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OPTIMAL TORQUE CONTROL OF A SYNCHRONOUS MACHINE

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Abstract: The optimal torque control of a synchronous machine is a classical problem. This paper presents an alternative to SVM (Space Vector Modulation which belongs to PWM approaches) based upon direct boolean control. The result is a predictive method allowing an optimal torque control of the synchronous machine. A comparison between both approaches will be proposed . *Copyright © 2002 IFAC*

Keywords: synchronous machine, boolean control, predictive control.

1. INTRODUCTION

The optimal torque control of a synchronous machine is a classical problem in power electronics. A common solution to this problem is to control the stator currents in the Park transformation. Usually, the three-phase voltage supplying the machine is built from a continuous voltage source using an inverter made up of six switches. The control strategy consists in driving the six switches of the inverter in order to obtain an optimal torque control. The purpose of this paper is to present a new approach based on the Boolean predictive control (Holderbaum, 1999) and to compare it to the Space Vector Modulation strategy (SVM) (Bühler, 1997).

The paper is organized as follows:

In a first part, the state model of the system and the principles of optimal torque control are presented. The second part is devoted to the SVM which is based on a continuous point of view, some simulation results are presented. The third part proposes an original predictive boolean control method and presents some simulation results. The fourth part concerns the stability robustness of the predictive boolean control method. The conclusion gives a comparison between both approaches.

2. SYSTEM MODELING – OPTIMAL TORQUE CONTROL

2.1 System modeling

The system includes a permanent magnet synchronous machine (PMSM) driving a load and supplied by an inverter. The PMSM has nb_p pairs of poles ; its statoric resistance is R and its magnetic induction flux is Φ_f. The constitutive relation between fluxes and currents is:

$$\begin{pmatrix}\Phi_d\\ \Phi_q\end{pmatrix}=\begin{pmatrix}L_d & 0\\ 0 & L_q\end{pmatrix}\begin{pmatrix}i_d\\ i_q\end{pmatrix}+\begin{pmatrix}\Phi_f\\ 0\end{pmatrix} \quad (1)$$

The load is an inertia J with a viscous friction R_f and a breaking torque C_{brk}. The control input of this system is the configuration of the switches. The system can be represented by the following figure:

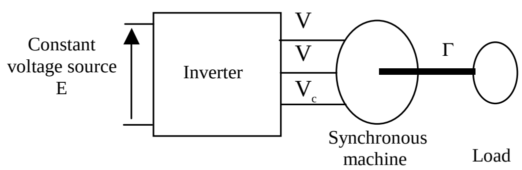

Fig. 1. Global scheme of the system

In order to simplify the PMSM model, Park transformation is used. In this matrix transformation, the three-phase voltage (V_a, V_b ,V_c) actually supplying the synchronous machine is changed into a new virtual three-phase voltage (V_d , V_q ,V_o). If the sum of the currents supplying the stator of the machine is null, as it will be the case in the following, the three-phase voltage (V_d , V_q ,V_o) can be reduced to a two-phase voltage (V_d , V_q).

The algebraic expression of Park transformation is described below :

$$\begin{pmatrix} V_d \\ V_q \end{pmatrix} = \sqrt{\frac{2}{3}} \begin{pmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{4\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{4\pi}{3}\right) \end{pmatrix} \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} = \sqrt{\frac{2}{3}} \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) \\ \cos\left(\theta - \frac{4\pi}{3}\right) & \sin\left(\theta - \frac{4\pi}{3}\right) \end{pmatrix} \begin{pmatrix} i_d \\ i_q \end{pmatrix} \tag{3}$$

After Park transformation, with the new inputs V_d and V_q , the state equations are (Barret, 1987):

$$\dot{\Phi}_d = -\frac{R}{L_d}\Phi_d + \frac{nb_p}{J}\Phi_q\, p + V_d \tag{4}$$

$$\dot{\Phi}_q = -\frac{R}{L_q}\Phi_q - \frac{nb_p}{J}\left(\Phi_d + \Phi_f\right)p + V_q \tag{5}$$

$$\dot{p} = nb_p\, \Phi_d\, \Phi_q \left(\frac{1}{L_d} - \frac{1}{L_q}\right) + nb_p\, \Phi_q\, \frac{\Phi_f}{L_q} - p\frac{R_f}{J} - C_{brk} \tag{6}$$

$$\dot{\theta} = nb_p\, \frac{p}{J} \tag{7}$$

where the state variables are : Φ_d , Φ_q the magnetic fluxes on d-q axes, *p*, the inertia momentum, θ , the angular position of the rotor and nb_p , the number of pairs of poles.

