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NONLINEAR OBSERVER FOR TAIL-CONTROLLED SKID-TO-TURN MISSILES[1]

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Abstract: This paper presents a nonlinear observer design method for acceleration control of tail-controlled Skid-to-Turn missiles. A nonlinear observer for wind angles (angle of attack and sideslip angle) is designed using a parametric affine missile model developed by the authors. Using the estimated wind angles, a desired acceleration tracking performance can be obtained by a nonlinear control based on a parametric affine missile model. The performance and stability of the proposed observer are analyzed. Simulation results are also included to show that the proposed approach can give a satisfactory performance for missile dynamics.

Keywords: observer, control, missiles, nonlinear, parametric.

1. INTRODUCTION

During the past decades, there has been much research on nonlinear autopilot designs for missiles with highly nonlinear characteristics. As the controller structure becomes more complex, it requires more information of state variables accordingly. In particular, it is necessary to know the wind angles, i.e., angle of attack and sidelisip angle since these values are used in a feedback control system. Actually, several states such as accelerations, angular rates, dynamic pressure, and missile speed can be measured rather accurately through instruments such as accelerometers, the Inertial Navigation System (INS) including rate gyros, and barometer. However, the measurement of the wind angles is not usual since the angle of attack and sideslip angle are not readily available due to the measurement noise and the vulnerability of sensor to mechanical damage (Stevens and Lewis, 1992). Thus, the practical design of autopilots should know the estimated values of wind angles from the measured angular rates and accelerations.

There have been some results on the state estimation for wind angles (Song *et al*. 1996; Song *et al*. 1997; Tahk and Briggs 1998). Actually, it is difficult to estimate wind angles accurately due to the nonlinear characteristics of aerodynamics and dynamics. In Song *et al*. (1996), the estimates of wind angles are obtained based on the simplified aerodynamic model in Oh (1989). Although the estimates can be easily obtained in this way, these estimated values obviously neglect the effects of control inputs. Moreover, these must be obtained at each flight condition in tabular form. In Song *et al*. (1997), a state observer design method was proposed using the functional approximation via neural network. This method is applied to nonlinear missile dynamics valid for all flight conditions, but yields the observer with complex structures requiring large memory and processing time. For plant inversion law via state feedback where accelerations are indirectly controlled (Tahk *et al*. 1988), an observer was also designed to provide an angle of attack and sideslip angle feedback in Tahk and Briggs (1998). Just as in the control law, the observer becomes effective only at each flight condition given in aerodynamic look-up table.

In this paper, a nonlinear observer design method is proposed using a parametric affine missile model (Chwa and Choi, 2000) for tail-controlled STT (Skid-to-Turn) missiles. Although the accelerations and angular rates are assumed to be measurable quantities, the states such as angle of attack, sideslip angle, and

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bank angle are estimated through the state observer using a parametric affine model. In particular, even with the coupling term between the lateral and longitudinal motion, the desired estimation performance can be guaranteed using the sliding surfaces. The states are estimated using the sliding mode observer which uses sliding mode control theory for nonlinear systems (Slotine *et al.*, 1987). Using these estimated states, a nonlinear control law in Chwa and Choi (2000) is employed for overall missile control system. The performance of the proposed observer is analyzed and simulation results are included to show the effectiveness of the proposed approach.

2. PARAMETRIC AFFINE MODELING

In this section, the parametric affine missile model in Chwa and Choi (2000) is briefly reviewed and formulated further as a model for state estimation. As the yaw and pitch dynamics are of the same form, variables are unified as $x = \beta$ (or α), $a_r = r$ (or q), $y = A_y$ (or A_z), $\delta = \delta_r$ (or δ_q), where β (α) is side-slip angle (angle of attack), r (q) is yaw (pitch) angular rate, A_y (A_z) is yaw (pitch) acceleration output, δ_r (δ_q) is deflection angle of yaw (pitch) control fin.

