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REGULAR JACOBIAN MOTION PLANNING ALGORITHMS FOR MOBILE MANIPULATORS

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Abstract: A mobile manipulator is defined as a robotic system composed of a nonholonomic mobile platform and a holonomic manipulator fixed to the platform. The kinematics of the mobile manipulator, describing velocity constraints imposed on the platform and determining actual position and orientation of the end effector, are represented by a driftless control system with outputs. The motion planning problem considered in the paper consists in computing a control of the platform and a joint position of the manipulator, that produce a desirable position and orientation of the end effector in the taskspace at a prescribed instant of time. The motion planning problem can be solved globally, using methods of optimal control theory, or locally, by Jacobian methods employing the concept of analytic Jacobian. In the paper we concentrate on local methods, and present three Jacobian motion planning algorithms effective at regular configurations of the mobile manipulator. The performance and convergence of these algorithms is illustrated with computer simulations. *Copyright 2002 ©IFAC*

Keywords: mobile manipulator, kinematics, endogenous configuration, control system, motion planning

1. INTRODUCTION

We define a mobile manipulator as a system built of a mobile platform having a manipulator aboard. The platform is assumed nonholonomic. Important role played in robotics by mobile manipulators results from their exquisite mobility and manipulability characteristics. In the last decade contributions to theory and design of mobile manipulators have been made by quite many authors (Papadopoulos and Dubowsky, 1991; Yamamoto and Yun, 1994; Khatib *et al.*, 1996; Seraji, 1998; Tchoń and Muszyński, 2000; Gardner and Velinsky, 2000; Bayle *et al.*, 2001); for an extensive review the reader is directed to (Tchoń *et al.*, 2001).

Recently, in (Tchoń and Muszyński, 2000; Tchoń *et al.*, 2001) we have set forth a new approach to mobile manipulators, based on a control theoretic concept of endogenous configuration. As a result we have been able to reconstruct mobile manipulator counterparts of basic concepts and algorithms known for stationary manipulators, including the concept of kinematics, analytic Jacobian, regular and singular configurations, kinematic dexterity measures, and regular as well as singular Jacobian inverse kinematics algorithms. It is worth mentioning that singular configurations of mobile manipulators have been characterized as singular controls that may be detected by examining the output reachability Gramian of a variational control system. In the case when the mobile manipulator consists of a mobile platform exclusively, Jacobian pseudoinverse motion planning algorithms based on the analogy be-

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tween stationary and mobile manipulators specify to nonholonomic motion planning algorithms developed within the continuation approach (Sussmann, 1992; Chitour and Sussmann, 1998; Divelbiss *et al.*, 1998).

The motion planning problem for mobile manipulators consists in computing an endogenous configuration (i.e. a control for the platform and a joint position of the aboard manipulator) that allows the end effector to assume a desirable location in the taskspace. Principally, we study the problem without obstacles (Laumond, 1993; Chitour and Sussmann, 1998), although potentially our algorithms can be adapted to obstacle-populated environments (Divelbiss *et al.*, 1998). The motion planning problem is solved either globally, e.g. using methods of optimal control (Desai and Kumar, 1997; Tchoń, 2001), or locally, by Jacobian methods relying on the concept of analytic Jacobian for the mobile manipulator. In this paper we concentrate on local algorithms working at regular configurations, and present three Jacobian algorithms of motion planning for mobile manipulators: Jacobian pseudoinverse, extended Jacobian pseudoinverse, and adjoint Jacobian. The convergence issue is addressed, and exemplary computer simulations accomplished to illustrate algorithms performance. Distinct superiority of Jacobian pseudoinverse-based algorithms over the adjoint Jacobian algorithm has been proved theoretically, and confirmed in simulations.

The composition of this paper is the following. Section 2 introduces basic concepts focused around endogenous configuration and analytic Jacobian. Section 3 is devoted to motion planning algorithms. Their performance and convergence are studied in section 4. Section 5 concludes the paper.