The torque supplied to the load is related to Φ_d and Φ_q by:

$$\Gamma = -nb_p\, \frac{\Phi_d\, \Phi_q}{L_d} + nb_p\left(\Phi_d + \psi_f\right)\frac{\Phi_q}{L_q} \tag{8}$$

The inverter is made up of 6 switches that commutate by pairs. (T1 with T4, T3 with T6 and T5 with T2). The structure is identical to a Graetz bridge (cf. Fig. 2.). If the switches are considered as ideal, three elementary commutation cells can be isolated (C1, C2 and C3, cf. Fig. 2.) (Buisson, *et al.*, 2001). For each cell, a boolean is defined (respectively *m*, *n* and *p*) whose value depends on the state of the switches in this commutation cell. Thus, the booleans m_1, m_2 and m_3 define the state of the inverter.

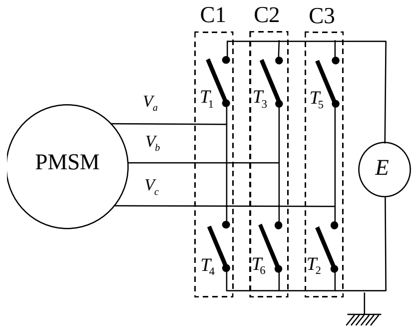

Fig. 2. Inverter

The simple following input/ouput relationships for the inverter are obtained:

$$V_a = m_1\, E \tag{9}$$

$$V_b = m_2\, E \tag{10}$$

$$V_c = m_3\, E \tag{11}$$

and the expression of V_d and V_q can be deduced from the configuration of the inverter (cf. relations (12) and (13)).

$$V_d = E\sqrt{\frac{2}{3}}\left(m_1 \cos(\theta) + m_2 \cos(\theta - \frac{2\pi}{3}) + m_3 \cos(\theta - \frac{4\pi}{3}) \right) \tag{12}$$

$$V_q = E\sqrt{\frac{2}{3}}\left(m_1 \sin(\theta) + m_2 \sin(\theta - \frac{2\pi}{3}) + m_3 \sin(\theta - \frac{4\pi}{3}) \right) \tag{13}$$

The voltage output of the inverter is denoted by: $V_i = \begin{pmatrix} V_d \\ V_q \end{pmatrix}_i$. Index *i* indicates the configuration of the inverter (it is the integer conversion of the binary number $m_1\ m_2\ m_3$). The 8 possible switches configurations lead to distinct non zero output voltages except for $V_0 = V_7 = 0$.

2.2 Optimal torque control

The control purpose is to make the torque Γ follow a reference Γ_c. Since Γ depends on currents i_d and i_q (cf. relations (1) and (8)), there exists a degree of freedom. Let recall that optimal torque control is based on Joule losses minimization. In others words $i_s = \sqrt{i_d^{\ 2} + i_q^{\ 2}}$ is the criterion to minimize (Bühler, 1997) and this condition is realized when :

$$i_d = 0 \tag{14}$$

$$i_q = \frac{\Gamma_c}{nb_p\, \Phi_f} \tag{15}$$

with the condition $L_d = L_q$.

3. SPACE VECTOR MODULATION STRATEGY

3.1 Description of the method

This approach is a continuous one, the first step of which consists in determining a multivariable controller (for example a PID). The inputs of the controller are the respective errors on i_d and i_q, its outputs are V_d and V_q.

The state equations of the operative part being non linear (cf. eq. (4) to (7)), a possible controller design method consists in applying the non linear compensation.

Using the new inputs :

$$U_1 = \frac{nb_p}{J}\Phi_q\ p + V_d \qquad (16)$$

$$U_2 = -\frac{nb_p}{J}\left(\Phi_d + \Phi_f\right)p + V_q \qquad (17)$$

in equations (4) and (5) leads to two linear first order decoupled systems.

As a consequence, two independent controllers (PI) can be elaborated in order to regulate i_d and i_q.

