Consider a superstable system (4). Then:

a) If $w_k \equiv 0$ for all $k \geq 0$ then

$$\|x_k\| \leq (1 - \mu_d)^k \|x_0\| \tag{5}$$

in the discrete-time case and

$$\|x(t)\| \leq e^{-\mu_c t} \|x(0)\| \tag{6}$$

for the continuous-time counterpart.

b) If $\|w\|_\infty \leq 1$ and $\|x_0\| \leq \gamma \doteq \|B\|_1/\mu$ then

$$\|x\|_\infty \leq \gamma \tag{7}$$

for both continuous and discrete time systems.

Property **a)** above shows that system (4) is exponentially stable and admits a polyhedral Lyapunov function (Blanchini, 1999) of the form $V(x) = \|x\|$. In the case of discrete-time systems, this property is well known and similar results can be found in a number of textbooks on linear algebra. For continuous-time systems, estimates of the form (6), (7) have, apparently, been first obtained by Lozinskii (1953). They were also widely used in the works by Newman (1959), Coppel (1965), McKenzie (1966), Siljak (1978), and Kaszkurevich and Bhaya (2000). It is worth noticing that for stable (but not superstable) systems, one can only obtain an estimate of the form:

$$\|x_k\| \leq C(\varepsilon)(\rho+\varepsilon)^k \|x_0\|, \qquad \varepsilon > 0, \quad \rho+\varepsilon < 1,$$

where $C(\varepsilon)$ is a constant which may take very large values. As a result, the norm $\|x_k\|$ does not decay monotonically as k grows (as it holds for superstable systems), but can rather increase at initial iterations. For instance, for the system with the same matrix $A = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$ with $\rho = 0$ and initial conditions $x_0 = (0, 1)$, we have $x_1 = (2, 0)$, i.e., $\|x_1\|$ becomes twice as large as $\|x_0\|$. Superstable systems do not experience such an undesired "peak effect" (as well as a similar overshoot effect) at the initial part of the trajectory.

Property **b)** is related to input–output (BIBO) stability and disturbance rejection. It shows that the hypercube $\{x \in \mathcal{R}^n : \|x\| \leq \gamma\}$ is d–invariant (Blanchini, 1999), that is, all trajectories initiating inside this cube remain there for all admissible perturbations.

An additional important property of superstable systems is their robustness against time varying and nonlinear disturbances. We consider a generalization of system (4):

$$\sigma x(t) = A(t)x(t) + f[x(t), t],$$

where the matrix $A(t)$ may be time-dependent, and the disturbances $f(\cdot,\cdot)$ may depend both on time t and state.

For a superstable system the following properties hold.

Fact 2. Let $\mu(A(t)) \geq \mu > 0$ and $\|f(x,t)\| \leq \alpha + \beta\|x(t)\|$, $0 \leq \beta < \mu$, for all $0 \leq t < \infty$. Then,

a) For $\alpha = 0$, we have

$$\|x(t)\| \leq e^{-(\mu-\beta)t}\|x(0)\|, \qquad 0 \leq t < \infty$$

in the continuous-time case and

$$\|x_k\| \leq q^k\|x_0\|, \qquad q = 1 - \mu + \beta < 1$$

in the discrete-time case.

b) For $\alpha > 0$, $\|x(0)\| \leq \gamma \doteq \alpha/(\mu - \beta)$, we have (for both continuous and discrete systems)

$$\|x(t)\|_\infty \leq \gamma.$$

Similar results can be found in the above-mentioned works (Lozinskii, 1953), (Newman, 1959), (Coppel, 1965), (McKenzie, 1966), (Siljak, 1978), and (Kaszkurevich and Bhaya, 2000). Note that there is no counterpart of these results for stable systems: solutions of the system $x_{k+1} = A_k x_k$ may not tend to zero even if all matrices A_k are stable. For instance, given $A_0 = A_2 = A_4 = \ldots = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$, $A_1 = A_3 = A_5 = \ldots = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$, for $x_0 = (0,1)^{\mathrm{T}}$ we have $x_{2k} = (0, 2^{2k})^{\mathrm{T}} \to \infty$, although all matrices A_k satisfy $\rho(A_k) = 0$, i.e., condition (3) does hold.

2.3 Spectral Properties

In this section we briefly discuss the spectral properties of superstable matrices. Recently Bobyleva and Pyatnitskii (2001) obtained a characterization of the eigenvalues of a (Hurwitz) matrix such that the associated continuous time system admits a piecewise linear Lyapunov function. Using this results and the fact that $\|x\|$ is a Lyapunov function for a superstable system (see above), leads to the following result:

Lemma 1. Let a matrix A satisfy conditions (2). Then its eigenvalues belong to the following sector:

$$\lambda_i \in \Lambda \doteq \left\{\lambda \in \mathcal{C} : |\arg\lambda - \pi| \leq (1 - n^{-1})\pi/2\right\}.$$

In particular, for $n = 2$, the eigenvalues belong to the right angle with the negative semiaxis being its bisectrix. As n grows, this sector tends to the whole left half plane.