Then, the parametric affine missile model can be expressed as

$$\begin{cases} \dot{x} = -a_r + \dfrac{QS}{Um}(w_1 x + w_2 x^3 + w_3 \delta + \Delta_y) \\ y = -\dfrac{Q}{h_v}(v_1 x + v_2 x^3) - \dfrac{Q}{h_v}\Delta_a \end{cases} \tag{1}$$

where Q is dynamic pressure, U is linear velocity about x-axis, S is aerodynamic reference area, m is missile mass, respectively; w_1, w_2, w_3, v_1, v_2 are slowly time-varying parameters, which depend on Mach number M and bank angle ϕ_A, and defined by

$$w_j = \sum_{i=1}^{N} \mu_i(M)(c_{ij}^{f1} + c_{ij}^{f2} \sin^2(2\phi_A)), \; v_1 = c_1 + c_2|\phi_A|$$

$$v_2 = c_3, \mu_i(M) = \mu_i^0(M) \Big/ \sum_{j=1}^{N} \mu_j^0(M)$$

$\mu_i^0(M) = \exp(-(M - M_i)^2 / \sigma_i^2)$, $i = 1, \cdots, N$, $j = 1,2,3$; c_{ij}^{f1}, c_{ij}^{f2}, c_1, c_2, c_3 are all fitting parameters obtained by a curve fitting technique from a look-up table of aerodynamic coefficients for each $M = M_i$; $h_v = \dfrac{(l_f - l_g)m}{I_M}$, $I_M = I_z$ is moment of inertia about z-axis, l_f and l_g are distances from the nose of a missile to the center-of-pressure of control fins and the center-of-gravity; Δ_y, Δ_a are approximation errors. The variables to be estimated in (1) are x and ϕ_A (contained in w_1, w_2, w_3). y and a_r are assumed to be measurable by accelerometers and rate gyros, respectively.

To formulate (1) for observer-based control further definition is introduced as

$$f_1 = \frac{QS}{Um}\sum_{i=1}^{N}\mu_i(M)c_{i1}^{f1}, \; f_2 = \frac{QS}{Um}\sum_{i=1}^{N}\mu_i(M)c_{i1}^{f2},$$

$$f_3 = \frac{QS}{Um}\sum_{i=1}^{N}\mu_i(M)c_{i2}^{f1}, \; f_4 = \frac{QS}{Um}\sum_{i=1}^{N}\mu_i(M)c_{i2}^{f2}$$

$$f_5 = \frac{QS}{Um}\sum_{i=1}^{N}\mu_i(M)c_{i3}^{f1}, \; f_6 = \frac{QS}{Um}\sum_{i=1}^{N}\mu_i(M)c_{i3}^{f2}$$

$$h_1 = -\frac{Q}{h_v}c_1, \; h_2 = -\frac{Q}{h_v}c_2, \; h_3 = -\frac{Q}{h_v}c_3, \; \Delta_x = \frac{QS}{Um}\Delta_y$$

$$\Delta_h = -\frac{Q}{h_v}\Delta_a$$

Then, (1) can be rewritten by

$$\begin{cases} \dot{x} = -a_r + f_1 x + f_2 \sin^2(2\phi_A)x + f_3 x^3 \\ \quad + f_4 \sin^2(2\phi_A)x^3 + f_5\delta + f_6 \sin^2(2\phi_A)\delta + \Delta_x \\ y = h_1 x + h_2|\phi_A|x + h_3 x^3 + \Delta_h. \end{cases} \tag{2}$$

For notational simplicity, (2) is expressed by

$$\begin{cases} \dot{x} = f(x,\phi_A) + g(x,\phi_A)\delta + \Delta_x \\ y = h(x,\phi_A) + \Delta_h \end{cases} \tag{3}$$

where

$$f(x,\phi_A) = -a_r + f_1 x + f_2 \sin^2(2\phi_A)x + f_3 x^3 + f_4 \sin^2(2\phi_A)x^3$$

$$g(x,\phi_A) = f_5 + f_6 \sin^2(2\phi_A)$$

$$h(x,\phi_A) = h_1 x + h_2|\phi_A|x + h_3 x^3 .$$

3. STATE OBSERVER DESIGN FOR WIND ANGLES

In Chwa and Choi (2000), parametric affine modeling and control method is proposed with the assumption that the full state is measurable, which is not the case in practical implementation. The design of the state observer for a parametric affine missile dynamics (3) implies the state estimation for both x and ϕ_A. As the bank angle is physically bounded as $|\phi_A| \le \pi/4$, estimated bank angle is obtained by using the estimates of angle of attack $\hat{\alpha}$ and sideslip angle $\hat{\beta}$ as

$$\hat{\phi}_A = \begin{cases} \pi/4, & \hat{\phi}_A \ge \pi/4 \\ \arctan(\tan\hat{\alpha}/\tan\hat{\beta}), & |\hat{\phi}_A| \le \pi/4 \\ -\pi/4, & \hat{\phi}_A \le -\pi/4 \end{cases} \tag{4}$$

For the simultaneous estimation of these states, the structure of the standard Luenberger observer is not enough to guarantee the performance of the observer and, accordingly, it is modified here as a sliding mode observer. Overall performance of the sliding mode observer can have the advantages of robustness of the sliding surface against disturbance and uncertainties.