2. BASIC CONCEPTS

Given a mobile manipulator, we let variables $q \in R^n$, $x \in R^p$, $y \in R^r$ denote posture of the platform, joint positions of the manipulator, and position and orientation of the end effector. Under assumption that the platform velocities are subject to $l \leq n$ independent constraints, and that the taskspace of the mobile manipulator consists of position and orientation coordinates of the end effector relative to an inertial coordinate frame, the kinematics of the mobile manipulator can be represented by a driftless control system with outputs

$$\begin{cases} \dot{q} = G(q)u = \sum_{i=1}^{m} g_i(q)u_i \\ y = k(q,x), \end{cases} \quad (1)$$

with analytic vector fields $g_1(q), g_2(q), \ldots, g_m(q)$, $m = n - l$. Control signals entering the system (1) encompass control functions of the platform $u(\cdot)$, assumed to be (Lebesgue) square integrable on the interval $[0,T]$, and manipulator joint positions $x \in R^p$ that remain constant over $[0,T]$. It is assumed that for any $u(\cdot)$ the solution $q(t) = \varphi_{q_0,t}(u(\cdot))$ of (1) initialized at q_0 and driven by $u(\cdot)$ exists for every $t \in [0,T]$. The Hilbert space $\mathcal{X} = L^2_m[0,T] \times R^p$ equipped with inner product

$$\langle (u_1(\cdot),x_1),(u_2(\cdot),x_2)\rangle = \int_0^T u_1^T(t)u_2(t)dt + x_1^T x_2, \quad (2)$$

and the corresponding norm $||(u(\cdot),x)||$ will be referred to as the **endogenous configuration space** of the mobile manipulator.

For the control system representation (1), we fix the initial posture q_0 of the platform as well as the control horizon T, and define the instantaneous kinematics of the mobile manipulator (Tchoń and Muszyński, 2000) as the output reachability map $K_{q_0,T} : \mathcal{X} \longrightarrow R^r$ of (1) such that

$$K_{q_0,T}(u(\cdot),x) = k(\varphi_{q_0,T}(u(\cdot)),x). \quad (3)$$

Given the kinematics (3), the motion planning problem for the mobile manipulator consists in determining an endogenous configuration $(u(\cdot),x) \in \mathcal{X}$ such that the end effector, placed initially at a certain $y_0 = k(q_0,x_0)$, reaches at T a desirable position and orientation y_d, so that

$$K_{q_0,T}(u(\cdot),x) = y_d. \quad (4)$$

The motion planning problem will be solved provided that the kinematics (3) are accompanied with an analytic Jacobian. At a configuration $(u(\cdot),x) \in \mathcal{X}$, the analytic Jacobian $J_{q_0,T}(u(\cdot),x) : \mathcal{X} \longrightarrow R^r$ is a linear map defined with reference to the so-called variational system associated with (1). The variational system describes a linear approximation to (1) along a triple $(u(t),x,q(t))$, and is represented by a linear, time-dependent control system

$$\begin{cases} \dot{\xi} = A(t)\xi + B(t)v \\ \eta = C(t,x)\xi + D(t,x)w, \end{cases} \quad (5)$$

whose matrices

$$A(t) = \frac{\partial (G(q(t))u(t))}{\partial q},\ B(t) = G(q(t)), \\ C(t,x) = \frac{\partial k(q(t),x)}{\partial q}, \quad D(t,x) = \frac{\partial k(q(t),x)}{\partial x}. \quad (6)$$

Eventually, we let $\Phi(t,s)$ denote the fundamental matrix of (5), satisfying the equation

$$\frac{\partial}{\partial t}\Phi(t,s) = A(t)\Phi(t,s),\ \ \Phi(s,s) = I_n. \quad (7)$$

After this introduction, we are ready to define the analytic Jacobian by the following formula

$$J_{q_0,T}(u(\cdot),x)(v(\cdot),w) = C(T,x)\int_0^T \Phi(T,s)B(s)v(s)ds + D(T,x)w. \quad (8)$$

Thanks to the introduction of the analytic Jacobian, the nonlinear equation (4) can be replaced by a family of linear Jacobian equations of the form

$$J_{q_0,T}(u(\cdot),x)(v(\cdot),w) = \eta, \quad (9)$$

and solved iteratively. A necessary and sufficient condition for solvability of the Jacobian equation is surjectivity of the map $J_{q_0,T}(u(\cdot),x)$. A presence or an absence of surjectivity leads to distinguishing regular and singular endogenous configurations of the mobile manipulator. Configurations at which the Jacobian is surjective are regular. Other configurations of the mobile manipulator are singular. It may be proved that the variational system (5) at a regular configuration is output controllable.