First controller: $K_1 = 10R_1$ and $Ti_1 = \frac{Ld_1}{R_1}$ (19)

Second controller: $K_2 = 10R_1$ and $Ti_2 = \frac{Lq_1}{R_1}$ (20)

Remark : K_1 and K_2 have been chosen so that the rise times be the same as in the boolean approach (cf. §4). Ti_1 and Ti_2 are such that both closed loops be first order ones.

The second step of the approach consists in applying the PWM strategy, in order to approximate the continuous voltage output $V = \begin{pmatrix} V_d \\ V_q \end{pmatrix}$ of the controller at best from the constant voltage E using the 8 possible configurations of the inverter. This method is called Space Vector Modulation.

Let V_{ech} be the sampled signal derived from V using a period T_{ech}. At each sample time, V_{ech} can be expressed as a linear combination of V_0 and two voltages V_k, V_l chosen between V_1 and V_6, with positive coefficients T_0, T_k and T_l corresponding to the respective durations of V_0, V_k and V_l voltages application within a sample period:

$$V_{ech} = \frac{T_0\ V_0 + T_k\ V_k + T_l\ V_l}{T_{ech}} \qquad (21)$$

with : $T_0 + T_k + T_l = T_{ech}$ (22)

To determine k and l as well as the durations T_0, T_k and T_l, an algebraic transformation $(d,q) \rightarrow (\alpha,\beta)$ is first elaborated that locates the outputs V_i of the inverter on the vertices of an hexagon and on its centre (for $V_0 = V_7 = 0$**).** The output V_{ech} is located in a particular sector of the hexagon between two vertices that define k and l. For more details, see (Bühler, 1997 ; Zare and Ledwich, 1999).

Then an algorithm allows the determination of the durations T_0, T_k and T_l.

With such a technique the number of commutations of switches by second is equal to $12\,F_{ech}$.

A complete simulation (cf. the figure thereafter) of this control technique has been realized using MATLAB/SIMULINK.

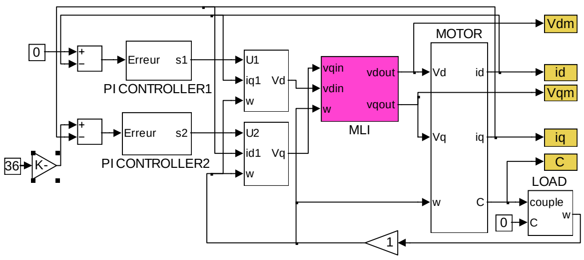

Fig. 3. Optimal torque control of a synchronous machine uing a PWM technique

3.2 Some results using the SVM strategy

Some results are presented using the SVM technique exposed in the previous paragraph. The following experimental parameters have been adopted :

$L_d = 112.6 10^{-6}\ H$	$L_q = 112.6 10^{-6}\ H$
$R = 0.02\,\Omega$	$\Phi_f = 60.39 10^{-3}\ S.I$
$nb_p = 6$	$J = 1\,kg\,m$
$R_f = 0\ \Omega$	$C_{brk} = 0\ \ N\,m$

The torque must be regulated to the value : $\Gamma_c = 36\,N.m$ corresponding to $i_q = 100\,A$ and $i_d = 0\,A$. The criterion to test the performance of the control, F_{ech} being fixed, is the mean square error on the torque and the current i_d respectively denoted by E_Γ and E_{Id}. Two simulations have been realized. The only difference is the sample frequency F_{ech}.

- Simulation1: $F_{ech} = 1000Hz$

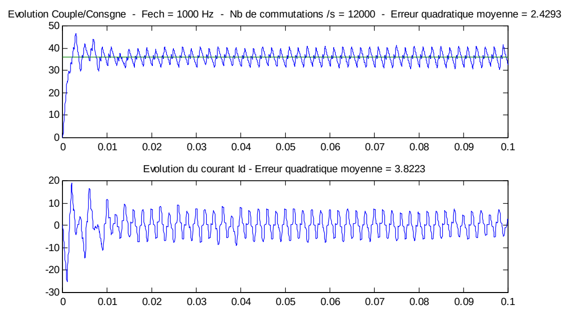

Fig. 4. Torque and current i_d profiles

- Simulation2: $F_{ech} = 2500Hz$

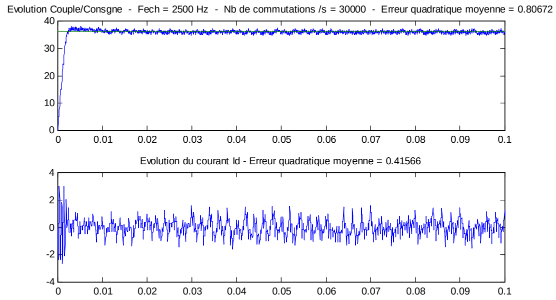

Fig. 5. Torque and current i_d profiles

We obtain the following results for the mean square errors :