As output y contains ϕ_A, one can choose estimated output $\hat{y}$ as $h(\hat{x},\hat{\phi}_A)$. However, to design control law it is necessary to obtain the time derivative of $\hat{y}$ and $\hat{\phi}_A$, which is hard to obtain. Thus, $\hat{y}$ is chosen to be $h(\hat{x},\hat{\phi}_A=0)$ instead of $h(\hat{x},\hat{\phi}_A)$. Following the standard form of sliding mode observer for nonlinear system in Slotine *et al.* (1987), the observer for (3) is proposed as

$$\begin{cases}\dot{\hat{x}} = f(\hat{x},\hat{\phi}_A) + g(\hat{x},\hat{\phi}_A)\delta + k_e(y-\hat{y}) - D(\hat{x},\delta)\operatorname{sgn}(y-\hat{y}) \\ \hat{y} = h(\hat{x},\hat{\phi}_A = 0)\end{cases} \tag{5}$$

where k_e and $D(\hat{x},\delta)$ are observer gains specified for the estimation convergence. Here, k_e is chosen to satisfy

$$|f_1|+|f_2| < k_e h_1,\ k_e h_2 > 0,\ |f_3|+|f_4| < k_e h_3 \tag{6}$$

and $D(\hat{x},\delta)$ is chosen as

$$D(\hat{x},\delta) = |f_2\hat{x}| + |f_4\hat{x}^3| + |k_e h_2(\pi/4)\hat{x}| + |f_6\delta|. \tag{7}$$

Now, the following assumptions are further made, which holds for the missile systems in Chwa and Choi (2000) employed in this paper.

Assumption 3.1: $h_1, h_2, h_3 < 0$.

Assumption 3.2: The approximation errors Δ_x and Δ_h are bounded and smooth as $|\Delta_x| \le D_x$, $|\Delta_h| \le D_h$.

Remark 3.1: In the missile system employed in this paper, h_1, h_2, and h_3 are shown to be always negative in Chwa and Choi (2000). Thus, k_e satisfying (6) should be negative. In case of other missiles, Assumption 3.1 may not hold and the observer in (5) may have to be modified depending on the signs of h_1, h_2, and h_3.

Here, the state estimation error $\tilde{x} = x - \hat{x}$ and the output estimation error $\tilde{y} = y - \hat{y}$ are defined. The stability and performance of the state observer is summarized in the following theorem.

Theorem 3.1 (Nonlinear Observer I):
The state estimation error between the actual state of the nominal parametric affine missile in (3) and the estimated one by the state observer in (5) with the observer gains k_e and $D(\hat{x},\delta)$ satisfying (6) and (7) under Assumptions 3.1 and 3.2 is stable in the sense that
i) the estimation $\tilde{x}$ are uniformly ultimately bounded, meaning that they remain in a small neighborhood of zero after some time.
ii) furthermore, when the coupling effect due to ϕ_A and the modeling errors Δ_x and Δ_h are zeros, $\tilde{x}$ asymptotically converges to zero, i.e., $\lim_{t\to\infty}\tilde{x} = 0$ if $\phi_A = \hat{\phi}_A = \Delta_x = \Delta_h = 0$.