Suppose that a configuration $(u(\cdot),x) \in \mathcal{X}$ is regular. In such a case, the Jacobian equation may be solved using the least squares method that yields a (right) pseudoinverse $J^{\#}_{q_0,T}(u(\cdot),x) : R^r \longrightarrow \mathcal{X}$ of the Jacobian such that

$$\left(J^{\#}_{q_0,T}(u(\cdot),x)\eta\right)(t) = \begin{pmatrix} v(t) \\ w \end{pmatrix} = \begin{bmatrix} B^T(t)\Phi^T(T,t)C^T(T,x) \\ D^T(T,x) \end{bmatrix} \mathcal{D}^{-1}_{q_0,T}(u(\cdot),x)\eta. \quad (10)$$

The Jacobian pseudoinverse depends on a matrix

$$\mathcal{D}_{q_0,T}(u(\cdot),x) = C(T,x)\mathcal{M}_{q_0,T}C^T(T,x) + D(T,x)D^T(T,x) \quad (11)$$

known as the output reachability Gramian of the variational system, and called the dexterity matrix of the mobile manipulator (Tchoń and Muszyński, 2000). The state reachability Gramian

$$\mathcal{M}_{q_0,T}(u(\cdot)) = \int_0^T \Phi(T,s)B(s)B^T(s)\Phi^T(T,s)ds$$

incorporated in (11) denotes the mobility matrix of the platform. A dexterity of configuration $(u(\cdot),x)$, measuring its degree of regularity, is defined as

$$d_{q_0,T}(u(\cdot),x) = \sqrt{\det \mathcal{D}_{q_0,T}(u(\cdot),x)}. \quad (12)$$

It follows that a configuration $(u(\cdot),x)$ is regular, if and only if $\mathcal{D}_{q_0,T}(u(\cdot),x)$ is positive definite or the dexterity (12) is positive.

The pseudoinverse (10) is right, what means that it satisfies the identity

$$J_{q_0,T}(u(\cdot),x)J^{\#}_{q_0,T}(u(\cdot),x) = I_r. \quad (13)$$

Moreover, the form of (13) entails that the map

$$J^{\#}_{q_0,T}(u(\cdot),x)J_{q_0,T}(u(\cdot),x) - id_{\mathcal{X}}, \quad (14)$$

where $id_{\mathcal{X}}$ denotes the identity on $\mathcal{X}$, is a projection of the configuration space onto the null-space $\operatorname{Ker} J_{q_0,T}(u(\cdot),x)$.

3. INVERSE KINEMATICS ALGORITHMS

Assuming regularity of configurations, below we shall present three local motion planning algorithms employing the concept of analytic Jacobian. In each case our aim will consists in providing an iterative solution to equation (4).

3.1 *General Jacobian pseudoinverse*

The Jacobian pseudoinverse (10) can be plugged into the Newton algorithm in the configuration space, resulting in the basic Jacobian pseudoinverse motion planning algorithm. In accordance with this algorithm a solution of the motion planning problem is sought for as the limit

$$\begin{pmatrix} u_d(t) \\ x_d \end{pmatrix} = \lim_{\theta\to+\infty} \begin{pmatrix} u_\theta(t) \\ x(\theta) \end{pmatrix}$$

of a curve in the configuration space, initialized at a given configuration $(u_0(\cdot),x_0)$, and satisfying the differential equation.

$$\frac{d}{d\theta}\begin{pmatrix} u_\theta(t) \\ x(\theta) \end{pmatrix} = -\gamma J^{\#}_{q_0,T}(u_\theta(\cdot),x(\theta)) (K_{q_0,T}(u_\theta(\cdot),x(\theta)) - y_d)(t), \quad (15)$$

where $J^{\#}_{q_0,T}(u(\cdot),x)$ stands for the Jacobian pseudoinverse (10), and $\gamma > 0$. In order to compute the left hand side of (15), for every subsequent configuration $(u_\theta(\cdot),x(\theta))$ we first find out a trajectory $q_\theta(t)$ of the control system $\dot{q}_\theta = G(q_\theta)u_\theta$ starting from q_0, and then derive equations of the variational system along $(u_\theta(t),x(\theta),q_\theta(t))$ whose matrices $A_\theta(t)$, $B_\theta(t)$, $C_\theta(t,x)$, $D_\theta(t,x)$ define the Jacobian pseudoinverse.