In the discrete time, for $n = 2$, the eigenvalues of A belong to the diamond $\|\lambda\|_1 < 1$; in the general case (for $n > 2$) the result is not known.

3. SUPERSTABILIZATION

The previous section was devoted to system analysis, i.e., we considered superstable systems without control inputs. In this section we show how to apply these tools to controller synthesis. Consider an LTI system of the form system

$$\begin{aligned} \sigma x(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{8}$$

with output y and control u. Our goal is to find a static output feedback law $u = Ky$ such that the resulting closed-loop system

$$\sigma x(t) = A_c x(t), \qquad A_c = A + BKC$$

be superstable[2]. In this case we refer to the matrix K as *superstabilizing*. Note that, in contrast to state feedback (when we seek a controller in the form $u = Kx$), static output feedback is known to be an extremely hard problem. In spite of considerable efforts and many papers available on the subject, the problem remains open (Blondel and Tsitsiklis, 2000), (V. Blondel *et. al.*, 1999). Moreover, since it reduces to a rank minimization problem, it is conjectured to be NP–hard. On the other hand, as we show next, output superstabilization admits a very simple solution.

Theorem 1. For a system of the form (8), a static output superstabilizing controller $u = Ky$ exists iff the set of inequalities in K

$$\mu(A + BKC) > 0 \tag{9}$$

admits a solution (for both continuous-time and discrete-time case).

Note that solvability of (9) can be checked via LP, since it can be recast into a system of linear inequalities with respect to the entries of the matrix K using standard tools and passing from the problem involving absolute values to a linear one. However, the simplicity of solution is gained at the expense of restricting the class of stabilizable systems to a smaller class of superstabilizable systems. For example, it is well known that any controllable pair (A, B) is stabilizable via state feedback. On the other hand, it may not be superstabilizable. Indeed, if a system is specified in the controllable canonical form, then only elements in the last row of the closed-loop matrix can be affected by changing the parameters of the controllers. Clearly in this case condition (9) cannot be achieved. Loosely speaking, in order to superstabilize a system, it is required that the number of control inputs be large enough so that any row of the matrix $A + BKC$ can be affected by choice of K.

[2] Clearly the same formulation can be used to design fixed structure controllers.

4. REJECTION OF BOUNDED DISTURBANCES

Assume that a system is superstabilizable using output or state feedback. Then, one can attempt to find superstabilizing controllers that optimize some performance index. In particular, two problems are of our interest here: (i) optimal rejection of bounded exogenous disturbances, and (ii) minimization of an integral performance index.

We consider a linear system specified in the state-space form

$$\begin{aligned} \sigma x &= Ax(t) + Bu(t) + D_1 w(t) \\ y(t) &= Cx(t) + D_2 w(t), \end{aligned} \tag{10}$$

where w is a persistent, bounded exogenous disturbance:

$$\|w(t)\| \leq 1, \qquad 0 \leq t < \infty. \tag{11}$$

The goal is to design a stabilizing controller $u = Ky$ which minimizes

$$J \doteq \max_w \max_t \|x(t)\|$$

(as above, we use the convention about the operator σ and, in the discrete-time case, we replace $x(t)$ with x_k and $\max_w \max_t \|x(t)\|$ with $\max_w \max_k \|x_k\|$).

In other words, our goal is to make the state of the system as small as possible for the worst-case perturbations. Problems of this type are the subject of the so-called l_1-optimization theory, see (Dahleh and Pearson, 1987), (Dahleh and Diaz-Bobillo, 1995), (Barabanov, 1996). Such problems are hard, and satisfactory solutions are obtained just in a number of particular cases. Let us take the superstabilization standpoint; namely, we first require that the closed-loop matrix be superstable:

$$\begin{aligned} \sigma x(t) &= A_c x(t) + Dw(t), \qquad A_c = A + BKC \\ D &= D_1 + BKD_2, \quad \mu(A_c) > 0. \end{aligned}$$

We then can make use of estimate (7) $\|x(t)\| \leq \|D\|_1/\mu(A_c)$ and try to minimize the right-hand side of this inequality. Hence, we arrive at the following problem:

$$\begin{gathered} \min_{K,\nu} \|D_1 + BKD_2\|_1/\nu \doteq J^* \\ \text{subject to } \mu(A + BKC) \geq \nu > 0. \end{gathered} \tag{12}$$

As above, this problem is seen to be an LP with respect to the entries of the controller matrix K, and ν is a scalar parameter. In other words, design of a superstabilizing controller that optimally attenuates bounded disturbances reduces to solving a parametric linear program (12). We formalize this result below.