Proof: The estimation error equation between (3) and (5) becomes

$$\begin{aligned}
\dot{\tilde{x}} &= \dot{x} - \dot{\hat{x}} \\
&= \{f(x,\phi_A) - f(\hat{x},\hat{\phi}_A)\} + \{g(x,\phi_A) - g(\hat{x},\hat{\phi}_A)\}\delta - k_e\tilde{y} \\
&\quad + \Delta_x + D(\hat{x},\delta)\operatorname{sgn}(\tilde{y}) \\
&= f_1(x-\hat{x}) + f_2\{\sin^2(2\phi_A)x - \sin^2(2\hat{\phi}_A)\hat{x}\} \\
&\quad + f_3(x^3 - \hat{x}^3) \\
&\quad + f_4\{\sin^2(2\phi_A)x^3 - \sin^2(2\hat{\phi}_A)\hat{x}^3\} \\
&\quad + f_6\{\sin^2(2\phi_A) - \sin^2(2\hat{\phi}_A)\}\delta - k_e\cdot\{h_1(x-\hat{x}) + h_2|\phi_A|x \\
&\quad + h_3(x^3-\hat{x}^3)\} + \Delta_x + D(\hat{x},\delta)\operatorname{sgn}(\tilde{y}) \\
&= \{f_1 + f_3(x^2 + x\hat{x} + \hat{x}^2)\}\cdot(x-\hat{x}) + f_2\sin^2(2\phi_A) \\
&\quad \cdot(x-\hat{x}) + f_2\{\sin^2(2\phi_A) - \sin^2(2\hat{\phi}_A)\}\hat{x} \\
&\quad + f_4\sin^2(2\phi_A)\cdot(x^3-\hat{x}^3) + f_4\{\sin^2(2\phi_A) - \sin^2(2\hat{\phi}_A)\}\hat{x}^3 \\
&\quad + f_6\{\sin^2(2\phi_A) - \sin^2(2\hat{\phi}_A)\}\delta \\
&\quad - k_e\{h_1(x-\hat{x}) + h_2|\phi_A|(x-\hat{x}) + h_2|\phi_A|\hat{x} + h_3(x^2 + x\hat{x} + \hat{x}^2) \\
&\quad \cdot(x-\hat{x})\} + \Delta_x + D(\hat{x},\delta)\operatorname{sgn}(\tilde{y}) \\
&= \{f_1 + f_3(x^2 + x\hat{x} + \hat{x}^2) + f_2\sin^2(2\phi_A) + f_4\sin^2(2\phi_A) \\
&\quad \cdot(x^2 + x\hat{x} + \hat{x}^2)\}\cdot\tilde{x} \\
&\quad - k_e\{h_1 + h_2|\phi_A| + h_3(x^2 + x\hat{x} + \hat{x}^2)\}\cdot\tilde{x} + D_e + \Delta_x \\
&\quad + D(\hat{x},\delta)\operatorname{sgn}(\tilde{y})
\end{aligned}$$

where

$$\begin{aligned}
D_e &= f_2\{\sin^2(2\phi_A) - \sin^2(2\hat{\phi}_A)\}\hat{x} + f_4\{\sin^2(2\phi_A) \\
&\quad - \sin^2(2\hat{\phi}_A)\}\hat{x}^3 + f_6\{\sin^2(2\phi_A) - \sin^2(2\hat{\phi}_A)\}\delta - k_e h_2|\phi_A|\hat{x}
\end{aligned}$$

Here, D_e can be easily shown to satisfy the inequality

$$|D_e| \le D(\hat{x},\delta) \tag{8}$$

by using $|\phi_A| \le \pi/4$. Defining

$$\begin{aligned}
A_e &= f_1 + f_2\sin^2(2\phi_A) + f_3(x^2 + x\hat{x} + \hat{x}^2) + f_4\sin^2(2\phi_A) \\
&\quad \cdot(x^2 + x\hat{x} + \hat{x}^2) \\
C_e &= h_1 + h_2|\phi_A| + h_3(x^2 + x\hat{x} + \hat{x}^2),
\end{aligned}$$

estimation error equation can be further rearranged as

$$\dot{\tilde{x}} = (A_e - k_e C_e)\tilde{x} + D_e + \Delta_x + D(\hat{x},\delta)\operatorname{sgn}(\tilde{y}) \tag{9}$$

In addition, using $h_1, h_2, h_3 < 0$ in Assumption 3.1 it follows that

$$C_e < 0. \tag{10}$$

The condition (6) yields

$$\begin{aligned}
A_e - k_e C_e &\le |f_1| + |f_2| + (|f_3| + |f_4|)(x^2 + x\hat{x} + \hat{x}^2) \\
&\quad - k_e\{h_1 + h_2|\phi_A| + h_3(x^2 + x\hat{x} + \hat{x}^2)\} \\
&= (|f_1| + |f_2| - k_e h_1) + (-k_e h_2)\cdot|\phi_A| \\
&\quad + (|f_3| + |f_4| - k_e h_3)\cdot(x^2 + x\hat{x} + \hat{x}^2) < 0.
\end{aligned} \tag{11}$$