The basic algorithm may be augmented by a projection of a vector $(\omega(\cdot)\zeta) \in \mathcal{X}$ into the Jacobian null-space, resulting in the following general Jacobian pseudoinverse algorithm

$$\frac{d}{d\theta}\begin{pmatrix} u_\theta(t) \\ x(\theta) \end{pmatrix} = -\gamma J^{\#}_{q_0,T}(u_\theta(\cdot),x(\theta)) (K_{q_0,T}(u_\theta(\cdot),x(\theta)) - y_d)(t) + (J^{\#}_{q_0,T}(u_\theta(\cdot),x(\theta)) J_{q_0,T}(u_\theta(\cdot),x(\theta)) - id_{\mathcal{X}})\begin{pmatrix} \omega(\cdot) \\ \zeta \end{pmatrix}(t). \quad (16)$$

The vector $(\omega(\cdot),\zeta)$ may be chosen e.g. to point out into a direction of pushing off from obstacles or increasing dexterity (12) of the mobile manipulator. The latter means that the differential

$$Dd^2_{q_0,T}(u(\cdot),x)(v(\cdot),w) = \langle(\omega(\cdot),\zeta),(v(\cdot),w)\rangle,$$

where $\langle\cdot,\cdot\rangle$ stands for the inner product (2).

To better apprehend the convergence of the general Jacobian pseudoinverse algorithm, we compute the derivative of taskspace error

$$e(\theta) = K_{q_0,T}(u_\theta(\cdot),x(\theta)) - y_d, \quad (17)$$

along a solution of (16)

$$\frac{d}{d\theta}e(\theta) = J_{q_0,T}(u_\theta(\cdot),x(\theta))\frac{d}{d\theta}\begin{pmatrix} u_\theta(\cdot) \\ x(\theta) \end{pmatrix} = -\gamma J_{q_0,T}(u_\theta(\cdot),x(\theta))J^{\#}_{q_0,T}(u_\theta(\cdot),x(\theta))e(\theta) = -\gamma e(\theta)$$

that implies exponential convergence of the error with the rate γ, $e(\theta) = e(0)\exp(-\gamma\theta)$.

3.2 *Extended Jacobian pseudoinverse*

Right pseudoinverses of the analytic Jacobian, alternative to (10), may be introduced by suitable extensions of the output map of system (1). To this aim, let us add to $k(q,x)$ new output functions $\bar{y}_j = h_j(q,x),\ j = 1,2,\ldots,s$, written vectorially as

$$\bar{y} = h(q,x), \qquad (18)$$

where the number s, $0 \le s \le n+p-r$, is called a degree of output extension. For the extended control system representation, comprising (1) and (18), we compute the extended Jacobian

$$J^E_{q_0,T}(u(\cdot),x)(v(\cdot),w) = \begin{bmatrix} C(T,x) \\ \bar{C}(T,x) \end{bmatrix} \int_0^T \Phi(T,s)B(s)v(s)ds + \begin{bmatrix} D(T,x) \\ \bar{D}(T,x) \end{bmatrix} w, \qquad (19)$$

where $\bar{C}(T,x) = \frac{\partial h(q(T),x)}{\partial q}$, $\bar{D}(T,x) = \frac{\partial h(q(T),x)}{\partial x}$, whereas remaining terms come from (6) and (7). If regular configurations of $J^E_{q_0,T}$ and $J_{q_0,T}$ coincide, we get the pseudoinverse of extended Jacobian