Simulation1: $E_\Gamma = 2.4293$, $E_{Id} = 3.8223$, $Nb_c = 12000$

Simulation2: $E_\Gamma = 0.8067$, $E_{Id} = 0.4157$, $Nb_c = 30000$

The third simulation allows to test the dynamic performances of the controller. Now $\Gamma_c = 36\sin(10\pi t)$ and $F_{ech} = 1000Hz$. The following results are obtained :

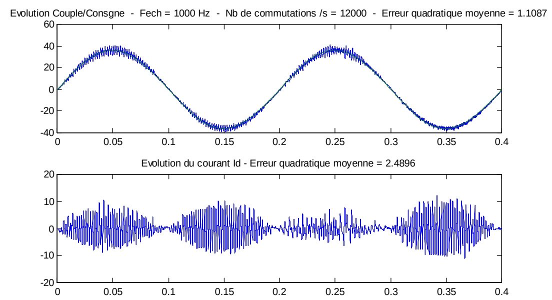

Fig. 6. Torque and current i_d profiles

The mean square errors are mentioned below.

Simulation3: $E_\Gamma = 1.1087$, $E_{Id} = 2.4896$, $Nb_c = 12000$

4. PREDICTIVE CONTROL

4.1 Principle

In the previous approach, the system has been first considered as continuous in order to determine a controller. A method (the SVM strategy) has been applied to approximate the output voltage *V* profile as well as possible.

For a predictive control, the discrete nature of the system is directly taken into account to elaborate the controller. The idea is to realize a predictive control on an horizon equal to one sample period. At each sample time (*k-1*), ones determines the state of the switches that minimizes the following criterion:

$$Crit = \alpha\left(\frac{\Phi_q}{Lq} - \frac{\Gamma_c}{nb_p\Phi_f}\right)_k^2 + \beta\left(\frac{\Phi_d}{Ld}\right)_k^2 + \gamma\sum_{i=1}^{3}\left(Conf_{k\,i} - Conf_{k-1\,i}\right)^2 \quad (23)$$

The first two terms represent the prediction on both errors (on the torque and the current i_d) at sample time *k*.

Since an optimal torque control must be realized, the currents i_d and i_q must be regulated to the values defined in (14) and (15), that is why the two first terms of the criterion have been introduced.

$Conf_{k-1}$ is a vector with three components (*m*, *n* and *p*) defining the current inverter configuration. $Conf_k$ defines the next configuration.

Thus, a minimum-loss strategy in the switches can be elaborated if the coefficient γ is not equal to zero.In order to do the prediction, the relations (3) to (6) are discretized using the Euler algorithm. The criterion is computed for each of the 8 possible values of $Conf_k$. Coefficients α, β and γ are the weighting coefficients of the criterion.

4.2 MATLAB/SIMULINK implementation

The global system (operative part and control part) modeled using MATLAB/SIMULINK is represented on the figure below:

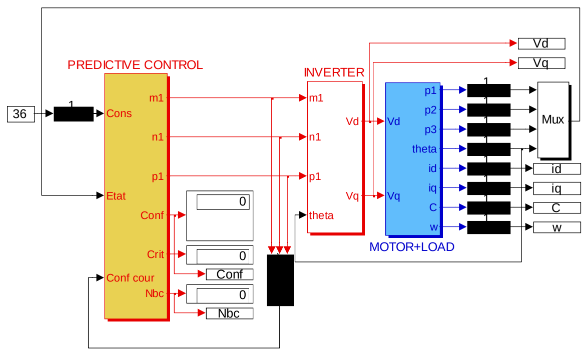

Fig. 7. SIMULINK scheme

The structure of the predictive controller which determines $Conf_k$ at each sample time is organized as follows. For the 8 possible configurations of the inverter, the criterion is computed using a parallel architecture. Then, a comparison is realized in order to keep the only configuration (i.e. m_1 m_2 m_3 booleans) that minimizes the criterion.