Theorem 2. If the parametric LP problem (12) admits a solution K, ν, then the controller $u = Ky$ superstabilizes system(10), (11), and for all $\|w(t)\| \leq 1$, $0 \leq t < \infty$, we have:

$$\|x(t)\| \leq J^*, \qquad 0 \leq t < \infty.$$

We stress that the estimate J^* is an upper bound for the true value J, i.e., the proposed method yields suboptimal solutions.

Some properties of this approach are worth noting. First, the norm of the state vector is minimized uniformly over time rather than just asymptotically. This prevents "peak effects," or overshoot, at the initial part of the trajectory. Second, we seek a controller of the given, fixed structure $u = Ky$ so that its order does not depend on initial problem data. In contrast, l_1-optimization theory leads to controllers with arbitrarily high order (discrete-time case), or infinite-dimensional (continuous-time case).

5. THE LINEAR-LINEAR REGULATOR

The following Linear Quadratic Regulator problem is considered classical in the control theory: For the linear system

$$\dot{x} = Ax + Bu, \qquad x(0) = x_0, \tag{13}$$

find a stabilizing control in the form $u = Kx$, which minimizes the *quadratic* integral performance index

$$J_q \doteq \int_0^\infty (x^{\mathrm{T}} P x + u^{\mathrm{T}} Q u) dt.$$

However, in a number of situations, the *linear* integral performance

$$J \doteq \int_0^\infty (\|x\| + \alpha \|u\|) dt \tag{14}$$

may be more appropriate. Problems of such form are sometimes considered in the literature; e.g., as IAE criterion (Integral of absolute value of the tracking error) (Ogata, 1990), or as induced ℓ^1-norm minimization (Yu and Sideris, 1997); however, there are no systematic ways for design. Below, we propose a solution method based on the estimates given in Sec. 2. Again, we require that the closed-loop matrix $A + BK$ be superstable rather than just stable. Then $\|x(t)\| \leq e^{-\mu_c t}\|x_0\|$ (see (6)) with $\mu_c \doteq \mu_c(A + BK)$, whence the estimate $J \leq (1 + \alpha\|K\|_1)\|x_0\|/\mu_c$ follows. The right-hand side of this inequality is an upper bound for J, and it is linear in K so that minimizing it reduces to a parametric linear programming.

Theorem 3. If there exists a solution K, ν to the parametric LP

$$\min_{K,\nu}(1+\alpha\|K\|_1)/\nu \doteq J^*$$

$$\text{subject to } \mu_c(A+BK) \geq \nu > 0,$$

then the controller $u = Ky$ superstabilizes system (13), (14), and $J \leq J^*$.

Note that if we drop the term $\alpha\|u\|$, then in some cases the solution may degenerate. For example, if the inequality $\mu_c(A+BK) \geq \nu > 0$ is solvable for any ν (in particular, for B square and nonsingular), then μ can take arbitrarily large values, and J can be made arbitrarily small. The resulting matrix K and control u, however, will be extremely large in that case. The term $\alpha\|u\|$ is incorporated in the integral to prevent this effect. Similar results hold for discrete-time systems.

6. ROBUSTNESS AND SUPERSTABILITY

Previous sections were devoted to problems involving exact description of the plant, i.e., the matrices A, B, etc., were given. In real-life problems, uncertainty unavoidably appears in the plant description, and systems must be designed to retain certain performance (in particular, be stable) in the presence of uncertainty, i.e., they must be robust. There is a large volume of literature on robustness (e.g., see (Bhattacharyya *et. al.*, 1995) and the references therein); nevertheless, some of the problems are very hard, and so far, solutions are unknown. In particular, robust stability of interval matrices, i.e., checking the stability of the matrix family

$$A = ((a_{ij})), \qquad \underline{a}_{ij} \leq a_{ij} \leq \overline{a}_{ij}, \qquad (15)$$

is proven to be NP-hard (Blondel and Tsitsiklis, 2000) and admits no efficient solution methods. We show that the problem of robust *superstability* of interval matrices is nearly trivial. Let us rewrite family (15) in a somewhat different form:

$$\begin{array}{c} A = ((a_{ij})),\ |a_{ij} - a^0_{ij}| \leq \gamma m_{ij} \\ i,j = 1,\dots,n, \end{array} \qquad (16)$$

where A^0 is the nominal matrix, $\gamma \geq 0$ is a scalar parameter, and $m_{ij} \geq 0$ are given numbers. Assume now that the nominal A^0 is superstable, i.e., $\mu(A^0) > 0$. We then easily arrive at the following criterion of robust superstability of family (16), i.e., at the conditions that $\mu(A) > 0$ holds for all matrices of the family.