$$\left(J^{E\#}_{q_0,T}(u(\cdot),x)\begin{pmatrix} \eta \\ \bar{\eta} \end{pmatrix}\right)(t) = \begin{bmatrix} B^T(t)\Phi^T(T,t)\left[C^T(T,x)\,\bar{C}^T(T,x)\right] \\ \left[D^T(T,x)\,\bar{D}^T(T,x)\right] \end{bmatrix} \left(\mathcal{D}^e_{q_0,T}\right)^{-1}(u(\cdot),x)\begin{pmatrix} \eta \\ \bar{\eta} \end{pmatrix}, \qquad (20)$$

where $\eta \in R^r$, $\bar{\eta} \in R^s$, whereas the extended dexterity matrix

$$\mathcal{D}^e_{q_0,T}(u(\cdot),x) = \begin{bmatrix} C(T,x)\ D(T,x) \\ \bar{C}(T,x)\ \bar{D}(T,x) \end{bmatrix} \begin{bmatrix} \mathcal{M}_{q_0,T}(u(\cdot)) & 0 \\ 0 & I_p \end{bmatrix} \begin{bmatrix} C(T,x)\ D(T,x) \\ \bar{C}(T,x)\ \bar{D}(T,x) \end{bmatrix}^T. \qquad (21)$$

Expression (20), restricted to η subspace, yields the extended Jacobian pseudoinverse

$$J^{e\#}_{q_0,T}(u(\cdot),x)\eta = J^{E\#}_{q_0,T}(u(\cdot),x)\begin{pmatrix} \eta \\ 0 \end{pmatrix}, \qquad (22)$$

defined as

$$\left(J^{e\#}_{q_0,T}(u(\cdot),x)\eta\right)(t) = \begin{bmatrix} B^T(t)\Phi^T(T,t)\left[C^T(T,x)\,\bar{C}^T(T,x)\right] \\ \left[D^T(T,x)\,\bar{D}^T(T,x)\right] \end{bmatrix} \left(\mathcal{D}^e_{q_0,T}\right)^{-1}(u(\cdot),x)|_{r\,cls}\eta, \qquad (23)$$

where "$r\ cls$" refers to the first r columns of matrix $\left(\mathcal{D}^e_{q_0,T}\right)^{-1}(u(\cdot),x)$. For the degree of extension s may be varied, (23) actually provides us with a collection of right pseudoinverses such that

$$J_{q_0,T}(u(\cdot),x)J^{e\#}_{q_0,T}(u(\cdot),x) = I_r.$$

The Newton algorithm coupled with the extended Jacobian pseudoinverse results in the extended Jacobian pseudoinverse algorithm. Hence, a solution of the motion planning problem arises as the limit

$$\begin{pmatrix} u_d(t) \\ x_d \end{pmatrix} = \lim_{\theta\to+\infty} \begin{pmatrix} u_\theta(t) \\ x(\theta) \end{pmatrix}$$

of a curve $(u_\theta(\cdot), x(\theta))$ in the configuration space, satisfying the differential equation

$$\frac{d}{d\theta}\begin{pmatrix} u_\theta(t) \\ x(\theta) \end{pmatrix} = -\gamma J^{e\#}_{q_0,T}(u_\theta(\cdot),x(\theta)) (K_{q_0,T}(u_\theta(\cdot),x(\theta)) - y_d)(t), \qquad (24)$$

where $J^{e\#}_{q_0,T}(u(\cdot),x)$ is given by (23), and $\gamma > 0$.

Proceeding similarly to the case of the basic Jacobian pseudoinverse algorithm, by differentiation of the taskspace error (17), and using (24) as well as the property of $J^{e\#}_{q_0,T}$ being a right pseudoinverse, we obtain

$$\frac{d}{d\theta}e(\theta) = J_{q_0,T}(u_\theta(\cdot),x(\theta))\frac{d}{d\theta}\begin{pmatrix} u_\theta(\cdot) \\ x(\theta) \end{pmatrix} = -\gamma J_{q_0,T}(u_\theta(\cdot),x(\theta))J^{e\#}_{q_0,T}(u_\theta(\cdot),x(\theta))e(\theta) = -\gamma e(\theta),$$

and conclude that again convergence of the taskspace error is exponential, $e(\theta) = e(0)\exp(-\gamma\theta)$.