4.3 Some results for the Boolean predictive control

The experimental parameters are the same as in the SVM strategy. The torque must be regulated to the

value $\Gamma_c = 36\,N.m$, corresponding to $i_q = 100\,A$ and $i_d = 0\,A$. As previously, the number Nb_c of commutations by second being fixed, the performance criteria of the control are the mean square errors E_Γ on the torque and E_{Id} on current i_d.
Several simulations have been performed, with different sample frequencies and weighting coefficients α, β and γ.
Since $\gamma = 0$ in the first two simulations, the control achieved does not minimize the number of switches commutations between two successive sample times.

- Simulation1: $F_{ech} = 4000\,Hz$, $\alpha = 1$, $\beta = 1$, $\gamma = 0$

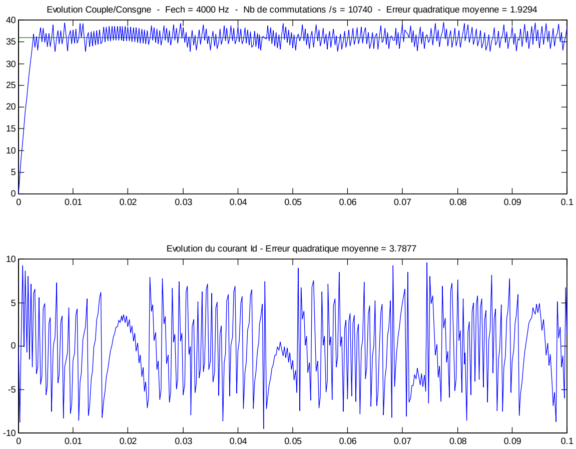

Fig. 8. Torque and current i_d profiles

- Simulation2: $F_{ech} = 12000\ Hz$, $\alpha = 1$, $\beta = 1$, $\gamma = 0$

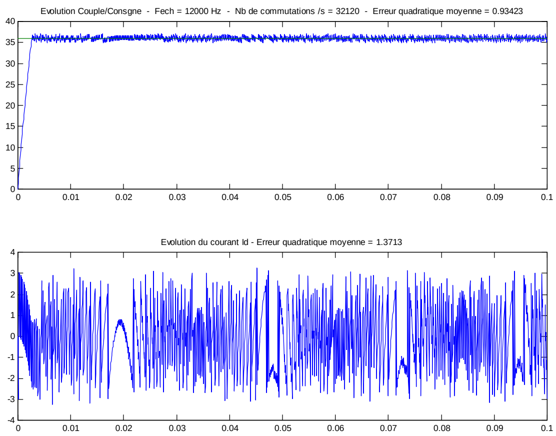

Fig. 9. Torque and current i_d profiles

The following results for the mean square errors ai obtained:
Simulation1: $E_\Gamma = 1.9294$, $E_{Id} = 3.7877$, $Nb_c = 10740$
Simulation2: $E_\Gamma = 0.9342$, $E_{Id} = 1.3713$, $Nb_c = 32120$

In the third simulation, $\gamma \neq 0$, thus the number of switches commutations is taken into account.

- Simulation3: $F_{ech} = 10000\,Hz$, $\alpha = 1$, $\beta = 1$, $\gamma = 35$

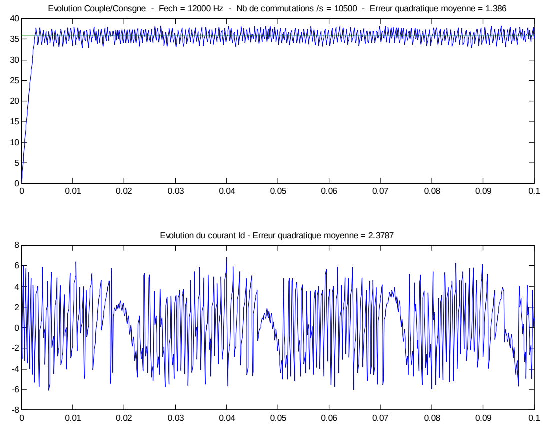

Fig. 10. Torque and current i_d profiles

The mean square errors are mentioned below.