Theorem 4. Let A^0 be superstable, $\mu(A^0) > 0$, then family (16) is robustly superstable iff

$$\gamma < \gamma^* \doteq \min_i \frac{1 - \sum_j |a^0_{ij}|}{\sum_j m_{ij}}$$

in the discrete-time case and

$$\gamma < \gamma^* = \min_i \frac{-a^0_{ii} - \sum_{j\neq i} |a^0_{ij}|}{\sum_j m_{ij}}$$

in the continuous-time case. In particular, for $m_{ij} \equiv 1$, the robust superstability radius is given by (for both continuous-time and discrete-time cases)

$$\gamma^* = \mu(A^0)/n.$$

Therefore, γ^* is the maximal possible range of perturbations for the entries of A^0 which retains superstability of $A = A^0 + \gamma\Delta$, where $\Delta = ((\Delta_{ij}))$, $|\Delta_{ij}| \leq m_{ij}$.

The estimates obtained can also be used when solving problems of robust superstabilization and robust optimal control having the form considered above.

7. SOME OPEN PROBLEMS

To conclude the paper, in this section we briefly comment on some open problems. The first relates to the SISO versions of the results presented in this paper. Consider a SISO system described by the nth order difference equation:

$$x_k = a_1 x_{k-1} + \ldots + a_n x_{k-n} + w_k, \quad (17)$$

where $x_k \in \mathcal{R}^1, w_k \in \mathcal{R}^1$. Then, introducing the delay operator $zx_k = x_{k-1}$, we obtain

$$P(z)x_k = w_k,\ P(z) = 1 - a_1 z - \ldots - a_n z^n.$$

Definition 3. A polynomial $P(z)$ of the form above is said to be superstable if its coefficients satisfy the condition $\sum_i |a_i| < 1$.

Such polynomials were introduced by Cohn in 1922 and were used by Blanchini and Sznaier (1997) and Polyak and Halpern (2001) to synthesize low order suboptimal controllers. It turns out that results completely analogous to Facts 1 and 2 hold for a SISO system if its associated polynomial is superstable. On the other hand, it is worth noticing that the standard conversion of a SISO system in the form (17), e.g. an nth order difference equation, to the state space form (4) does not yield a superstable matrix. This suggest that the results presented here for MIMO systems can be expanded to larger classes of systems and that in some cases (at least SISO) superstability is a structural property, independent of the coordinate system chosen for the state-space realization.

Secondly, the issue of SISO analogs of superstability remains unsolved for continuous-time systems: it looks like there is no any meaningful counterpart for superstability of continuous-time polynomials (with left half plane roots).

Finally, a third open problem relates to estimating the conservatism incurred with the proposed approach. For example, inequalities (5), (7), (6) are just upper bounds for the corresponding quantities, and it is not clear how large the gap is from the true values (i.e., from $\sup\|x\|$). Examples illustrate that this difference may be quite large, e.g., for the system $x_{k+1} = Ax_k$ with $A = \begin{pmatrix} 0 & q \\ 0 & 0 \end{pmatrix}$, $|q| < 1$, and arbitrary x_0 we have $x_k = 0$, $k \geq 2$, while estimate (5) yields $\|x_k\| \leq |q|^k \|x_0\|$. On the other hand, if the same matrix A is considered in the nonhomogeneous system $x_{k+1} = Ax_k + w_k$, estimate (7) results in $\|x_k\| \leq 1/(1-|q|)$, while $\sup\|x_k\| = 1 + |q|$, i.e., the difference is not that dramatic provided that $|q|$ is not too close to 1. Numerical modeling shows that the ratios between the upper bound and the true value constitute 1.22, 2.24, 3.57, and 7.1 for $n = 2, 5, 10, 20$, respectively (in the experiments, superstable matrices were generated randomly with the succeeding calculation of estimates (7) and $\sup\|x_k\|$). Finally, the results in (Blanchini and Sznaier, 1997) show this bound to be tight for SISO systems having a Finite Impulse Response.

8. CONCLUSIONS

The results presented in this paper demonstrate that the superstability concept is very useful in control theory; it allows for solving many problems which do not admit solutions with the classical approaches. At the same time, the method elaborated is subject to certain limitations, which we mention above. At large, we believe that this line of research is very promising as a tool to design simple controllers in cases where the classical concept of stability leads to hard computational problems.

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