3.3 *Adjoint Jacobian*

The Newton algorithm may be replaced by the steepest descent algorithm in the configuration space of a mobile manipulator, directed toward decreasing the taskspace error norm

$$e^2(u(\cdot),x) = \frac{1}{2}||K_{q_0,T}(u(\cdot),x) - y_d||^2. \qquad (25)$$

Differentiation of the error yields

$$De^2(u(\cdot),x)(v(\cdot),w) = (K_{q_0,T}(u(\cdot),x) - y_d)^T J_{q_0,T}(u(\cdot),x)(v(\cdot),w) = \left\langle J^*_{q_0,T}(u(\cdot),x)\left(K_{q_0,T}(u(\cdot),x) - y_d\right), (v(\cdot),w)\right\rangle, \qquad (26)$$

where $J^*_{q_0,T}(u(\cdot),x) : (R^r)^* \longrightarrow (\mathcal{X})^*$ denotes the adjoint Jacobian defined as

$$\langle J^*_{q_0,T}(u(\cdot),x)p,(v(\cdot),w)\rangle = \langle (B^T(\cdot)\Phi^T(T,\cdot)C^T(T,x)p, D^T(T,x)p),(v(\cdot),w)\rangle, \qquad (27)$$

for $p \in (R^r)^* \cong R^r$, $(v(\cdot),w) \in \mathcal{X}$, and the inner product (2). By definition,

$$\left(J^*_{q_0,T}(u(\cdot),x)p\right)(t) = \begin{pmatrix} B^T(t)\Phi^T(T,t)C^T(T,x)p \\ D^T(T,x)p \end{pmatrix}. \qquad (28)$$

In consequence, we have derived the adjoint Jacobian motion planning algorithm providing a solution in the form of the limit

$$\begin{pmatrix} u_d(t) \\ x_d \end{pmatrix} = \lim_{\theta\to+\infty} \begin{pmatrix} u_\theta(t) \\ x(\theta) \end{pmatrix}$$

of a curve $(u_\theta(\cdot), x(\theta)) \in \mathcal{X}$ satisfying the differential equation

$$\frac{d}{d\theta}\begin{pmatrix} u_\theta(t) \\ x(\theta) \end{pmatrix} = -\gamma J^*_{q_0,T}(u_\theta(\cdot),x(\theta)) (K_{q_0,T}(u_\theta(\cdot),x(\theta)) - y_d)(t), \qquad (29)$$

with adjoint Jacobian (27), and $\gamma > 0$.

In order to assess convergence of the adjoint Jacobian algorithm, we consider the taskspace error (17) that is governed by the differential equation

$$\frac{d}{d\theta}e(\theta) = J_{q_0,T}(u_\theta(\cdot), x(\theta))\frac{d}{d\theta}\begin{pmatrix}u_\theta(\cdot)\\ x(\theta)\end{pmatrix}, \qquad (30)$$

and use $V(\theta) = \frac{1}{2}||e(\theta)||^2$ as a Lyapunov function. By differentiation we deduce

$$\frac{d}{d\theta}V(\theta) = -\gamma e^T(\theta)J_{q_0,T}(u_\theta(\cdot),x(\theta))J^*_{q_0,T}(u_\theta(\cdot),x(\theta))e(\theta) = -\gamma e^T(\theta)\mathcal{D}_{q_0,T}(u_\theta(\cdot),x(\theta))e(\theta) \leq 0,$$

and conclude via La Salle invariance principle that outside singular configurations the takspace error converges asymptotically to 0.