Now, $\tilde{y}$ can be expressed as

$$\begin{aligned}
\tilde{y} &= y - \hat{y} \\
&= h(x,\phi_A) + \Delta_h - h(\hat{x},\hat{\phi}_A = 0). \\
&= C_e\tilde{x} + h_2|\phi_A|\hat{x} + \Delta_h.
\end{aligned} \tag{12}$$

Since $|h_2|\phi_A|\hat{x}+\Delta_h| \le (\pi/4)\cdot|h_2|\cdot|\hat{x}|+D_h$ holds, (9) can be used to obtain the relation between the sign of $\tilde{y}$ and that of $\tilde{x}$ as follows:

- Case (i) : $|C_e\tilde{x}| \le (\pi/4)\cdot|h_2|\cdot|\hat{x}|+D_h$;

this becomes

$$|\tilde{x}| \le \frac{1}{|C_e|}\{(\pi/4)\cdot|h_2|\cdot|\hat{x}|+D_h\} . \qquad (13a)$$

- Case (ii) : $|C_e\tilde{x}| \ge (\pi/4)\cdot|h_2|\cdot|\hat{x}|+D_h$;

this becomes

$$|\tilde{x}| \ge \frac{1}{|C_e|}\{(\pi/4)\cdot|h_2|\cdot|\hat{x}|+D_h\} \qquad (13b)$$

and yields

$$\mathrm{sgn}(\tilde{y}) = \mathrm{sgn}(C_e\tilde{x}) . \qquad (14)$$

In order to verify the stability of $\tilde{x}$ - dynamics, the Lyapunov function candidate is chosen as

$$V = \frac{1}{2}\tilde{x}^2 \qquad (15)$$

and its time derivative becomes

$$\begin{aligned}\dot{V} &= \tilde{x}\dot{\tilde{x}} \\ &= (A_e - k_eC_e)\tilde{x}^2 + \tilde{x}\{D_e + \Delta_x + D(\hat{x},\delta)\,\mathrm{sgn}(\tilde{y})\}\end{aligned} \qquad (16)$$

from (9). Also, (8), (10), and (14) yields

$$\begin{aligned}\dot{V} &\le (A_e - k_eC_e)|\tilde{x}|^2 + |\tilde{x}|\cdot D(\hat{x},\delta)\{1+\mathrm{sgn}(C_e)\} + D_x|\tilde{x}| \\ &= -A_{oe}|\tilde{x}|^2 + D_x|\tilde{x}|\end{aligned} \qquad (17)$$

where $A_{oe} = -(A_e - k_eC_e) > 0$ from (11). This implies that $\tilde{x}$ converges to a set

$$\{\ \tilde{x}\in R \mid |\tilde{x}| \le D_x/A_{oe}\ \}. \qquad (18)$$

That is, (18) holds for Case (ii) and (13a) for Case (i). Thus, it can be concluded that in any case $\tilde{x}$ converges to a set

$$\{\ \tilde{x}\in R \mid |\tilde{x}| \le \max[\{(\pi/4)\cdot|h_2|\cdot|\hat{x}|+D_h\}/|C_e|, D_x/A_{oe}]\ \} \qquad (19)$$

where $\max[a,b]=a$ for $a \ge b$ and $\max[a,b]=b$ for $a<b$. Now, the boundedness of $\tilde{x}$ in (19) must be guaranteed irrespective of $|\hat{x}|$. Simple differential calculus shows that $n|\hat{x}|/\{d_1+d_2|\hat{x}|^2\}$ has the maximum value at $|\hat{x}| = n\sqrt{(d_1/d_2)}/(2d_1)$ for any $n, d_1, d_2 > 0$. Since $|h_1+h_2|\phi_A|+h_3(x^2+x\hat{x}+\hat{x}^2)| =$ $|h_1|+|h_2|\cdot|\phi_A|+|h_3|\cdot(x^2+x\hat{x}+\hat{x}^2)$ holds for $h_1,h_2,h_3<0$ in Assumption 3.1, it follows that

$$\begin{aligned}|h_2|\cdot|\hat{x}|/|C_e| &= |h_2|\cdot|\hat{x}|\Big/\left|h_1+h_2|\phi_A|+h_3(x^2+x\hat{x}+\hat{x}^2)\right| \\ &= |h_2|\cdot|\hat{x}|/\{|h_1|+|h_2|\cdot|\phi_A|+|h_3|[(x+0.5\hat{x})^2 \\ &\quad + 0.75|\hat{x}|^2]\} \\ &\le |h_2|\cdot|\hat{x}|/\{|h_1|+0.75|h_3|\cdot|\hat{x}|^2]\}.\end{aligned} \qquad (20)$$