Simulation 3 : $E_\Gamma = 1.386$, $E_{Id} = 2.3787$, $Nb_c = 10500$

Comparing the first and the third simulations, it appears that better static performances are reached with fewer switches commutations by taking a non zero value of gamma in the criterion.
Remark : for these three first simulations, the same value has been chosen for the coefficients α and β in order not to give priority to the i_q or i_d control.

The fourth simulation uses a non constant reference $\Gamma_c = 36\sin(10\pi t)$ to test the dynamic performances of the controller.

- Simulation4: $F_{ech} = 4000\,Hz$, $\alpha = 10$, $\beta = 1$, $\gamma = 0$

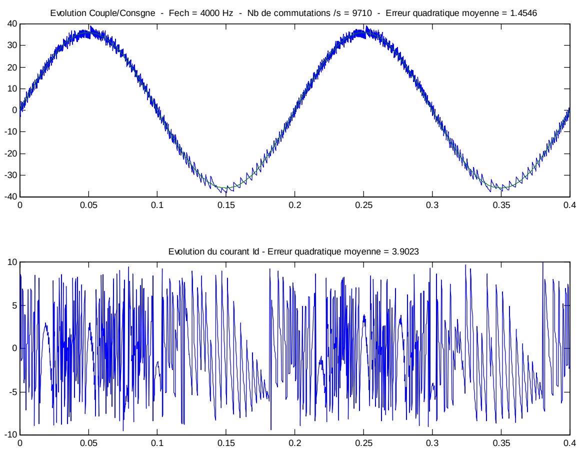

Fig. 11. Torque and current i_d profiles

The following results for the mean square errors are obtained:
Simulation 4: $E_{\Gamma} = 1.386$, $E_{Id} = 2.3787$, $Nb_c = 10500$

5. STABILITY ROBUSTNESS

Because of the presence of the inverter, residual oscillations on the currents i_d and i_q always exist (cf. the simulation results). Nevertheless, the stability robustness of such a system can be studied from an empirical point of view.
In the simulation2 of §3.3, the oscillations magnitudes are equal to 1 for the torque and 3 for the current i_d.
In order to analyse the stability robustness on this particular example, a time delay and a gain will be applied on the state variables Φ_d and Φ_q. The resulting variations of both oscillations magnitudes allows to measure the robustness. An empirical limit of stability is fixed to a maximal oscillation of 10.

On the time delay and the gain

The simulation results allow to conclude that a time delay on Φ_d (respectively on Φ_q) has no influence on the magnitude of the torque oscillations (respectively on i_d oscillations). Similarly, a gain on Φ_d (respectively on Φ_q) has no influence on the magnitude of the torque oscillations (respectively on i_d oscillations).

The limit of stability is reached with a gain equal to 2 and with a time delay equal to 0.2 *ms* (which is equal to $2.4T_{ech}$).

6. CONCLUSION

In this paper, two particular control strategies are compared.
The aim is to compare the 2 control strategies, SVM strategy versus predictive control, with respect to the number of commutation time Nb_c. For the SVM strategy Nb_c is directly proportional to F_{ech} whereas in the case of the predictive control, Nb_c can not be determined. Nevertheless, some simulations using predictive control have been realized with parameter Nb_c almost equal to the simulations in the SVM strategy so a that comparison can be realized.
For $Nb_c \approx 10000$ (number of switches commutations by second), the predictive control leads to better performances particularly if γ is not equal to 0. In that case the criterion tends to minimize the number of switches commutations. (the criterion being the mean square error).
For $Nb_c \approx 30000$, the performances associated with the torque are the same, but results about current i_d are better with SVM approach. (0.4 vs 1.37 for the mean square error).

Actually, it appears that the main difference between both approaches lies in the control design itself. Indeed, the execution of MATLAB/SIMULINK simulations are more complex in the case of SVM technique since it implies multiple non trivial procedures:

- Algebraic transformation $(d,q) \to (\alpha,\beta)$
- Sector determination
- Determination of T_0, T_k and T_l
- Determination of the control profile of the switches….

Whereas in the case of predictive boolean control, the implementation is simpler. At each sample time, one just has to define a criterion, to discretize state equations (using Euler method for instance), to elaborate the parallel architecture of the controller allowing to compute the criterion for the 8 possible configurations of the inverter and lastly to select the configuration minimizing that criterion.

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