4. SIMULATIONS

Simulations have been run for a mobile manipulator portrayed in figure 1, consisting of a kinematic car type platform with an RTR manipulator on board. The platform posture vector $q = (q_1, q_2, q_3, q_4)$ in-

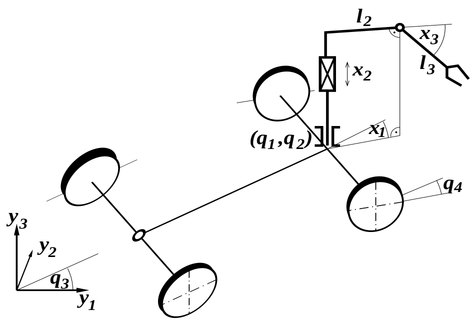

Fig. 1. RTR manipulator atop of kinematic car.

cludes position coordinates in the plane (q_1, q_2), the orientation angle q_3, and the direction angle q_4 of front wheels. The vector $x = (x_1, x_2, x_3)$ denotes manipulator joint position. Taskspace coordinates $y = (y_1, y_2, y_3)$ describe Cartesian coordinates of the end effector. The length of the car serves as a measure unit. If side-slipping of the platform wheels is not permitted, the control system representation (1) of the mobile manipulator kinematics assumes the following form

$$\begin{cases}\dot{q}_1 = u_1\cos q_3\cos q_4, & \dot{q}_2 = u_1\sin q_3\cos q_4,\\ \dot{q}_3 = u_1\sin q_4, & \dot{q}_4 = u_2,\end{cases}$$

$$y = k(q,x) = \begin{pmatrix}q_1 + (l_2 + l_3\cos x_3)\cos(q_3 + x_1)\\ q_2 + (l_2 + l_3\cos x_3)\sin(q_3 + x_1)\\ x_2 + l_3\sin x_3\end{pmatrix}. \qquad (31)$$

The time horizon is fixed as $T = 1$. Controls driving the system (31), that consist of the linear velocity of the platform (u_1) and the angular velocity of directing front wheels (u_2), have been chosen in the form of a truncated Fourier series

$$u_1(t) = \lambda_{01} + \sqrt{2}(\lambda_{11}\sin 2\pi t + \lambda_{21}\cos 2\pi t + \lambda_{31}\sin 4\pi t + \lambda_{41}\cos 4\pi t),$$

$$u_2(t) = \lambda_{02} + \sqrt{2}(\lambda_{12}\sin 2\pi t + \lambda_{22}\cos 2\pi t + \lambda_{32}\sin 4\pi t + \lambda_{42}\cos 4\pi t).$$

After parameterization, the endogenous configuration space $\mathcal{X} \cong R^{13}$ encompasses 10-dimensional platform control space and 3-dimensional manipulator jointspace. When applicable, the output extension includes two of ZYZ Euler angles characterizing the end effector orientation, so $\overline{y_1} = \theta = x_3$, $\overline{y_2} = \varphi = q_3 + x_1$.

For three initial positions (q_{01}, q_{02}) of the platform given in polar coordinates as $P(5,0), Q(5,\frac{\pi}{4}), R(5,\frac{5\pi}{4})$, we study the following motion planning problem: given zero initial platform orientation $q_{03} = 0$, initial front wheel direction $q_{04} = \frac{\pi}{6}$, initial configuration $\lambda_0 = (0.01, -0.01)$, $\lambda_1 = \cdots = \lambda_4 = 0$, $x_{01} = 0$, $x_{02} = 2$, $x_{03} = \frac{\pi}{4}$, and the desirable end effector position $y_d = (0,0,1)$, find a configuration $(\lambda, x) \in \mathcal{X}$ such that y_d is reached in time $T = 1$ with admissible taskspace error norm $\leq 10^{-12}$. Lengths of manipulator links are $l_2 = 0.6$, $l_3 = 0.4$. Convergence rate γ is set to 1.

Figures 2, 3 and 4 show a sample of takspace paths of the platform, and of the end effector, for basic and extended Jacobian pseudoinverse as well as for adjoint Jacobian motion planning algorithms. A comparison of convergence of these algorithms is displayed in figures 5, 6 and 7. The adjoint Jacobian algorithm started from point A has not converged.

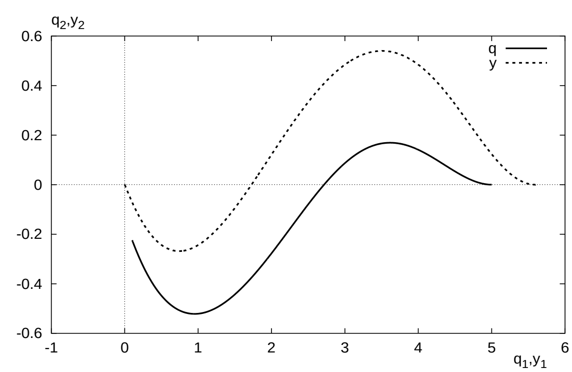

Fig. 2. Point P, basic Jacobian pseudoinverse.