Thus, $|h_2|\cdot|\hat{x}|/|C_e|$ can be shown to be bounded by replacing n, d_1, d_2 with $|h_2|$, $|h_1|$, $0.75|h_3|$, respectively. Thus, (19) yields that $\tilde{x}$ is uniformly ultimate bounded, that is, $\tilde{x}$ converges to a bounded set in (19) irrespective of the magnitude of the control input. The bound in (19) is a conservative one and the above analysis shows that $\tilde{x}$ remains sufficiently small compared with the magnitude of x when the approximation error Δ_x and Δ_h also become sufficiently small. In fact, the magnitude of Δ_x and Δ_h can be expected to be very small compared with $|A_{oe}|$ and $|C_e|$ by adjusting k_e in (6). When ϕ_A, D_x, and D_h become zero, $h_2|\phi_A|\hat{x}+\Delta_h = 0$ holds and it can be proceeded as Case (ii). This, in turn, shows that $\tilde{x}$ is guaranteed to converge zero as $\dot{V}_1 \le -A_{oe}|\tilde{x}|^2$ holds instead of (17). This implies that the estimation error is critically dependent on the coupling effect caused by ϕ_A and the modeling errors Δ_x and Δ_h. (Q.E.D.)

The estimator in (5) is useful in that the estimated states are used in the control law. Although the chattering phenomenon of the sliding mode term is filtered through the observer, the direct use of these states can induce the similar chattering phenomenon.

To alleviate the chattering of estimated states further than that of (5), an additional observer given by

$$\begin{cases}\dot{\hat{x}}_0 = f(\hat{x}_0,\hat{\phi}_A) + g(\hat{x}_0,\hat{\phi}_A)\delta + k_e(y-\hat{y}) \\ \quad - D(\hat{x},\delta)\,\mathrm{sgn}(y-\hat{y}) + k_{e0}(\hat{y}-\hat{y}_0) \\ \hat{y}_0 = h(\hat{x}_0,\hat{\phi}_A=0)\end{cases} \qquad (21)$$

is used, where the gains k_e and $D(\hat{x},\delta)$ are the same as in (6) and (7), and k_{e0} is an additional observer gain satisfying

$$|f_1|+|f_2| < k_{e0}h_1\ ,\ k_{e0}h_2 > 0\ ,\ |f_3|+|f_4| < k_{e0}h_3 \qquad (22)$$

to guarantee that estimated states $\hat{x}_0$ and $\hat{y}_0$ converge to $\hat{x}$ and $\hat{y}$.

Here, $\tilde{x}_e = \hat{x}-\hat{x}_0$ and $\tilde{y}_e = \hat{y}-\hat{y}_0$ are defined. The stability and convergence of the additional state observer is summarized in the following theorem.

Theorem 3.2 (Nonlinear Observer II):
The relation between the sliding mode observer in (5) and that in (19) with the observer gains k_e, $D(\hat{x},\delta)$, and k_{e0} satisfying (6), (7), and (22) under Assumptions 3.1 and 3.2 is such that $\tilde{y}_e$ as well as $\tilde{x}_e$ converge to zero as time goes on.

Proof: The estimation error equation between (5) and (21) becomes

$$\begin{aligned}\dot{\tilde{x}}_e &= \dot{\hat{x}} - \dot{\hat{x}}_0 \\ &= \{f(\hat{x},\hat{\phi}_A) - f(\hat{x}_0,\hat{\phi}_A)\} + \{g(\hat{x},\hat{\phi}_A) - g(\hat{x}_0,\hat{\phi}_A)\}\delta \\ &\quad - k_{e0}\{h(\hat{x},0) - h(\hat{x}_0,0)\} \\ &= f_1(\hat{x}-\hat{x}_0) + f_2\{\sin^2(2\hat{\phi}_A)\hat{x} - \sin^2(2\hat{\phi}_A)\hat{x}_0\} \\ &\quad + f_3(\hat{x}^3-\hat{x}_0^3) + f_4\{\sin^2(2\hat{\phi}_A)\hat{x}^3 - \sin^2(2\hat{\phi}_A)\hat{x}_0^3\} \\ &\quad - k_{e0}\cdot\{h_1(\hat{x}-\hat{x}_0) + h_3(\hat{x}^3-\hat{x}_0^3)\} \\ &= \{f_1 + f_3(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)\}\cdot(\hat{x}-\hat{x}_0) + f_2\sin^2(2\hat{\phi}_A) \\ &\quad \cdot(\hat{x}-\hat{x}_0) + f_4\sin^2(2\hat{\phi}_A)\cdot(\hat{x}^3-\hat{x}_0^3)\end{aligned}$$

$$
\begin{aligned}
&-k_{e0}\{h_1(\hat{x}-\hat{x}_0)+h_3(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)\cdot(\hat{x}-\hat{x}_0)\}\\
&=\{f_1+f_3(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)+f_2\sin^2(2\hat{\phi}_A)\\
&\quad+f_4\sin^2(2\hat{\phi}_A)\cdot(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)\}\cdot\tilde{x}_e\\
&\quad-k_{e0}\{h_1+h_3(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)\}\cdot\tilde{x}_e\\
&=(A_{e0}-k_{e0}C_{e0})\,\tilde{x}_e
\end{aligned}
$$

where

$$
\begin{aligned}
A_{e0}&=f_1+f_3(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)+f_2\sin^2(2\hat{\phi}_A)\\
&\quad+f_4\sin^2(2\hat{\phi}_A)\cdot(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)\\
C_{e0}&=h_1+h_3(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)\ .
\end{aligned}
$$

From (22), it follows that

$$
\begin{aligned}
A_{e0}-k_{e0}C_{e0}&\le|f_1|+|f_2|+(|f_3|+|f_4|)\cdot(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)\\
&\quad-k_{e0}\{h_1+h_2|\phi_A|+h_3(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)\}\\
&=(|f_1|+|f_2|-k_{e0}h_1)+(-k_{e0}h_2)\\
&\quad+(|f_3|+|f_4|-k_{e0}h_3)\cdot(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)\\
&<0
\end{aligned}
$$

This implies that $\tilde{x}_e$ converges to zero. Furthermore, $\tilde{y}_e$ can be expressed as

$$
\begin{aligned}
\tilde{y}_e&=\hat{y}-\hat{y}_0\\
&=h(\hat{x},\hat{\phi}_A=0)-h(\hat{x}_0,\hat{\phi}_A=0)\ .\\
&=C_{e0}\tilde{x}_e
\end{aligned}
$$

where $C_{e0}=h_1+h_3(\hat{x}^2+\hat{x}\hat{x}_0+\hat{x}_0^2)<0$. Accordingly, $\tilde{y}_e$ also converges to zero as time goes on. (Q.E.D.)

4. SIMULATION RESULTS

This section presents the simulation results on the state observer and the control law using estimated states. The proposed design technique will be shown to satisfy the performance requirements sufficiently. A full six-degree-of-freedom nonlinear model with assumptions only (A1-3) is used for simulation missile model.

4.1 Simulation conditions and Design parameters

Design parameters of the observer in (6) and (22) are selected as

$$k_e=-0.5\ ,\ k_{e0}=-1\ . \qquad (23)$$

Estimates of the nonlinear observer are substitued into the full state-feedback control law in Chwa and Choi (2000) to form a missile control system.

As an actuator model, the following low pass filter are included:

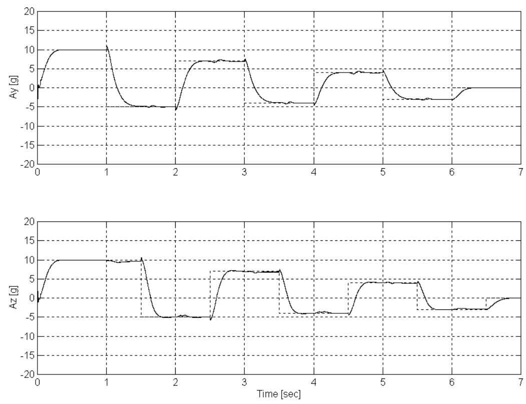
$$\tau\dot{\delta}_r=-\delta_r+\delta_r^c\ ,\ \tau\dot{\delta}_q=-\delta_q+\delta_q^c \qquad (24)$$

where the time constant $\tau=0.01\sec$. The tracking performance for square wave acceleration commands is evaluated with the forward velocity is initially $U=884\,m/\sec$ and decreasing due to the drag effect. The initial estimated states for $\hat{x}$ and $\hat{x}_0$ and actual ones for x in each yaw and pitch axis are given as $\hat{\beta}(t=0)=\hat{\beta}_0(t=0)=-\hat{\alpha}(t=0)=-\hat{\alpha}_0(t=0)=5.73\,\deg$, $\beta(t=0)=0\,\deg$ and $\alpha(t=0)=0\,\deg$. That is, the initial estimation errors are $\tilde{\beta}(t=0)=\tilde{\beta}_0(t=0)=-\tilde{\alpha}(t=0)=-\tilde{\alpha}_0(t=0)=-5.73\,\deg$.