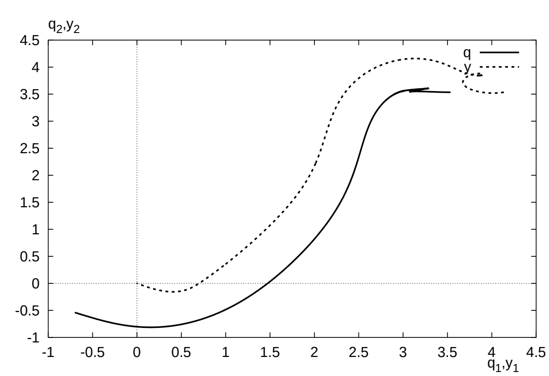

Fig. 3. Point Q, extended Jacobian pseudoinverse.

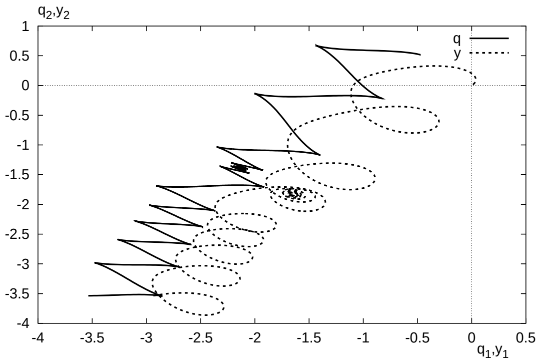

Fig. 4. Point R, adjoint Jacobian.

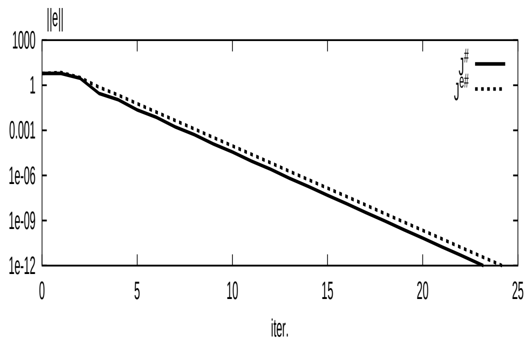

Fig. 5. Convergence comparison, point P.

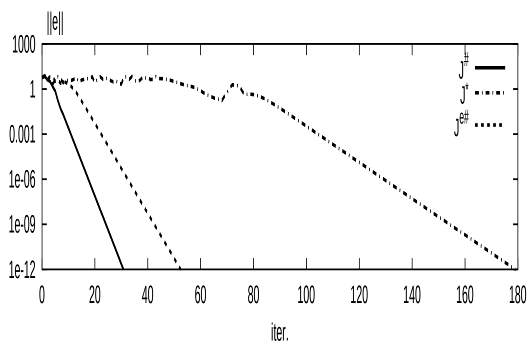

Fig. 6. Convergence comparison, point Q.

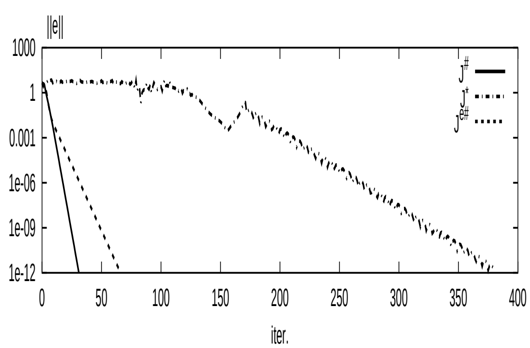

Fig. 7. Convergence comparison, point R.

5. CONCLUSION

We have investigated a sample performance of regular Jacobian motion planning algorithms for mobile manipulators. Our computer simulations show a distinct superiority of Jacobian pseudoinverse and extended Jacobian pseudoinverse algorithms over the adjoint Jacobian algorithm. More comprehensive results will be presented in a separate study.

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