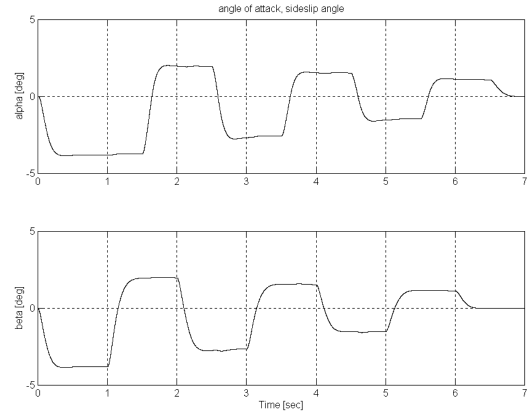
4.2 Performance of the state observer

In this subsection, the performance of the state observer is evaluated. Also, the tracking performance for square wave acceleration commands will be checked by substituting the estimates of the nonlinear observer into the full state-feedback control law in Chwa and Choi (2000).

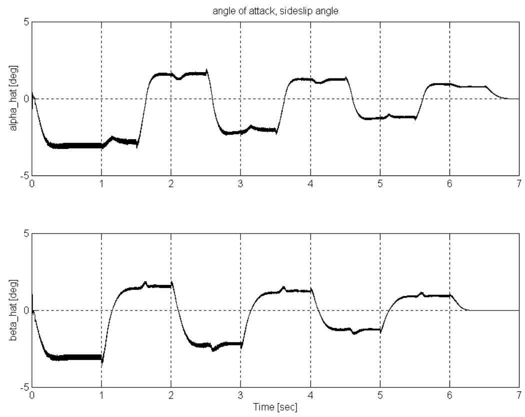
Fig. 1 shows the performance of the nonlinear observer and nonlinear controller using state estimates. In Fig. 1 (a), are shown the trajectories of achieved accelerations for each yaw and pitch channel, where the coupling effects due to bank angles are included. The tracking performance including the rise time, steady state error, and overshoot is satisfactory. Also, Fig. 1 (b), (c), and (d) show that the angle of attack and sideslip angle can be estimated with sufficient accuracy even with the initial estimation errors. In particular, the chattering phenomenon in estimates $\hat{y}$ and $\hat{x}$ occurs due to the sliding mode observer term, which is alleviated in $\hat{y}_0$ and $\hat{x}_0$, respectively, and further in actual states y and x.



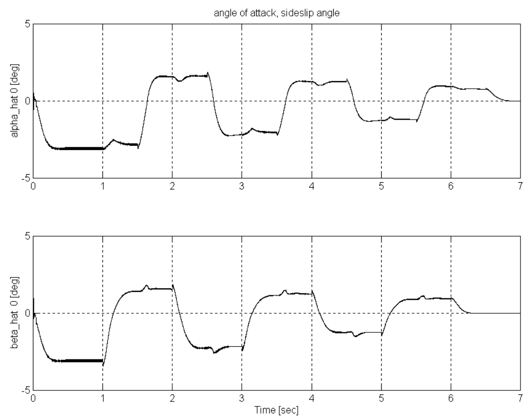
(a) Actual accelerations (y)



(b) Actual wind angles (x)



(c) First estimates of wind angles ($\hat{x}$)



(d) Second estimates of wind angles ($\hat{x}_0$)

Fig. 1. Performance of the state observer and the observer-based controller.

5.CONCLUSION

In this paper, the observer-based control approach is proposed for acceleration control using the parametric affine missile dynamics. The state observer for wind angles is necessary for practical implementation. Through simulation results as well as theoretical analysis, it is shown that the proposed approach can achieve satisfactory performance. Further study can be the observer-based control law where the effects of estimation errors on the control law are taken into account to guarantee the overall missile control system in more analytic way.

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